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In 1960, Belgian Geologist Jean de Heinzelein de Braucourt discovered the Ishango bone in central Africa. This bone, dated to be more than 20,000 years old, is believed to be the oldest known artifact indicating the use of arithmetic. The first written records indicate the Egyptians and Babylonians used arithmetic as early as 2000 BC. The Mayans used arithmetic to make astronomical computations, and developed the concept of zero over 2,000 years ago. The word “arithmetic” is derived from the Greek word arithmos (translated as “number”). It is the oldest and most elementary branch of mathematics and is used for a variety of tasks ranging from simple counting to advanced science and business calculations.
1.1 An Introduction to the Integers

We begin with the set of counting numbers, formally called the set of natural numbers.

**The Natural Numbers.** The set

\[ \mathbb{N} = \{1, 2, 3, 4, 5, \ldots\} \]

is called the set of natural numbers.

If we add the number zero to the set of natural numbers, then we have a set of numbers that are called the whole numbers.

**The Whole Numbers.** The set

\[ \mathbb{W} = \{0, 1, 2, 3, 4, 5, \ldots\} \]

is called the set of whole numbers.

The number 0 is special, in that whenever you add it to another whole number, you get the identical number as an answer.

**Additive Identity Property.** If \( a \) is any whole number, then

\[ a + 0 = a. \]

For this reason, the whole number 0 is called the additive identity.

Thus, for example, \( 3 + 0 = 3 \), \( 15 + 0 = 15 \), and \( 123 + 0 = 123 \). These are all examples of the additive identity property.

Every natural number has an opposite, so that when you add them together, their sum is zero.

**Additive Inverse Property.** If \( a \) is any natural number, then define the opposite of \( a \), symbolized by \(-a\), so that

\[ a + (-a) = 0. \]

The number \(-a\) is called the “opposite of \( a \),” or more formally, the additive inverse of \( a \).

For example, the opposite (additive inverse) of 3 is \(-3\), and \(3 + (-3) = 0\). The opposite (additive inverse) of 12 is \(-12\), and \(12 + (-12) = 0\). The opposite of
254 is $-254$, and $254 + (-254) = 0$. These are all examples of additive inverses and the additive inverse property.

Because $7 + (-7) = 0$, we’ve said that $-7$ is the opposite (additive inverse) of $7$. However, we can also turn that around and say that $7$ is the opposite of $-7$. If we translate the phrase “the opposite of $-7$ is $7$” into mathematical symbols, we get $-(−7) = 7$.

The opposite of the opposite. Because $a + (-a) = 0$, we can say that $a$ is the opposite of $-a$. In symbols, we write:

$$-(-a) = a$$

Thus, for example, $-(-11) = 11$, $-(-103) = 103$, and $-(-1255) = 1255$.

The Integers

If we collect all the natural numbers and their additive inverses, then include the number zero, we have a collection of numbers called the integers.

The Integers. The set

$$\mathbb{Z} = \{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$$

is called the set of integers.

The integers can be made to correspond to points on a line in a very natural manner. First, draw a line, then locate the number zero anywhere you wish. Secondly, place the number one to the right of zero. This determines the length of one unit. Finally, locate the numbers $1$, $2$, $3$, $4$, $5$, $\ldots$ to the right of zero, then their opposites (additive inverses) $-1$, $-2$, $-3$, $-4$, $-5$, $\ldots$ to the left of zero (see Figure 1.1).

Figure 1.1: Each integer corresponds to a unique position on the number line.

Note that as we move to the right on the number line, the integers get larger. On the other hand, as we move to the left on the number line, the integers get smaller.

Positive and negative integers. On the number line, some integers lie to the right of zero and some lie to the left of zero.
• If \( a \) is an integer that lies to the right of zero, then \( a \) is called a \textit{positive} integer.

• If \( a \) is an integer that lies to the left of zero, then \( a \) is called a \textit{negative} integer.

Thus, 4, 25, and 142 are positive integers, while \(-7\), \(-53\), and \(-435\) are negative integers.

**Absolute Value**

The absolute value (or magnitude) of an integer is defined as follows.

**The Absolute Value of an Integer.** If \( a \) is an integer, then the absolute value of \( a \), written \(|a|\), is defined as the distance between the integer and zero on the number line.

**You Try It!**

**EXAMPLE 1.** Simplify \(|-4|\).

**Solution:** Consider the position of \(-4\) on the number line. Note that \(-4\) lies four units away from zero.

Because the absolute value (magnitude) of an integer equals its distance from zero, \(|-4| = 4\).

Answer: \(|-23| = 23\)

In similar fashion:

• The integer 5 lies five units away from zero. Hence, \(|5| = 5\).

• The integer 0 lies zero units away from zero, Hence, \(|0| = 0\).

Note that the absolute value of any number is either positive or zero. That is, the absolute value of a number is \textit{nonnegative} (not negative).
**Integer Addition**

This section is designed to provide a quick review of integer addition. For a more thorough introduction to integer addition, read section two of chapter two of our prealgebra textbook, provided online at the following URL:

http://msenux.redwoods.edu/PreAlgText/contents/chapter2/chapter2.pdf

We consider the first of two cases.

**Adding Integers with Like Signs.** To add two integers with like signs (both positive or both negative), add their magnitudes (absolute values), then prefix their common sign.

---

**EXAMPLE 2.** Simplify $7 + 12$.

**Solution:** We have like signs. The magnitudes (absolute values) of 7 and 12 are 7 and 12, respectively. If we add the magnitudes, we get 19. If we prefix the common sign, we get 19. That is:

$7 + 12 = 19$

Answer: 41

---

**EXAMPLE 3.** Simplify $-8 + (-9)$.

**Solution:** We have like signs. The magnitudes (absolute values) of $-8$ and $-9$ are 8 and 9, respectively. If we add the magnitudes, we get 17. If we prefix the common sign, we get $-17$. That is:

$-8 + (-9) = -17$

Answer: $-33$

---

Next, we consider the case where we have unlike signs.

**Adding Integers with Unlike Signs.** To add two integers with unlike signs (one positive and one negative), subtract the integer with the smaller magnitude (absolute value) from the number with the larger magnitude, then prefix the sign of the integer with the larger magnitude.
EXAMPLE 4. Simplify \(-14 + 11\).

**Solution:** We have unlike signs. The magnitudes (absolute values) of \(-14\) and 11 are 14 and 11, respectively. If we subtract the smaller magnitude from the larger, we get 3. The number \(-14\) has the larger magnitude, so we prefix our answer with its negative sign. That is:

Answer: \(-17\)

\[-14 + 11 = \boxed{-3}\]

EXAMPLE 5. Simplify \(40 + (-25)\).

**Solution:** We have unlike signs. The magnitudes (absolute values) of 40 and \(-25\) are 40 and 25, respectively. If we subtract the smaller magnitude from the larger, we get 15. The number 40 has the larger magnitude, so we prefix our answer with its positive sign. That is:

Answer: \(-58\)

\[40 + (-25) = \boxed{15}\]

**Mathematical Properties of Addition**

The order in which we add integers does not matter. That is, \(-20 + 34\) gives an answer identical to the sum \(34 + (-20)\). In both cases, the answer is 14. This fact is called the *commutative property of addition*.

**The Commutative Property of Addition.** If \(a\) and \(b\) are any two integers, then:

\[a + b = b + a\]

Next, when we add three integers, it does not matter which two we add first. For example, if we add the second and third of three numbers first, we get:

\[-11 + (-2 + 5) = -11 + 3\]  Parentheses first: \(-2 + 5 = 3\)

\[= -8\]  Add: \(-11 + 3 = -8\]
1.1. AN INTRODUCTION TO THE INTEGERS

On the other hand, if we add the first and second of three numbers first, we get:

\[ (-11 + (-2)) + 5 = -13 + 5 \]
\[ = -8 \quad \text{Parentheses first: } -11 + (-2) = -13 \]
\[ \text{Add: } -13 + 5 = -8 \]

Thus, \(-11 + (-2 + 5) = (-11 + (-2)) + 5\). This fact is called the associative property of addition.

**The Associative Property of Addition.** If \(a\), \(b\), and \(c\) are any three integers, then:

\[ a + (b + c) = (a + b) + c \]

**Integer Subtraction**

Subtraction is the inverse, or the opposite, of addition.

**Subtracting Integers.** If \(a\) and \(b\) are any two integers, then:

\[ a - b = a + (-b) \]

Subtracting \(b\) is identical to adding the opposite (additive inverse) of \(b\).

**You Try It!**

**EXAMPLE 6.** Simplify: \(-13 - 27\)

**Solution:** The “opposite” (additive inverse) of 27 is \(-27\). So, subtracting 27 is the same as adding \(-27\).

\[ -13 - 27 = -13 + (-27) \quad \text{Subtracting 27 is the same} \]
\[ \text{as adding } -27. \]
\[ = -50 \quad \text{Add the magnitudes, then} \]
\[ \text{prefix the common negative sign.} \]

Answer: \(-26\)

**You Try It!**

**EXAMPLE 7.** Simplify: \(-27 - (-50)\)

Simplify: \(-18 - (-54)\)
Solution: The “opposite” (additive inverse) of $-50$ is $-(-50)$, or $50$. So, subtracting $-50$ is the same as adding $50$.

\[-27 - (-50) = -27 + 50 \quad \text{Subtracting } -50 \text{ is the same as adding 50.}\]

\[= 23 \quad \text{Subtract the smaller magnitude from the larger magnitude, then prefix the sign of the larger magnitude.}\]

Answer: 36

Integer Multiplication

This section is designed to provide a quick review of multiplication and division of integers. For a more thorough introduction to integer multiplication and division, read section four of chapter two of our prealgebra textbook, provided online at the following URL:

http://msenux.redwoods.edu/PreAlgText/contents/chapter2/chapter2.pdf

Like Signs. If $a$ and $b$ are integers with like signs (both positive or both negative), then the product $ab$ and the quotient $a/b$ are positive.

\[(+)(+) = + \quad \text{or} \quad (+)/(+) = + \]
\[(-)(-) = + \quad \text{or} \quad (-)/(-) = + \]

You Try It!

EXAMPLE 8. Simplify each of the following expressions:

(a) $(2)(3)$
(b) $(-12)(-8)$
(c) $-14/(-2)$

Solution: When multiplying or dividing, like signs yield a positive result.

(a) $(2)(3) = 6$
(b) $(-12)(-8) = 96$
(c) $-14/(-2) = 7$

Answer: 90

Unlike Signs. If $a$ and $b$ are integers with unlike signs (one positive and one negative), then the product $ab$ and the quotient $a/b$ are negative.

\[(+)(-) = - \quad \text{or} \quad (+)/(-) = - \]
\[(-)(+) = - \quad \text{or} \quad (-)/(+) = - \]
**1.1. AN INTRODUCTION TO THE INTEGERS**

**You Try It!**

**EXAMPLE 9.** Simplify each of the following expressions: Simplify: \((-19)(3)\)

(a) \((2)(-12)\)  
(b) \((-9)(12)\)  
(c) \(24/(-8)\)

**Solution:** When multiplying or dividing, unlike signs yield a negative result.

(a) \((2)(-12) = -24\)  
(b) \((-9)(12) = -108\)  
(c) \(24/(-8) = -3\)  

Answer: \(-57\)

**Mathematical Properties of Multiplication**

The order in which we multiply integers does not matter. That is, \((-8)(5)\) gives an answer identical to \((5)(-8)\). In both cases, the answer is \(-40\). This fact is called the *commutative property of multiplication*.

**The Commutative Property of Multiplication.** If \(a\) and \(b\) are any two integers, then:

\[
a \cdot b = b \cdot a
\]

Next, when we multiply three integers, it does not matter which two we multiply first. If we multiply the second and third of three numbers first, we get:

\[
(-3)[(-4)(-5)] = (-3)(20) \quad \text{Brackets first: } (-4)(-5) = 20
\]

\[
= -60 \quad \text{Multiply: } (-3)(20) = -60
\]

On the other hand, if we multiply the first and second of three numbers first, we get:

\[
[(-3)(-4)](-5) = (12)(-5) \quad \text{Brackets first: } (-3)(-4) = 12
\]

\[
= -60 \quad \text{Multiply: } (12)(-5) = -60
\]

Thus, \((-3)[(-4)(-5)] = [(12)(-5)]\). This fact is called the *associative property of multiplication*.

**The Associative Property of Multiplication.** If \(a\), \(b\), and \(c\) are any three integers, then:

\[
a \cdot (b \cdot c) = (a \cdot b) \cdot c
\]

When you multiply an integer by 1, you get the identical number back as the product. For example, \((1)(5) = 5\) and \((-11)(1) = -11\). This fact is known as the *multiplicative identity property*. 

The Multiplicative Identity Property. If \( a \) is any integer, then:

\[
1 \cdot a = a \quad \text{and} \quad a \cdot 1 = a
\]

For this reason, the integer 1 is called the “multiplicative identity.”

Finally, note that \((-1)(5) = -5\). Thus, multiplying 5 by \(-1\) is identical to taking the “opposite” of 5 or negating 5.

The Multiplicative Property of \(-1\). Multiplying by minus one is identical to negating. That is:

\[
(-1)a = -a
\]

Exponents

In the exponential expression \(a^n\), the number \(a\) is called the base, while the number \(n\) is called the exponent. We now define what is meant by an exponent.

Exponents. Let \(a\) be an integer and let \(n\) be any whole number. If \(n \neq 0\), then:

\[
a^n = a \cdot a \cdot a \cdot \cdots \cdot a
\]

That is, to calculate \(a^n\), write \(a\) as a factor \(n\) times.

You Try It!

EXAMPLE 10. Simplify \((-2)^3\).

Solution: In the exponential expression \((-2)^3\), note that \(-2\) is the base, while 3 is the exponent. The exponent tells us to write the base as a factor three times. Simplify the result by performing the multiplications in order, moving from left to right.

\[
(-2)^3 = (-2)(-2)(-2) \quad \text{\(-2\) as a factor, three times.}
\]

\[
= (4)(-2) \quad \text{Multiply: \((-2)(-2) = 4\).}
\]

\[
= -8 \quad \text{Multiply: \((4)(-2) = -8\).}
\]

Answer: 4

Thus, \((-2)^3 = -8\).

In Example 10, note that the product of three negative factors is negative. Let’s try another example.
EXAMPLE 11. Simplify \((-2)^4\).

Solution: In the exponential expression \((-2)^4\), note that \(-2\) is the base, while 4 is the exponent. The exponent tells us to write the base as a factor four times. Simplify the result by performing the multiplications in order, moving from left to right.

\[
(-2)^4 = (-2)(-2)(-2)(-2)
\]

\(-2\) as a factor, four times.

\[
= (4)(-2)(-2)
\]

Multiply: \((-2)(-2) = 4\).

\[
= (-8)(-2)
\]

Multiply: \((4)(-2) = -8\).

\[
= 16
\]

Multiply: \((-8)(-2) = 16\).

Thus, \((-2)^4 = 16\).

In Example 11, note that the product of four negative factors is positive. Examples 10 and 11 reveal the following pattern.

Odd or Even Exponents.

1. When a negative integer is raised to an even exponent, the result is positive.
2. When a negative integer is raised to an odd exponent, the result is negative.

Graphing Calculator: Negating versus Subtracting

Consider the view of the lower half of the TI84 graphing calculator in Figure 1.2. Note that there are two keys that contain some sort of negative sign, one on the bottom row of keys, and another in the last column of keys on the right, positioned just above the plus symbol.

The first of these buttons is the unary “negation” operator. If you want to negate a single (thus the word “unary”) number, then this is the key to use. For example, enter \(-3\) by pressing the following button sequence. The result is shown in Figure 1.3.
The second button is the binary “subtraction” operator. If you want to subtract one number from another number (thus the word “binary”), then this is the key to use. For example, enter \(7 - 15\) by pressing the following button sequence. The result is shown in Figure 1.4.

\[
\begin{array}{c}
7 \\
- \\
1 \\
5 \\
\hline
\text{ENTER}
\end{array}
\]

Figure 1.3: Negating a number. Figure 1.4: Subtract two numbers.

**Important Point.** Do not interchange the roles of the unary negation operator and the binary subtraction operator.

1. To negate a number, use: \((-\))

2. To subtract one number from another, use: \((-\))

If you interchange the roles of these operators, the calculator will respond that you’ve made a “syntax error” (see Figures 1.5 and 1.6).

**You Try It!**

**EXAMPLE 12.** Use the TI84 graphing calculator to simplify each of the following expressions:

(a) \(-717 - 432\)  
(b) \((232)(-313)\)  
(c) \((-17)^3\)
1.1. AN INTRODUCTION TO THE INTEGERS

Solution: The minus sign in each of these examples looks exactly the same, but sometimes it is used as a “negative” sign and sometimes it is used as a “subtraction” sign.

a) The expression \(-717 - 432\) asks us to subtract 432 from “negative” 717. Enter the following sequence of keystrokes to produce the result shown in the first image in Figure 1.7.

\[
(-) \ 7 \ 1 \ 7 \ - \ 4 \ 3 \ 2 \ \text{ENTER}
\]

Hence, \(-717 - 432 = -1149\).

b) The expression \((232)(-313)\) asks us to find the product of 232 and “negative” 313. Enter the following sequence of keystrokes to produce the result shown in the second image in Figure 1.7.

\[
2 \ 3 \ 2 \ \times \ (-) \ 3 \ 1 \ 3 \ \text{ENTER}
\]

Hence, \((232)(-313) = -72616\).

c) The expression \((-17)^3\) asks us to raise “negative” to the third power. Enter the following sequence of keystrokes to produce the result shown in the third image in Figure 1.7. The “caret” symbol \(^\wedge\) is located just above the division key in the rightmost column of the TI84 graphing calculator.

\[
( (-) \ 1 \ 7 ) \ \wedge \ 3 \ \text{ENTER}
\]

Hence, \((-17)^3 = -4913\).

Answer: \(-11390625\)

Figure 1.7: Calculations made on the graphing calculator.
In Exercises 1-8, simplify each of the following expressions.

1. $|5|$. 
2. $|1|$. 
3. $|-2|$. 
4. $|-1|$. 
5. $|2|$. 
6. $|8|$. 
7. $|-4|$. 
8. $|-6|$. 

In Exercises 9-24, simplify each of the following expressions as much as possible.

9. $-91 + (-147)$ 
10. $-23 + (-13)$ 
11. $96 + 145$ 
12. $16 + 127$ 
13. $-76 + 46$ 
14. $-11 + 21$ 
15. $-59 + (-12)$ 
16. $-40 + (-58)$ 
17. $37 + (-86)$ 
18. $143 + (-88)$ 
19. $66 + (-85)$ 
20. $33 + (-41)$ 
21. $57 + 20$ 
22. $66 + 110$ 
23. $-48 + 127$ 
24. $-48 + 92$

In Exercises 25-32, find the difference.

25. $-20 - (-10)$ 
26. $-20 - (-20)$ 
27. $-62 - 7$ 
28. $-82 - 62$ 
29. $-77 - 26$ 
30. $-96 - 92$ 
31. $-7 - (-16)$ 
32. $-20 - (-5)$

In Exercises 33-40, compute the exact value.

33. $(-8)^6$ 
34. $(-3)^5$ 
35. $(-7)^5$ 
36. $(-4)^6$ 
37. $(-9)^2$ 
38. $(-4)^2$
In Exercises 41-52, use your graphing calculator to compute the given expression.

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<tr>
<td>43.</td>
<td>(-400 - (-8225))</td>
</tr>
<tr>
<td>44.</td>
<td>(-8176 + 578)</td>
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<tr>
<td>45.</td>
<td>((-856)(232))</td>
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<tr>
<td>46.</td>
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<td>47. 2916885</td>
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<tr>
<td>49. (-5832)</td>
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<tr>
<td>51. (-371293)</td>
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</tbody>
</table>
1.2 Order of Operations

The order in which we evaluate expressions can be ambiguous. Take, for example, the expression $-4 + 2 \cdot 8$. If we perform the addition first, then we get $-16$ as a result (the question mark over the equal sign indicates that the result is questionable).

$$-4 + 2 \cdot 8 \overset{?}{=} -2 \cdot 8$$

$$\overset{?}{=} -16.$$  

On the other hand, if we perform the multiplication first, then we get 12 as a result.

$$-4 + 2 \cdot 8 \overset{?}{=} -4 + 16$$

$$\overset{?}{=} 12.$$  

So, what are we to do?  

Of course, grouping symbols would remove the ambiguity.

**Grouping Symbols.** Parentheses, brackets, and absolute value bars can be used to group parts of an expression. For example:

$$3 + 5(9 - 11) \text{ or } -2 - [-2 - 5(1 - 3)] \text{ or } 6 - 3| -3 - 4|$$

In each case, the rule is “evaluate the expression inside the grouping symbols first.” If the grouping symbols are nested, evaluate the expression in the innermost pair of grouping symbols first.

Thus, if the example above is grouped as follows, we are forced to evaluate the expression inside the parentheses first.

$$(-4 + 2) \cdot 8 = -2 \cdot 8$$  

Parentheses first: $-4 + 2 = -2$  

$$= -16$$  

Multiply: $-2 \cdot 8 = -16$

Another way to avoid ambiguities in evaluating expressions is to establish an order in which operations should be performed. The following guidelines should always be strictly enforced when evaluating expressions.

**Rules Guiding Order of Operations.** When evaluating expressions, proceed in the following order.

1. Evaluate expressions contained in grouping symbols first. If grouping symbols are nested, evaluate the expression in the innermost pair of grouping symbols first.
1.2. ORDER OF OPERATIONS

2. Evaluate all exponents that appear in the expression.

3. Perform all multiplications and divisions in the order that they appear in the expression, moving left to right.

4. Perform all additions and subtractions in the order that they appear in the expression, moving left to right.

EXAMPLE 1. Simplify: \(-3 - 4 \cdot 8\)

Solution: Because of the established Rules Guiding Order of Operations, this expression is no longer ambiguous. There are no grouping symbols or exponents present, so we immediately go to rule three, evaluate all multiplications and divisions in the order that they appear, moving left to right. After that we invoke rule four, performing all additions and subtractions in the order that they appear, moving left to right.

\[
-3 - 4 \cdot 8 = -3 - 32 \\
= -3 + (-32) \\
= -35 
\]

Thus, \(-3 - 4 \cdot 8 = -35\).

You Try It! Simplify: \(-4 + 2 \cdot 8\)

Writing Mathematics. When simplifying expressions, observe the following rule to neatly arrange your work:

One equal sign per line. This means that you should not arrange your work horizontally.

\[
-2 - 4 \cdot (-8) = -2 - (-32) = -2 + 32 = 30 
\]

That’s three equal signs on a single line. Rather, arrange your work vertically, keeping equal signs aligned in a column.

\[
-2 - 4 \cdot (-8) = -2 - (-32) \\
= -2 + 32 \\
= 30 
\]
**EXAMPLE 2.** Simplify: $54/(-9)(2)$

**Solution:** There are no grouping symbols or exponents present, so we immediately go to rule three, evaluate all multiplications and divisions in the order that they appear, moving left to right.

\[
\frac{54}{(-9)(2)} = -6(2)
\]

Divide first: $54/(-9) = -6$

\[
= -12
\]

Multiply: $-6(2) = -12$

Thus, $54/(-9)(2) = -12$.

Answer: 16

Example 2 can be a source of confusion for many readers. Note that multiplication takes no preference over division, nor does division take preference over multiplication. Multiplications and divisions have the same level of preference and must be performed in the order that they occur, moving from left to right. If not, the wrong answer will be obtained.

**Warning!** Here is what happens if you perform the multiplication in Example 2 before the division.

\[
\frac{54}{(-9)(2)} = \frac{54}{(-18)}
\]

Multiply: $(-9)(2) = -18$

\[
= -3
\]

Divide: $54/(-18) = -3$

This is incorrect! Multiplications and divisions must be performed in the order that they occur, moving from left to right.

**EXAMPLE 3.** Simplify: (a) $(-7)^2$ and (b) $-7^2$

**Solution.** Recall that for any integer $a$, we have $(-1)a = -a$. Because negating is equivalent to multiplying by $-1$, the Rules Guiding Order of Operations require that we address grouping symbols and exponents before negation.

a) Because of the grouping symbols, we negate first, then square. That is,

\[
(-7)^2 = (-7)(-7)
\]

\[
= 49.
\]

b) There are no grouping symbols in this example. Thus, we must square first, then negate. That is,

\[
-7^2 = -(7 \cdot 7)
\]

\[
= -49.
\]

Thus, $(-7)^2 = 49$, but $-7^2 = -49$. Note: This example demonstrates that $(-7)^2$ is different from $-7^2$.

Answer: $-225$
1.2. ORDER OF OPERATIONS

Let’s try an example that has a mixture of exponents, multiplication, and subtraction.

### EXAMPLE 4. Simplify: $-3 - 2(-4)^2$

**Solution.** The *Rules Guiding Order of Operations* require that we address exponents first, then multiplications, then subtractions.

\[
-3 - 2(-4)^2 = -3 - 2(16) \quad \text{Exponent first: } (-4)^2 = 16
\]

\[
= -3 - 32 \quad \text{Multiply: } 2(16) = 32
\]

\[
= -3 + (-32) \quad \text{Add the opposite.}
\]

\[
= -35 \quad \text{Add: } -3 + (-32) = -35
\]

Thus, $-3 - 2(-4)^2 = -35$. Answer: 27

---

### Grouping Symbols

The *Rules Guiding Order of Operations* require that expressions inside grouping symbols (parentheses, brackets, or curly braces) be evaluated first.

### EXAMPLE 5. Simplify: $-2(3 - 4)^2 + 5(1 - 2)^3$

**Solution.** The *Rules Guiding Order of Operations* require that we first evaluate the expressions contained inside the grouping symbols.

\[
-2(3 - 4)^2 + 5(1 - 2)^3
\]

\[
= -2(3 + (-4))^2 + 5(1 + (-2))^3 \quad \text{Add the opposites.}
\]

\[
= -2(-1)^2 + 5(-1)^3 \quad \text{Parentheses first: } 3 + (-4) = -1
\]

\[
\text{and } 1 + (-2) = -1.
\]

Evaluate the exponents next, perform the multiplications, then add.

\[
= -2(1) + 5(-1) \quad \text{Exponents: } (-1)^2 = 1
\]

\[
\text{and } (-1)^3 = -1.
\]

\[
= -2 + (-5) \quad \text{Multiply: } -2(1) = -2
\]

\[
\text{and } 5(-1) = -5.
\]

\[
= -7 \quad \text{Add: } -2 + (-5) = -7
\]

Thus, $-2(3 - 4)^2 + 5(1 - 2)^3 = -7$. Answer: 373
CHAPTER 1. THE ARITHMETIC OF NUMBERS

Absolute Value Bars as Grouping Symbols
Like parentheses and brackets, you must evaluate what is inside them first, then take the absolute value of the result.

**EXAMPLE 6.** Simplify: \(-8 - |5 - 11|\)

**Solution.** We must first evaluate what is inside the absolute value bars.

\[-8 - |5 - 11| = -8 - |5 + (-11)|\]

Add the opposite.

\[-8 - |5 - 6|\]

Add: 5 + (-11) = -6.

The number -6 is 6 units from zero on the number line. Hence, \(|-6|=6|\).

\[-8 - 6\]

Add: \(|-6|=6|\).

\[-8 + (-6)\]

Add the opposite.

\[-14\]

Add.

Answer: -10

Thus, \(-8 - |5 - 11| = -14\).

### Nested Grouping Symbols

When grouping symbols are nested, the *Rules Guiding Order of Operations* tell us to evaluate the innermost expressions first.

**EXAMPLE 7.** Simplify: \(-3 - 4[-3 - 4(-3 - 4)]\)

**Solution.** The *Rules Guiding Order of Operations* require that we first address the expression contained in the innermost grouping symbols. That is, we evaluate the expression contained inside the brackets first.

\[-3 - 4[-3 - 4(-3 - 4)]\]

Add the opposite.

\[-3 - 4[-3 - 4(-7)]\]

Add: \(-3 + (-4) = -7\)

Next, we evaluate the expression contained inside the brackets.

\[-3 - 4[-3 - (-28)]\]

Multiply: \(4(-7) = -28\)

\[-3 - 4[-3 + 28]\]

Add the opposite.

\[-3 - 4[25]\]

Add: \(-3 + 28 = 25\)

Now we multiply, then subtract.
1.2. ORDER OF OPERATIONS

\[= -3 - 100 \quad \text{Multiply: } 4\{25\} = 100\]
\[= -3 + (-100) \quad \text{Add the opposite.}\]
\[= -103 \quad \text{Add: } -3 + (-100) = -103\]

Thus, \(-3 - 4[-3 - 4(-3 - 4)] = -103. \quad \text{Answer: } -14\]

Evaluating Algebraic Expressions

**Variable.** A variable is a symbol (usually a letter) that stands for an unknown value that may vary.

Let’s add the definition of an *algebraic expression*.

**Algebraic Expression.** When we combine numbers and variables in a valid way, using operations such as addition, subtraction, multiplication, division, exponentiation, the resulting combination of mathematical symbols is called an *algebraic expression*.  

Thus, 

\[2a, \quad x + 5, \quad \text{and} \quad y^2,\]

being formed by a combination of numbers, variables, and mathematical operators, are valid algebraic expressions.

An algebraic expression must be *well-formed*. For example,

\[2 + -5x\]

is *not a valid expression* because there is no term following the plus sign (it is not valid to write \(+\) with nothing between these operators). Similarly,

\[2 + 3(2)\]

is not well-formed because parentheses are not balanced.

In this section we will evaluate algebraic expressions for given values of the variables contained in the expressions. Here are some simple tips to help you be successful.

**Tips for Evaluating Algebraic Expressions.**
1. Replace all occurrences of variables in the expression with open parentheses. Leave room between the parentheses to substitute the given value of the variable.

2. Substitute the given values of variables in the open parentheses prepared in the first step.

3. Evaluate the resulting expression according to the Rules Guiding Order of Operations.

**You Try It!**

**EXAMPLE 8.** Evaluate the expression \( x^2 - 2xy + y^2 \) at \( x = -3 \) and \( y = 2 \).

**Solution.** Following *Tips for Evaluating Algebraic Expressions*, first replace all occurrences of variables in the expression \( x^2 - 2xy + y^2 \) with open parentheses. Next, substitute the given values of variables \((-3\) for \( x \) and \( 2 \) for \( y \)) in the open parentheses.

\[
x^2 - 2xy + y^2 = ( )^2 - 2( ) ( ) + ( )^2
\]

\[
= (-3)^2 - 2(-3)(2) + (2)^2
\]

Finally, follow the Rules Guiding Order of Operations to evaluate the resulting expression.

\[
x^2 - 2xy + y^2
\]

Original expression.

\[
= ( )^2 - 2( ) ( ) + ( )^2
\]

Replace variables with parentheses.

\[
= (-3)^2 - 2(-3)(2) + (2)^2
\]

Substitute \(-3\) for \( x \) and \( 2 \) for \( y \).

\[
= 9 - 2(-3)(2) + 4
\]

Evaluate exponents first.

\[
= 9 - (-6)(2) + 4
\]

Left to right, multiply: \( 2(-3) = -6 \)

\[
= 9 - (-12) + 4
\]

Left to right, multiply: \( (-6)(2) = -12 \)

\[
= 9 + 12 + 4
\]

Add the opposite.

\[
= 25
\]

Add.

**Answer:** \(-7\)

Thus, if \( x = -3 \) and \( y = 2 \), then \( x^2 - 2xy + y^2 = 25 \).

---

**Evaluating Fractions**

If a fraction bar is present, evaluate the numerator and denominator separately according to the Rules Guiding Order of Operations, then perform the division in the final step.
EXAMPLE 9. Evaluate the expression

\[ \frac{ad - bc}{a + b} \]

at \( a = 5, \ b = -3, \ c = 2, \) and \( d = -4. \)

Solution. Following Tips for Evaluating Algebraic Expressions, first replace all occurrences of variables in the expression \((ad - bc)/(a + b)\) with open parentheses. Next, substitute the given values of variables (5 for \( a \), -3 for \( b \), 2 for \( c \), and -4 for \( d \)) in the open parentheses.

\[
\frac{ad - bc}{a + b} = \frac{(5)(-4) - (-3)(2)}{(5) + (-3)}
\]

Finally, follow the Rules Guiding Order of Operations to evaluate the resulting expression. Note that we evaluate the expressions in the numerator and denominator separately, then divide.

\[
\frac{ad - bc}{a + b} = \frac{(5)(-4) - (-3)(2)}{(5) + (-3)} = \frac{-20 + 6}{2} = \frac{-14}{2} = -7
\]

Thus, if \( a = 5, \ b = -3, \ c = 2, \) and \( d = -4, \) then \((ad - bc)/(a + b) = -7.\) Answer: \(-2\)

Using the Graphing Calculator

The graphing calculator is a splendid tool for evaluating algebraic expressions, particularly when the numbers involved are large.
EXAMPLE 10. Use the graphing calculator to simplify the following expression.

\[-213 - 35[-18 - 211(15 - 223)]\]

Solution. The first difficulty with this expression is the fact that the graphing calculator does not have a bracket symbol for the purposes of grouping. The calculator has only parentheses for grouping. So we first convert our expression to the following:

\[-213 - 35(-18 - 211(15 - 223))\]

Note that brackets and parentheses are completely interchangeable.

The next difficulty is determining which of the minus signs are negation symbols and which are subtraction symbols. If the minus sign does not appear between two numbers, it is a negation symbol. If the minus sign appears between two numbers, it is a subtraction symbol. Hence, we enter the following keystrokes on our calculator. The result is shown in Figure 1.8.

![Keystrokes on Calculator](image)

Figure 1.8: Calculating \(-213 - 35[-18 - 211(15 - 223)].\)

Answer: \(-14\)

Thus, \(-213 - 35[-18 - 211(15 - 223)] = -1,535,663.\)
1.2. ORDER OF OPERATIONS

Solution. You might ask “Why do we need a calculator to evaluate this exceedingly simple expression?” After all, it’s very easy to compute.

\[
\frac{5 + 5}{5 + 5} = \frac{10}{10} = 1
\]

Simplify numerator and denominator.

Divide: \(10/10 = 1\).

Well, let’s enter the expression \(5+5/5+5\) in the calculator and see how well we understand the Rules Guiding Order of Operations (see first image in Figure 1.9). Whoa! How did the calculator get 11? The answer is supposed to be 1!

Let’s slow down and apply the Rules Guiding Order of Operations to the expression \(5+5/5+5\).

\[
5 + \frac{5}{5} + 5 = 5 + \frac{5}{5} + 5
\]

Divide first.

\[
= 5 + 1 + 5
\]

Divide: \(5/5 = 1\).

Add: \(5 + 1 + 5 = 11\).

Aha! That’s how the calculator got 11.

\[
5 + \frac{5}{5} + 5
\]

is equivalent to

\[
5 + \frac{5}{5} + 5
\]

Let’s change the order of evaluation by using grouping symbols. Note that:

\[
\frac{(5 + 5)}{(5 + 5)} = \frac{10}{10}
\]

Parentheses first.

\[
= 1
\]

Divide: \(10/10 = 1\).

That is:

\[
\frac{(5 + 5)}{(5 + 5)}
\]

is equivalent to

\[
\frac{5 + 5}{5 + 5}
\]

Enter \((5+5)/(5+5)\) and press the ENTER key to produce the output shown in the second image in Figure 1.9.

\[
\frac{5+5}{5+5} \quad 11
\]

\[
\frac{(5+5)}{(5+5)} \quad 1
\]

Figure 1.9: Calculating \(\frac{5+5}{5+5}\).

Answer: 1
The graphing calculator has memory locations available for “storing” values. They are lettered A–Z and appear on the calculator case, in alphabetic order as you move from left to right and down the keyboard. Storing values in these memory locations is an efficient way to evaluate algebraic expressions containing variables. Use the \textbf{ALPHA} key to access these memory locations.

**You Try It!**

**EXAMPLE 12.** Use the graphing calculator to evaluate $|a| - |b|$ at $a = -312$ and $b = -875$.

**Solution.** First store $-312$ in the variable \textbf{A} with the following keystrokes. To select the letter \textbf{A}, press the \textbf{ALPHA} key, then the \textbf{MATH} key, located in the upper left-hand corner of the calculator (see Figure 1.10).

\[
\begin{array}{c}
\text{(-)} \\
3 \quad 1 \quad 2 \quad \text{STO} > \quad \text{ALPHA} \quad \text{A} \quad \text{ENTER}
\end{array}
\]

Next, store $-875$ in the variable \textbf{B} with the following keystrokes. To select the letter \textbf{B}, press the \textbf{ALPHA} key, then the \textbf{APPS} key.

\[
\begin{array}{c}
\text{(-)} \\
8 \quad 7 \quad 5 \quad \text{STO} > \quad \text{ALPHA} \quad \text{B} \quad \text{ENTER}
\end{array}
\]

The results of these keystrokes are shown in the first image in Figure 1.11.

Now we need to enter the expression $|a| - |b|$. The absolute value function is located in the MATH menu. When you press the MATH key, you’ll notice submenus \textbf{MATH}, \textbf{NUM}, \textbf{CPX}, and \textbf{PRB} across the top row of the MATH menu. Use the right-arrow key to select the \textbf{NUM} submenu (see the second image in Figure 1.11). Note that \texttt{abs(} is the first entry on this menu. This is the absolute value function needed for this example. Enter the expression \texttt{abs(A)-abs(B)} as shown in the third image in Figure 1.11. Use the \textbf{ALPHA} key as described above to enter the variables \textbf{A} and \textbf{B} and close the parentheses using the right parentheses key from the keyboard. Press the \texttt{ENTER} key to evaluate your expression.

\[
\begin{array}{c}
-312 \rightarrow \text{A} \\
-875 \rightarrow \text{B}
\end{array}
\quad
\begin{array}{c}
\text{MATH} \\
\text{NUM} \\
\text{CPX} \\
\text{PRB}
\end{array}
\quad
\begin{array}{c}
\texttt{abs(A)-abs(B)} \\
-563
\end{array}
\]

Figure 1.11: Evaluate $|a| - |b|$ at $a = -312$ and $b = -875$.

Answer: 563

Thus, $|a| - |b| = -563$. 

\[\square\]
In Exercises 1-18, simplify the given expression.

1. \(-12 + 6(-4)\)  
2. \(11 + 11(7)\)  
3. \(-(-2)^5\)  
4. \(-(-5)^3\)  
5. \(-| - 40|\)  
6. \(-| - 42|\)  
7. \(-24/(-6)(-1)\)  
8. \(45/(-3)(3)\)  
9. \(-(-50)\)  
10. \(-(-30)\)  
11. \(-3^5\)  
12. \(-3^2\)  
13. \(48 \div 4(6)\)  
14. \(96 \div 6(4)\)  
15. \(-52 - 8(-8)\)  
16. \(-8 - 7(-3)\)  
17. \((-2)^4\)  
18. \((-4)^4\)  

In Exercises 19-42, simplify the given expression.

19. \(9 - 3(2)^2\)  
20. \(-4 - 4(2)^2\)  
21. \(17 - 10|13 - 14|\)  
22. \(18 - 3| - 20 - 5|\)  
23. \(-4 + 5(-4)^3\)  
24. \(3 + 3(-4)^3\)  
25. \(8 + 5(-1 - 6)\)  
26. \(8 + 4(-5 - 5)\)  
27. \((10 - 8)^2 - (7 - 5)^3\)  
28. \((8 - 10)^2 - (4 - 5)^3\)  
29. \(6 - 9(6 - 4(9 - 7))\)  
30. \(4 - 3(3 - 5(7 - 2))\)  
31. \(-6 - 5(4 - 6)\)  
32. \(-5 - 5(-7 - 7)\)  
33. \(9 + (9 - 6)^3 - 5\)  
34. \(12 + (8 - 3)^3 - 6\)  
35. \(-5 + 3(4)^2\)  
36. \(2 + 3(2)^2\)  
37. \(8 - (5 - 2)^3 + 6\)  
38. \(9 - (12 - 11)^2 + 4\)  
39. \(|6 - 15| - | - 17 - 11|\)  
40. \(| - 18 - 19| - | - 3 - 12|\)  
41. \(5 - 5(5 - 6(6 - 4))\)  
42. \(4 - 6(4 - 7(8 - 5))\)
In Exercises 43-58, evaluate the expression at the given values of \( x \) and \( y \).

43. \( 4x^2 + 3xy + 4y^2 \) at \( x = -3 \) and \( y = 0 \)
44. \( 3x^2 - 3xy + 2y^2 \) at \( x = 4 \) and \( y = -3 \)
45. \(-8x + 9\) at \( x = -9\)
46. \(-12x + 10\) at \( x = 2\)
47. \(-5x^2 + 2xy - 4y^2\) at \( x = 5\) and \( y = 0\)
48. \(3x^2 + 3xy - 5y^2\) at \( x = 0\) and \( y = 3\)
49. \(3x^2 + 3x - 4\) at \( x = 5\)
50. \(2x^2 + 6x - 5\) at \( x = 6\)
51. \(-2x^2 + 2y^2\) at \( x = 1\) and \( y = -2\)
52. \(-5x^2 + 5y^2\) at \( x = -4\) and \( y = 0\)
53. \(-3x^2 - 6x + 3\) at \( x = 2\)
54. \(-7x^2 + 9x + 5\) at \( x = -7\)
55. \(-6x - 1\) at \( x = 1\)
56. \(10x + 7\) at \( x = 9\)
57. \(3x^2 - 2y^2\) at \( x = -3\) and \( y = -2\)
58. \(-3x^2 + 2y^2\) at \( x = 2\) and \( y = 2\)

59. Evaluate \(\frac{a^2 + b^2}{a + b}\)
   at \( a = 27\) and \( b = -30\).

60. Evaluate \(\frac{a^2 + b^2}{a + b}\)
   at \( a = -63\) and \( b = 77\).

61. Evaluate \(\frac{a + b}{c - d}\)
   at \( a = -42\), \( b = 25\), \( c = 26\), and \( d = 43\).

62. Evaluate \(\frac{a + b}{c - d}\)
   at \( a = 38\), \( b = 42\), \( c = 10\), and \( d = 50\).

63. Evaluate \(\frac{a - b}{cd}\)
   at \( a = -7\), \( b = 48\), \( c = 5\), and \( d = 11\).

64. Evaluate \(\frac{a - b}{cd}\)
   at \( a = -46\), \( b = 46\), \( c = 23\), and \( d = 2\).

65. Evaluate the expressions \(a^2 + b^2\) and \((a + b)^2\) at \( a = 3\) and \( b = 4\). Do the expressions produce the same results?
66. Evaluate the expressions \(a^2b^2\) and \((ab)^2\) at \( a = 3\) and \( b = 4\). Do the expressions produce the same results?
67. Evaluate the expressions \(|a||b|\) and \(|ab|\) at \( a = -3\) and \( b = 5\). Do the expressions produce the same results?
68. Evaluate the expressions \(|a| + |b|\) and \(|a + b|\) at \( a = -3\) and \( b = 5\). Do the expressions produce the same results?
In Exercises 69-72, use a graphing calculator to evaluate the given expression.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>69.</td>
<td>$-236 - 324(-576 + 57)$</td>
</tr>
<tr>
<td>70.</td>
<td>$-443 + 27(-414 - 22)$</td>
</tr>
<tr>
<td>71.</td>
<td>$\frac{270 - 900}{300 - 174}$</td>
</tr>
<tr>
<td>72.</td>
<td>$\frac{3000 - 952}{144 - 400}$</td>
</tr>
</tbody>
</table>

73. Use a graphing calculator to evaluate the expression $\frac{a^2 + b^2}{a + b}$ at $a = -93$ and $b = 84$ by first storing $-93$ in the variable A and $84$ in the variable B, then entering the expression $(A^2+B^2)/(A+B)$.

74. Use a graphing calculator to evaluate the expression $\frac{a^2 + b^2}{a + b}$ at $a = -76$ and $b = 77$ by first storing $-76$ in the variable A and $77$ in the variable B, then entering the expression $(A^2+B^2)/(A+B)$.

75. The formula

$$F = \frac{9}{5}C + 32$$

will change a Celsius temperature to a Fahrenheit temperature. Given that the Celsius temperature is $C = 60^\circ$C, find the equivalent Fahrenheit temperature.

76. The surface area of a cardboard box is given by the formula

$$S = 2WH + 2LH + 2LW,$$

where $W$ and $L$ are the width and length of the base of the box and $H$ is its height. If $W = 2$ centimeters, $L = 8$ centimeters, and $H = 2$ centimeters, find the surface area of the box.

77. The kinetic energy (in joules) of an object having mass $m$ (in kilograms) and velocity $v$ (in meters per second) is given by

$$K = \frac{1}{2}mv^2.$$
<table>
<thead>
<tr>
<th>Answers</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $-36$</td>
<td>41. $40$</td>
</tr>
<tr>
<td>3. $32$</td>
<td>43. $36$</td>
</tr>
<tr>
<td>5. $-40$</td>
<td>45. $81$</td>
</tr>
<tr>
<td>7. $-4$</td>
<td>47. $-125$</td>
</tr>
<tr>
<td>9. $50$</td>
<td>49. $86$</td>
</tr>
<tr>
<td>11. $-243$</td>
<td>51. $6$</td>
</tr>
<tr>
<td>13. $72$</td>
<td>53. $-21$</td>
</tr>
<tr>
<td>15. $12$</td>
<td>55. $-7$</td>
</tr>
<tr>
<td>17. $16$</td>
<td>57. $19$</td>
</tr>
<tr>
<td>19. $-3$</td>
<td>59. $-543$</td>
</tr>
<tr>
<td>21. $7$</td>
<td>61. $1$</td>
</tr>
<tr>
<td>23. $-324$</td>
<td>63. $-1$</td>
</tr>
<tr>
<td>27. $-4$</td>
<td>67. Yes.</td>
</tr>
<tr>
<td>29. $24$</td>
<td>69. $167920$</td>
</tr>
<tr>
<td>31. $4$</td>
<td>71. $-5$</td>
</tr>
<tr>
<td>33. $31$</td>
<td>73. $-1745$</td>
</tr>
<tr>
<td>35. $43$</td>
<td>75. $140^\circ F$</td>
</tr>
<tr>
<td>37. $-13$</td>
<td>77. $8750$ joules</td>
</tr>
<tr>
<td>39. $-19$</td>
<td></td>
</tr>
</tbody>
</table>
1.3 The Rational Numbers

We begin with the definition of a rational number.

**Rational Numbers.** Any number that can be expressed in the form \( p/q \), where \( p \) and \( q \) are integers, \( q \neq 0 \), is called a rational number. The letter \( \mathbb{Q} \) is used to represent the set of rational numbers. That is:

\[
\mathbb{Q} = \left\{ \frac{p}{q} : p \text{ and } q \text{ are integers, } q \neq 0 \right\}
\]

Because \(-2/3, 4/5, \) and \(123/(-12)\) have the form \( p/q \), where \( p \) and \( q \) are integers, each is an example of a rational number. If you think you hear the word “fraction” when we say “rational number,” you are correct in your thinking. Any number that can be expressed as a fraction, where the numerator and denominator are integers, is a rational number.

Every integer is also a rational number. Take, for example, the integer \(-12\). There are a number of ways we can express \(-12\) as a fraction with integer numerator and denominator, \(-12/1, 24/(-2),\) and \(-36/3\) being a few.

**Reducing Fractions to Lowest Terms**

First, we define what is meant by the greatest common divisor of two integers.

**The Greatest Common Divisor.** Given two integers \( a \) and \( b \), the greatest common divisor of \( a \) and \( b \) is the largest integer that divides evenly (with no remainder) into both \( a \) and \( b \). The notation \( \text{GCD}(a, b) \) is used to represent the greatest common divisor of \( a \) and \( b \).

For example, \( \text{GCD}(12, 18) = 6 \), \( \text{GCD}(32, 40) = 8 \), and \( \text{GCD}(18, 27) = 9 \).

We can now state when a fraction is reduced to lowest terms.

**Lowest Terms.** A fraction \( a/b \) is said to be reduced to lowest terms if and only if \( \text{GCD}(a, b) = 1 \).

A common technique used to reduce a fraction to lowest terms is to divide both numerator and denominator by their greatest common divisor.

**You Try It!**

**EXAMPLE 1.** Reduce \( 8/12 \) to lowest terms.

Reduce: \( -48/60 \)
CHAPTER 1. THE ARITHMETIC OF NUMBERS

**Solution:** Note that GCD(8, 12) = 4. Divide both numerator and denominator by 4.

\[
\frac{8}{12} = \frac{8 \div 4}{12 \div 4}
\]

Divide numerator and denominator by GCD(8, 12) = 4.

\[
= \frac{2}{3}
\]

Simplify numerator and denominator.

Thus, \( \frac{8}{12} = \frac{2}{3} \).

Answer: \(-\frac{4}{5}\)

Recall the definition of a prime number.

**Prime Number.** A natural number greater than one is prime if and only if its only divisors are one and itself.

For example, 7 is prime (its only divisors are 1 and 7), but 14 is not (its divisors are 1, 2, 7, and 14). In like fashion, 2, 3, and 5 are prime, but 6, 15, and 21 are not prime.

---

**You Try It!**

Reduce 18/24 to lowest terms.

**EXAMPLE 2.** Reduce 10/40 to lowest terms.

**Solution:** Note that GCD(10, 40) = 10. Divide numerator and denominator by 10.

\[
\frac{10}{40} = \frac{10 \div 10}{40 \div 10}
\]

Divide numerator and denominator by GCD(10, 40) = 10.

\[
= \frac{1}{4}
\]

Simplify numerator and denominator.

**Alternate solution:** Use factor trees to express both numerator and denominator as a product of prime factors.

\[
\begin{array}{c}
10 \\
\text{2} \\
\text{5}
\end{array}
\quad
\begin{array}{c}
40 \\
\text{2} \\
\text{2} \\
\text{10}
\end{array}
\quad
\begin{array}{c}
40 \\
\text{2} \\
\text{2} \\
\text{2} \\
\text{5}
\end{array}
\]

Hence, 10 = 2 \cdot 5 and 40 = 2 \cdot 2 \cdot 2 \cdot 5. Now, to reduce 10/40 to lowest terms, replace the numerator and denominator with their prime factorizations, then cancel factors that are in common to both numerator and denominator.
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\[
\frac{10}{40} = \frac{2 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 5} \quad \text{Prime factor numerator and denominator.}
\]

\[
= \frac{2 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 5}
\]

\[
= \frac{2}{2 \cdot 2 \cdot 5}
\]

\[
= \frac{1}{4} \quad \text{Cancel common factors.}
\]

\[
= \frac{1}{4} \quad \text{Simplify numerator and denominator.}
\]

When we cancel a 2 from both the numerator and denominator, we’re actually dividing both numerator and denominator by 2. A similar statement can be made about canceling the 5. Canceling both 2 and a 5 is equivalent to dividing both numerator and denominator by 10. This explains the 1 in the numerator when all factors cancel. 

\[
\text{Answer: } \frac{3}{4}
\]

Example 2 demonstrates an important point.

**When all factors cancel.** When all of the factors cancel in either numerator or denominator, the resulting numerator or denominator is equal to one.

### Multiplying Fractions

First, the definition.

**Multiplication of Fractions.** If \( \frac{a}{b} \) and \( \frac{c}{d} \) are two fractions, then their product is defined as follows:

\[
\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}
\]

Thus, to find the product of \( \frac{a}{b} \) and \( \frac{c}{d} \), simply multiply numerators and multiply denominators. For example:

\[
\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8} \quad \text{and} \quad \frac{2}{5} \cdot \frac{7}{3} = \frac{14}{15} \quad \text{and} \quad -\frac{5}{8} \cdot \frac{1}{6} = \frac{5}{48}
\]

Like integer multiplication, like signs yield a positive answer, unlike signs yield a negative answer.

Of course, when necessary, remember to reduce your answer to lowest terms.

**You Try It!**

**EXAMPLE 3.** Simplify:

\[
\frac{14}{20} \cdot \frac{10}{21}
\]

Simplify:

\[
\frac{8}{9} \cdot \left( -\frac{27}{20} \right)
\]
CHAPTER 1. THE ARITHMETIC OF NUMBERS

Solution: Multiply numerators and denominators, then reduce to lowest terms.

\[
\frac{-14}{20} \cdot \frac{10}{21} = \frac{-140}{420} \quad \text{Multiply numerators and denominators.}
\]

\[
= -\frac{2 \cdot 2 \cdot 5 \cdot 7}{2 \cdot 2 \cdot 3 \cdot 5 \cdot 7} \quad \text{Prime factor.}
\]

\[
= -\frac{2 \cdot 5 \cdot 7}{2 \cdot 3 \cdot 5 \cdot 7} \quad \text{Cancel common factors.}
\]

\[
= -\frac{1}{3} \quad \text{Simplify.}
\]

Note that when all the factors cancel from the numerator, you are left with a 1. Thus, \((-14/20) \cdot (10/21) = -1/3.

Answer: \(6/5\)

Cancellation Rule. When multiplying fractions, cancel common factors according to the following rule: “Cancel a factor in a numerator for an identical factor in a denominator.”

The rule is “cancel something on the top for something on the bottom.”

Thus, an alternate approach to multiplying fractions is to factor numerators and denominators in place, then cancel a factor in a numerator for an identical factor in a denominator.

You Try It!

EXAMPLE 4. Simplify: \(\frac{15}{8} \cdot \left(-\frac{14}{9}\right)\)

Solution: Factor numerators and denominators in place, then cancel common factors in the numerators for common factors in the denominators.

\[
\frac{15}{8} \cdot \left(-\frac{14}{9}\right) = \frac{3 \cdot 5}{2 \cdot 2 \cdot 2} \cdot \left(-\frac{2 \cdot 7}{3 \cdot 3}\right) \quad \text{Factor numerators and denominators.}
\]

\[
= \frac{3 \cdot 5}{2 \cdot 2 \cdot 2} \cdot \left(-\frac{7}{3 \cdot 3}\right) \quad \text{Cancel a factor in a numerator for a common factor in a denominator.}
\]

\[
= -\frac{35}{12} \quad \text{Multiply numerators and denominators.}
\]

Note that unlike signs yield a negative product. Thus, \((15/8) \cdot (-14/9) = -35/12.

Answer: \(1/3\)
1.3. THE RATIONAL NUMBERS

Dividing Fractions

Every nonzero rational number has what is called a multiplicative inverse or reciprocal.

The Reciprocal. If \( a \) is any nonzero rational number, then \( \frac{1}{a} \) is called the multiplicative inverse or reciprocal of \( a \), and:

\[
 a \cdot \frac{1}{a} = 1
\]

Note that:

\[
 2 \cdot \frac{1}{2} = 1 \quad \text{and} \quad \frac{3}{5} \cdot \frac{5}{3} = 1 \quad \text{and} \quad -\frac{4}{7} \cdot \left( -\frac{7}{4} \right) = 1.
\]

Thus, the reciprocal of 2 is 1/2, the reciprocal of 3/5 is 5/3, and the reciprocal of −4/7 is −7/4. Note that to find the reciprocal of a number, simply invert the number (flip it upside down).

Now we can define the quotient of two fractions.

Division of Fractions. If \( \frac{a}{b} \) and \( \frac{c}{d} \) are two fractions, then their quotient is defined as follows:

\[
 \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}
\]

That is, dividing by \( \frac{c}{d} \) is the same as multiplying by the reciprocal \( \frac{d}{c} \).

The above definition of division is summarized by the phrase “invert and multiply.”

EXAMPLE 5. Simplify: \( -\frac{35}{21} \div \left( -\frac{10}{12} \right) \)

Solution: Invert and multiply, then factor in place and cancel common factors in a numerator for common factors in a denominator.

\[
 -\frac{35}{21} \div \left( -\frac{10}{12} \right) = -\frac{35}{21} \cdot \left( -\frac{12}{10} \right) \quad \text{Invert and multiply.}
\]

\[
 = -\frac{5 \cdot 7}{3 \cdot 7} \cdot \left( -\frac{2 \cdot 2 \cdot 3}{2 \cdot 5} \right) \quad \text{Prime factor.}
\]

\[
 = -\frac{5 \cdot 7}{3 \cdot 7} \cdot \left( -\frac{2 \cdot 2 \cdot 3}{2 \cdot 5} \right) \quad \text{Cancel common factors.}
\]

\[
 = \frac{2}{1} \quad \text{Multiply numerators and denominators.}
\]

\[
 = 2 \quad \text{Simplify.}
\]
CHAPTER 1. THE ARITHMETIC OF NUMBERS

Note that when all the factors in a denominator cancel, a 1 remains. Thus, \((-35/21) \div (-10/12) = 2\). Note also that like signs yield a positive result.

### Adding Fractions

First the definition.

**Addition of Fractions.** If two fractions have a denominator in common, add the numerators and place the result over the common denominator. In symbols:

\[
\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}
\]

For example:

\[
-\frac{3}{5} + \frac{7}{5} = \frac{4}{5} \quad \text{and} \quad -\frac{4}{3} + \left(-\frac{7}{3}\right) = -\frac{11}{3} \quad \text{and} \quad \frac{4}{7} + \left(-\frac{5}{7}\right) = -\frac{1}{7}
\]

If the fractions do not possess a common denominator, first create equivalent fractions with a least common denominator, then add according to the rule above.

**Least Common Denominator.** If the fractions \(a/b\) and \(c/d\) do not share a common denominator, the least common denominator for \(b\) and \(d\), written \(\text{LCD}(b,d)\), is defined as the smallest number divisible by both \(b\) and \(d\).

---

**You Try It!**

**EXAMPLE 6.** Simplify: \(-\frac{3}{8} + \frac{5}{12}\)

**Solution:** The least common denominator in this case is the smallest number divisible by both 8 and 12. In this case, \(\text{LCD}(8,12) = 24\). We first need to make equivalent fractions with a common denominator of 24.

\[
-\frac{3}{8} + \frac{5}{12} = -\frac{3 \cdot 3}{8 \cdot 3} + \frac{5 \cdot 2}{12 \cdot 2} \quad \text{Make equivalent fraction with a common denominator of 24.}
\]

\[
= \frac{9}{24} + \frac{10}{24} \quad \text{Multiply numerators and denominators.}
\]

\[
= \frac{1}{24} \quad \text{Add: } -9 + 10 = 1.
\]

**Answer:** \(-\frac{13}{18}\)
Order of Operations

Rational numbers obey the same Rules Guiding Order of Operations as do the integers.

Rules Guiding Order of Operations. When evaluating expressions, proceed in the following order.

1. Evaluate expressions contained in grouping symbols first. If grouping symbols are nested, evaluate the expression in the innermost pair of grouping symbols first.

2. Evaluate all exponents that appear in the expression.

3. Perform all multiplications and divisions in the order that they appear in the expression, moving left to right.

4. Perform all additions and subtractions in the order that they appear in the expression, moving left to right.

EXAMPLE 7. Given \(x = \frac{2}{3}, y = -\frac{3}{5},\) and \(z = \frac{10}{9},\) evaluate \(xy + yz.\)

Solution: Following Tips for Evaluating Algebraic Expressions, first replace all occurrences of variables in the expression \(xy + yz\) with open parentheses. Next, substitute the given values of variables (\(2/3\) for \(x\), \(-3/5\) for \(y\), and \(10/9\) for \(z\)) in the open parentheses.

\[
xy + yz = \left( \frac{2}{3} \right) \left( -\frac{3}{5} \right) + \left( -\frac{3}{5} \right) \left( \frac{10}{9} \right)
\]

Replace variables with parentheses.

\[
= \left( \frac{2}{3} \right) \left( -\frac{3}{5} \right) + \left( -\frac{3}{5} \right) \left( \frac{10}{9} \right)
\]

Substitute: \(2/3\) for \(x\), \(-3/5\) for \(y\), and \(10/9\) for \(z\).
Use the Rules Guiding Order of Operations to simplify.

\[
\begin{align*}
\frac{-6}{15} + \left( \frac{30}{45} \right) & = \text{Multiply.} \\
\frac{2}{5} + \left( -\frac{2}{3} \right) & = \text{Reduce.} \\
\frac{2}{5} \cdot \frac{3}{3} + \left( -\frac{2}{3} \cdot \frac{5}{3} \right) & = \text{Make equivalent fractions with a least common denominator.} \\
\frac{-6}{15} + \left( \frac{10}{15} \right) & = \text{Add.}
\end{align*}
\]

Answer: \(-1\)

Thus, if \(x = \frac{2}{3}, y = -\frac{3}{5},\) and \(z = \frac{10}{9},\) then \(xyz = -\frac{16}{15}\)

---

**You Try It!**

EXAMPLE 8. Given \(x = -\frac{3}{5},\) evaluate \(-x^3.\)

**Solution:** First, replace each occurrence of the variable \(x\) with open parentheses, then substitute \(-\frac{3}{5}\) for \(x.\)

\[
\begin{align*}
-x^3 &= - \left( \frac{3}{5} \right)^3 \\
&= - \left( \frac{3}{5} \right) \left( \frac{3}{5} \right) \left( \frac{3}{5} \right) \\
&= - \left( \frac{27}{125} \right) \\
&= \frac{27}{125}
\end{align*}
\]

The opposite of \(-27/125\) is \(27/125.\)

Answer: \(1/81\)

Hence, \(-x^3 = 27/125,\) given \(x = -3/5.\)

---

**You Try It!**

Given \(x = -\frac{3}{4}\) and \(y = -\frac{4}{5},\) evaluate \(x^2 - y^2.\)

EXAMPLE 9. Given \(a = -\frac{4}{3}\) and \(b = -\frac{3}{2},\) evaluate \(a^2 + 2ab - 3b^2.\)

**Solution:** Following *Tips for Evaluating Algebraic Expressions*, first replace all occurrences of variables in the expression \(a^2 + 2ab - 3b^2\) with open parentheses.
1.3. THE RATIONAL NUMBERS

Next, substitute the given values of variables \((-4/3 \text{ for } a \text{ and } -3/2 \text{ for } b\) in the open parentheses.

\[
a^2 + 2ab - 3b^2 = \left( \frac{4}{3} \right)^2 + 2 \left( \frac{4}{3} \right) \left( \frac{-3}{2} \right) - 3 \left( \frac{-3}{2} \right)^2
\]

Next, evaluate the exponents: \((-4/3)^2 = 16/9\) and \((-3/2)^2 = 9/4\).

\[
= \frac{16}{9} + \frac{2}{1} \left( \frac{-4}{3} \right) \left( \frac{-3}{2} \right) - 3 \left( \frac{-3}{2} \right)^2
\]

Next, perform the multiplications and reduce.

\[
= \frac{16}{9} + \frac{24}{6} - \frac{27}{4}
\]

\[
= \frac{16}{9} + 4 - \frac{27}{4}
\]

Make equivalent fractions with a common denominator, then add.

\[
= \frac{16}{9} \cdot \frac{4}{4} + 4 \cdot \frac{36}{36} - \frac{27}{4} \cdot \frac{9}{9}
\]

\[
= \frac{64}{36} + \frac{144}{36} - \frac{243}{36}
\]

\[
= \frac{-35}{36}
\]

Thus, if \(a = -4/3\) and \(b = -3/2\), then \(a^2 + 2ab - 3b^2 = -35/36\) \hspace{1cm} \text{Answer: } -31/400

Fractions on the Graphing Calculator

We must always remember that the graphing calculator is an “approximating machine.” In a small number of situations, it is capable of giving an exact answer, but for most calculations, the best we can hope for is an approximate answer.

However, the calculator gives accurate results for operations involving fractions, as long as we don’t use fractions with denominators that are too large for the calculator to respond with an exact answer.
**EXAMPLE 10.** Use the graphing calculator to simplify each of the following expressions:

(a) \( \frac{2}{3} + \frac{1}{2} \)  
(b) \( \frac{5}{3} \cdot \frac{7}{7} \)  
(c) \( \frac{3}{5} \div \frac{1}{3} \)

**Solution:** We enter each expression in turn.

a) The *Rules Guiding Order of Operations* tell us that we must perform divisions before additions. Thus, the expression \( \frac{2}{3} + \frac{1}{2} \) is equivalent to:

\[
\frac{2}{3} + \frac{1}{2} = \frac{2}{3} + \frac{1}{2} \quad \text{Divide first.}
\]

\[
= \frac{4}{6} + \frac{3}{6} \quad \text{Equivalent fractions with LCD.}
\]

\[
= \frac{7}{6} \quad \text{Add.}
\]

Enter the expression \( \frac{2}{3} + \frac{1}{2} \) on your calculator, then press the ENTER key. The result is shown in the first image in Figure 1.12. Next, press the MATH button, then select 1:►Frac (see the second image in Figure 1.12) and press the ENTER key again. Note that the result shown in the third image in Figure 1.12 matches the correct answer of \( \frac{7}{6} \) found above.

![Figure 1.12: Calculating 2/3 + 1/2.](image)

b) The *Rules Guiding Order of Operations* tell us that there is no preference for division over multiplication, or vice-versa. We must perform divisions and multiplications as they occur, moving from left to right. Hence:

\[
2/3 \times 5/7 = \frac{2}{3} \times \frac{5}{7} \quad \text{Divide: } 2/3 = \frac{2}{3}
\]

\[
= \frac{10}{21} \quad \text{Multiply: } \frac{10}{3} \times \frac{1}{7} = \frac{10}{21}
\]
This is precisely the same result we get when we perform the following calculation.

\[
\begin{aligned}
\frac{2}{3} \times \frac{5}{7} &= \frac{10}{21} \\
\text{Multiply numerators and denominators.}
\end{aligned}
\]

Hence:

\[
\frac{2}{3} \times \frac{5}{7} \quad \text{is equivalent to} \quad \frac{2}{3} \times \frac{5}{7}
\]

Enter the expression \(2/3 \times 5/7\) on your calculator, then press the ENTER key. The result is shown in the first image in Figure 1.13. Next, press the MATH button, then select \(1:\text{Frac}\) (see the second image in Figure 1.13) and press the ENTER key again. Note that the result shown in the third image in Figure 1.13 matches the correct answer of \(10/21\) found above.

\[
\begin{array}{c}
\text{Figure 1.13: Calculating } 2/3 \times 1/2.
\end{array}
\]

c) This example demonstrates that we need a constant reminder of the Rules Guiding Order of Operations. We know we need to invert and multiply in this situation.

\[
\begin{aligned}
\frac{3}{5} \div \frac{1}{3} &= \frac{3}{5} \times \frac{3}{1} \\
&= \frac{9}{5} \\
\text{Invert and multiply.}
\end{aligned}
\]

\[
\begin{aligned}
\frac{3}{5} \div \frac{1}{3} &= \frac{3}{5} \times \frac{3}{1} \\
&= \frac{9}{5} \\
\text{Multiply numerators and denominators.}
\end{aligned}
\]

So, the correct answer is \(9/5\).

Enter the expression \(3/5\div1/3\) on your calculator, then press the ENTER key. Select \(1:\text{Frac}\) from the MATH menu and press the ENTER key again. Note that the result in the first image in Figure 1.14 does not match the correct answer of \(9/5\) found above. What have we done wrong?

If we follow the Rules Guiding Order of Operations exactly, then:

\[
\begin{aligned}
3/5\div1/3 &= \frac{3}{5}/\frac{1}{3} \\
&= \frac{3}{5} \times \frac{1}{3} \\
&= \frac{1}{5} \\
\text{Invert and multiply.}
\end{aligned}
\]

\[
\begin{aligned}
3/5\div1/3 &= \frac{3}{5}/\frac{1}{3} \\
&= \frac{3}{5} \times \frac{1}{3} \\
&= \frac{1}{5} \\
\text{Multiply: } \frac{3}{5} \times \frac{1}{3} = \frac{1}{5}
\end{aligned}
\]
CHAPTER 1. THE ARITHMETIC OF NUMBERS

This explains the answer found in the first image in Figure 1.14. However, it also show that:

\[
\frac{3}{5} / \frac{1}{3} \quad \text{is not equivalent to} \quad \frac{3}{5} ÷ \frac{1}{3}
\]

We can cure the problem by using grouping symbols.

\[
\left(\frac{3}{5}\right) / \left(\frac{1}{3}\right) = \frac{3}{5} ÷ \frac{1}{3} \quad \text{Parentheses first.}
\]

\[
= \frac{3}{5} ÷ \frac{1}{3} \quad / \text{ is equivalent to ÷.}
\]

Hence:

\[
\left(\frac{3}{5}\right) / \left(\frac{1}{3}\right) \quad \text{is equivalent to} \quad \frac{3}{5} ÷ \frac{1}{3}
\]

Enter the expression \((3/5)/(1/3)\) on your calculator, then press the ENTER key. Select \(1: ▶ \text{Frac}\) from the MATH menu and press the ENTER key again. Note that the result in the second image in Figure 1.14 matches the correct answer of 9/5.

Answer: \(\frac{28}{15}\)
Exercises

In Exercises 1-6, reduce the given fraction to lowest terms by dividing numerator and denominator by their greatest common divisor.

1. \( \frac{20}{50} \)
   \( \frac{36}{38} \)
   \( \frac{10}{48} \)

2. \( \frac{36}{50} \)
   \( \frac{24}{45} \)

3. \( \frac{18}{48} \)
   \( \frac{21}{36} \)

In Exercises 7-12, reduce the given fraction to lowest terms by prime factoring both numerator and denominator and canceling common factors.

7. \( \frac{153}{170} \)
   \( \frac{198}{144} \)
   \( \frac{188}{141} \)

8. \( \frac{171}{144} \)
   \( \frac{159}{106} \)

9. \( \frac{140}{133} \)

In Exercises 13-18, for each of the following problems, multiply numerators and denominators, then prime factor and cancel to reduce your answer to lowest terms.

13. \( \frac{20}{8} \cdot \left( \frac{18}{13} \right) \)

14. \( \frac{18}{16} \cdot \left( \frac{2}{5} \right) \)

15. \( \frac{19}{4} \cdot \left( \frac{18}{13} \right) \)

16. \( -\frac{3}{2} \cdot \left( \frac{-14}{6} \right) \)

17. \( \frac{-16}{8} \cdot \frac{19}{6} \)

18. \( \frac{-14}{4} \cdot \frac{7}{17} \)

In Exercises 19-24, for each of the following problems, first prime factor all numerators and denominators, then cancel. After canceling, multiply numerators and denominators.

19. \( -\frac{5}{6} \cdot \left( \frac{-12}{49} \right) \)

20. \( \frac{36}{17} \cdot \left( \frac{-21}{46} \right) \)
21. $-\frac{21}{10} \cdot \frac{12}{55}$

22. $-\frac{49}{13} \div \frac{52}{51}$

23. $\frac{55}{29} \div \frac{-54}{11}$

24. $\frac{7}{13} \div \frac{-55}{49}$

In Exercises 25-30, divide. Be sure your answer is reduced to lowest terms.

25. $\frac{50}{39} \div \left( -\frac{5}{58} \right)$

26. $\frac{31}{25} \div \left( -\frac{4}{5} \right)$

27. $-\frac{60}{17} \div \frac{34}{31}$

28. $-\frac{27}{28} \div \frac{45}{23}$

29. $-\frac{7}{10} \div \left( -\frac{13}{28} \right)$

30. $-\frac{4}{13} \div \left( -\frac{48}{35} \right)$

In Exercises 31-38, add or subtract the fractions, as indicated, and simplify your result.

31. $-\frac{5}{6} + \frac{1}{4}$

32. $-\frac{1}{7} + \frac{5}{8}$

33. $-\frac{8}{9} + \left( -\frac{1}{3} \right)$

34. $-\frac{1}{3} + \left( -\frac{1}{2} \right)$

35. $-\frac{1}{4} - \left( -\frac{2}{9} \right)$

36. $-\frac{1}{2} - \left( -\frac{1}{8} \right)$

37. $-\frac{8}{9} - \frac{4}{5}$

38. $-\frac{4}{7} - \frac{1}{3}$

In Exercises 39-52, simplify the expression.

39. $\frac{8}{9} - \left| \frac{5}{2} - \frac{2}{5} \right|$ 44. $\left( -\frac{1}{3} \right) \left( -\frac{5}{7} \right) + \left( \frac{2}{3} \right) \left( -\frac{6}{7} \right)$

40. $\frac{8}{5} - \left| \frac{7}{6} - \frac{1}{2} \right|$ 45. $-\frac{5}{8} + \frac{7}{2} \left( -\frac{9}{2} \right)$

41. $\left( -\frac{7}{6} \right)^2 + \left( -\frac{1}{2} \right) \left( -\frac{5}{3} \right)$ 46. $\frac{3}{2} + \frac{9}{2} \left( -\frac{1}{4} \right)$

42. $\left( \frac{3}{2} \right)^2 + \left( -\frac{1}{2} \right) \left( \frac{5}{8} \right)$ 47. $\left( -\frac{7}{5} \right) \left( \frac{9}{2} \right) - \left( -\frac{2}{5} \right)^2$

43. $\left( -\frac{9}{5} \right) \left( \frac{9}{7} \right) + \left( \frac{8}{5} \right) \left( \frac{1}{2} \right)$ 48. $\left( \frac{3}{4} \right) \left( \frac{2}{3} \right) - \left( \frac{1}{4} \right)^2$
In Exercises 53-70, evaluate the expression at the given values.

53. \(xy - z^2\) at \(x = -1/2\), \(y = -1/3\), and \(z = 5/2\)
54. \(xy - z^2\) at \(x = -1/3\), \(y = 5/6\), and \(z = 1/3\)
55. \(-5x^2 + 2y^2\) at \(x = 3/4\) and \(y = -1/2\).
56. \(-2x^2 + 4y^2\) at \(x = 4/3\) and \(y = -3/2\).
57. \(2x^2 - 2xy - 3y^2\) at \(x = 3/2\) and \(y = -3/4\).
58. \(5x^2 - 4xy - 3y^2\) at \(x = 1/5\) and \(y = -4/3\).
59. \(x + yz\) at \(x = -1/3\), \(y = 1/6\), and \(z = 2/5\).
60. \(x + yz\) at \(x = 1/2\), \(y = 7/4\), and \(z = 2/3\).
61. \(ab + bc\) at \(a = -4/7\), \(b = 7/3\), and \(c = -5/2\)

62. \(ab + bc\) at \(a = -8/5\), \(b = 7/2\), and \(c = -9/7\)
63. \(x^3\) at \(x = -1/2\)
64. \(x^2\) at \(x = -3/2\)
65. \(x - yz\) at \(x = -8/5\), \(y = 1/3\), and \(z = -8/5\)
66. \(x - yz\) at \(x = 2/3\), \(y = 2/9\), and \(z = -3/5\)
67. \(-x^2\) at \(x = -8/3\)
68. \(-x^4\) at \(x = -9/7\)
69. \(x^2 + yz\) at \(x = 7/2\), \(y = -5/4\), and \(z = -5/3\)
70. \(x^2 + yz\) at \(x = 1/2\), \(y = 7/8\), and \(z = -5/9\)

71. \(a + b/c + d\) is equivalent to which of the following mathematical expressions?
   
   (a) \(a + b/c + d\)
   (b) \(a + b/c + d\)
   (c) \(a + b/c + d\)
   (d) \(a + b/c + d\)

72. \((a + b)/c + d\) is equivalent to which of the following mathematical expressions?
   
   (a) \(a + b/c + d\)
   (b) \(a + b/c + d\)
   (c) \(a + b/c + d\)
   (d) \(a + b/c + d\)

73. \(a + b/(c + d)\) is equivalent to which of the following mathematical expressions?
   
   (a) \(a + b/c + d\)
   (b) \(a + b/c + d\)
   (c) \(a + b/c + d\)
   (d) \(a + b/c + d\)

74. \((a + b)/(c + d)\) is equivalent to which of the following mathematical expressions?
   
   (a) \(a + b/c + d\)
   (b) \(a + b/c + d\)
   (c) \(a + b/c + d\)
   (d) \(a + b/c + d\)

75. Use the graphing calculator to reduce \(4125/1155\) to lowest terms.

76. Use the graphing calculator to reduce \(2100/945\) to lowest terms.
CHAPTER 1. THE ARITHMETIC OF NUMBERS

77. Use the graphing calculator to simplify

\[
\frac{45}{84} \cdot \frac{70}{33}.
\]

78. Use the graphing calculator to simplify

\[
\frac{34}{55} + \frac{13}{77}.
\]

79. Use the graphing calculator to simplify

\[
-\frac{28}{33} \div \left( -\frac{35}{44} \right).
\]

80. Use the graphing calculator to simplify

\[
-\frac{11}{84} - \left( -\frac{11}{36} \right).
\]

Answers

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>1. (\frac{2}{5})</td>
<td>25. (-\frac{580}{39})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. (\frac{5}{24})</td>
<td>27. (-\frac{930}{289})</td>
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<tr>
<td>5. (\frac{8}{15})</td>
<td>29. (\frac{98}{65})</td>
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<tr>
<td>7. (\frac{9}{10})</td>
<td>31. (-\frac{7}{12})</td>
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<td>9. (\frac{4}{3})</td>
<td>33. (-\frac{11}{9})</td>
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<tr>
<td>11. (\frac{3}{2})</td>
<td>35. (-\frac{1}{36})</td>
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<tr>
<td>13. (-\frac{45}{13})</td>
<td>37. (-\frac{76}{45})</td>
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<tr>
<td>15. (\frac{171}{26})</td>
<td>39. (-\frac{109}{90})</td>
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</tr>
<tr>
<td>17. (-\frac{19}{3})</td>
<td>41. (\frac{79}{36})</td>
<td></td>
<td></td>
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<tr>
<td>19. (\frac{10}{49})</td>
<td>43. (\frac{53}{35})</td>
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<tr>
<td>21. (-\frac{126}{270})</td>
<td>45. (-\frac{131}{8})</td>
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<tr>
<td>23. (-\frac{270}{29})</td>
<td>47. (-\frac{323}{50})</td>
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<tr>
<td>49. (\frac{62}{45})</td>
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</tbody>
</table>
1.3. **THE RATIONAL NUMBERS**

<p>| | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>51.</td>
<td>$-\frac{5}{42}$</td>
</tr>
<tr>
<td>53.</td>
<td>$-\frac{73}{12}$</td>
</tr>
<tr>
<td>55.</td>
<td>$-\frac{37}{16}$</td>
</tr>
<tr>
<td>57.</td>
<td>$\frac{81}{16}$</td>
</tr>
<tr>
<td>59.</td>
<td>$-\frac{2}{5}$</td>
</tr>
<tr>
<td>61.</td>
<td>$-\frac{43}{10}$</td>
</tr>
<tr>
<td>63.</td>
<td>$-\frac{1}{8}$</td>
</tr>
<tr>
<td>65.</td>
<td>$-\frac{16}{15}$</td>
</tr>
<tr>
<td>67.</td>
<td>$-\frac{64}{9}$</td>
</tr>
<tr>
<td>69.</td>
<td>$\frac{43}{3}$</td>
</tr>
<tr>
<td>71.</td>
<td>(a)</td>
</tr>
<tr>
<td>73.</td>
<td>(d)</td>
</tr>
<tr>
<td>75.</td>
<td>$\frac{25}{7}$</td>
</tr>
<tr>
<td>77.</td>
<td>$\frac{25}{22}$</td>
</tr>
<tr>
<td>79.</td>
<td>$\frac{16}{15}$</td>
</tr>
</tbody>
</table>
1.4 Decimal Notation

Every rational number can be expressed using *decimal notation*. To change a fraction into its decimal equivalent, divide the numerator of the fraction by its denominator. In some cases the process will terminate, leaving a zero remainder. However, in other cases, the remainders will begin to repeat, providing a decimal representation that repeats itself in blocks.

**You Try It!**

**EXAMPLE 1.** Change each of the following fractions into decimals.

(a) \( \frac{39}{80} \) (b) \( \frac{4}{11} \)

**Solution:** We perform two divisions, the one on the left to change 39/80 to a decimal, the one on the right to find a decimal representation for 4/11.

\[
\begin{align*}
0.4875 & \quad 0.3636 \\
80)39.0000 & \quad 11)4.0000 \\
32 & \quad 3 \\
70 & \quad 70 \\
64 & \quad 66 \\
60 & \quad 40 \\
0 & \quad 40 \\
600 & \quad 33 \\
400 & \quad 70 \\
0 & \quad 0 \\
400 & \quad 400 \\
0 & \quad 0 \\
\end{align*}
\]

On the left, the division process terminates with a zero remainder. Hence, 39/80 = 0.4875 is called a *terminating* decimal. On the right, the remainders repeat in a pattern and the quotient also repeats in blocks of two. Hence, 4/11 = 0.3636... is called a *repeating* decimal. We can also use a *repeating bar* to write 4/11 = 0.\overline{36} The block under the repeating bar repeats itself indefinitely.

Answer: 0.4875

Vice-versa, any terminating decimal can be expressed as a fraction. You need only count the number of digits after the decimal point and use the same number of zeros in your denominator.

**You Try It!**

**EXAMPLE 2.** Express each of the following decimals as fractions. Reduce your answers to lowest terms.

(a) 0.055 (b) 3.36

**Solution:** In each case, count the number of digits after the decimal point and include an equal number of zeros in the denominator.
In example (a), there are three digits after the decimal point, so we place the number over 1000, which has three zeros after the one.

\[
0.055 = \frac{55}{1000} = \frac{11}{200}
\]

In example (b), there are two digits after the decimal point, so we place the number over 100, which has two zeros after the one.

\[
3.36 = \frac{336}{100} = \frac{84}{25}
\]

Answer: 9/20

As we saw in Example 1, the repeating decimal \(0.3636\ldots\) is equivalent to the fraction \(4/11\). Indeed, any repeating decimal can be written as a fraction. For example, \(0.3 = 1/3\) and \(0.142857 = 1/7\). In future courses you will learn a technique for changing any repeating decimal into an equivalent fraction.

However, not all decimals terminate or repeat. For example, consider the decimal

\[
0.42424242242222\ldots,
\]

which neither terminates nor repeats. This number cannot be expressed using repeating bar notation because each iteration generates one additional 2. Because this number neither repeats nor terminates, it cannot be expressed as a fraction. Hence, \(0.424242242222\ldots\) is an example of an \textit{irrational number}.

\textbf{Irrational numbers.} If a number cannot be expressed in the form \(p/q\), where \(p\) and \(q\) are integers, \(q \neq 0\), then the number is called an \textit{irrational number}.

\textbf{Real numbers.} By including all of the rational and irrational numbers in one set, we form what is known as the set of \textit{real numbers}.

The set of real numbers includes every single number we will use in this textbook and course.

\textbf{Adding and Subtracting Decimals}

When adding signed decimals, use the same rules you learned to use when adding signed integers or fractions.

\textbf{Sign rules for addition.} When adding two decimal numbers, use the following rules:

- To add two decimals with like signs, add their magnitudes and prefix their common sign.
• To add two decimals with unlike signs, subtract the smaller magnitude from the larger, then prefix the sign of the decimal number having the larger magnitude.

You Try It!

EXAMPLE 3. Simplify: (a) $-2.3 + (-0.015)$ and (b) $-8.4 + 6.95$

Solution: In part (a), note that we have like signs. Hence, we add the magnitudes and prefix the common sign.

\[
\begin{array}{c}
-2.3 + (-0.015) = -2.315 \\
\hline
+0.015 \\
\hline
2.315
\end{array}
\]

In part (b), note that we have unlike signs. Thus, we first subtract the smaller magnitude from the larger magnitude, then prefix the sign of the decimal number with the larger magnitude.

\[
\begin{array}{c}
-8.4 + 6.95 = -1.45 \\
\hline
-6.95 \\
\hline
1.45
\end{array}
\]

Answer: $-4.13$

Hence, $-2.3 + (-0.015) = -2.315$ and $-8.4 + 6.95 = -1.45$.

Subtraction still means “add the opposite.”

You Try It!

EXAMPLE 4. Simplify: (a) $-5.6 - 8.4$ and (b) $-7.9 - (-5.32)$

Solution: In part (a), first we add the opposite. Then we note that we have like signs. Hence, we add the magnitudes and prefix the common sign.

\[
\begin{array}{c}
-5.6 - 8.4 = -5.6 + (-8.4) \\
\hline
+8.4 \\
\hline
14.0
\end{array}
\]

In part (b), first we add the opposite. Then we note that we have unlike signs. Thus, we first subtract the smaller magnitude from the larger magnitude, then prefix the sign of the decimal number with the larger magnitude.

\[
\begin{array}{c}
-7.9 - (-5.32) = -7.9 + 5.32 \\
\hline
5.32 \\
\hline
-2.58
\end{array}
\]

Answer: $-41.07$

Hence, $-5.6 - 8.4 = -14.0$ and $-7.9 - (-5.32) = -2.58$. 
1.4. DECIMAL NOTATION

Multiplication and Division of Decimals

The sign rules for decimal multiplication and division are the same as the sign rules used for integers and fractions.

**Sign Rules for multiplication and division.** When multiplying or dividing two decimal numbers, use the following rules:

- Like signs give a positive result.
- Unlike signs give a negative result.

Multiplication of decimal numbers is fairly straightforward. First multiply the magnitudes of the numbers, ignoring the decimal points, then count the number of digits to the right of the decimal point in each factor. Place the decimal point in the product so that the number of digits to the right of the decimal points equals the sum of number of digits to the right of the decimal point in each factor.

**EXAMPLE 5.** Simplify: \((-1.96)(2.8)\)

**Solution:** Multiply the magnitudes. The first decimal number has two digits to the right of the decimal point, the second has one digit to the right of the decimal point. Thus, we must place a total of three digits to the right of the decimal point in the product.

\[
\begin{align*}
\begin{array}{c}
1.96 \\
\times 2.8
\end{array}
\end{align*}
\]

\[
\begin{array}{c}
1 \, 568 \\
3 \, 92
\end{array}
\]

\[
\begin{array}{c}
5.488
\end{array}
\]

\((-1.96)(2.8) = -5.488\)

Note that unlike signs yield a negative product. Answer: 292.50

When dividing signed decimal numbers, ignore the signs and divide the magnitudes. Push the decimal point in the divisor to the end of the divisor, then move the decimal point in the dividend an equal number of spaces. This sets the decimal point in the quotient.

**EXAMPLE 6.** Simplify: \(-4.392 \div (-0.36)\)

**Solution.** Divide the magnitudes. Move the decimal in the divisor to the end of the divisor. Move the decimal in the dividend an equal number of places (two places) to the right.
0.36 \[ \begin{array}{c} \div 1.39 \end{array} \]

Place the decimal point in the quotient directly above the new position of the decimal point in the dividend, then divide.

\[
\begin{array}{c}
\phantom{12.2} \\
36)439.2 \\
\phantom{36}\phantom{3} \\
\phantom{36}\phantom{9} \\
\phantom{36}\phantom{7} \\
\phantom{36}\phantom{2} \\
\phantom{36}\phantom{7} \\
\phantom{36}\phantom{2} \\
\phantom{36}\phantom{0}
\end{array}
\]

Answer: \(-1.8\)

Like signs yield a positive result. Hence, \(-4.392 \div (-0.36) = 12.2\).

---

**Order of Operations**

Decimal numbers obey the same *Rules Guiding Order of Operations* as do the integers and fractions.

**Rules Guiding Order of Operations.** When evaluating expressions, proceed in the following order.

1. Evaluate expressions contained in grouping symbols first. If grouping symbols are nested, evaluate the expression in the innermost pair of grouping symbols first.

2. Evaluate all exponents that appear in the expression.

3. Perform all multiplications and divisions in the order that they appear in the expression, moving left to right.

4. Perform all additions and subtractions in the order that they appear in the expression, moving left to right.

---

**You Try It!**

Given \( y = -0.2 \), evaluate: \(-y^4\)

**EXAMPLE 7.** Given \( x = -0.12 \), evaluate \(-x^2\).

**Solution:** Following *Tips for Evaluating Algebraic Expressions*, first replace all occurrences of variable \( x \) in the expression \(-x^2\) with open parentheses. Next,
1.4. DECIMAL NOTATION

substitute \(-0.12\) for \(x\) in the open parentheses, then simplify.

\[
\begin{align*}
-x^2 &= -\left( \ ight)^2 \quad \text{Replace } x \text{ with open parentheses.} \\
 &= -(0.12)^2 \quad \text{Substitute } -0.12 \text{ for } x. \\
 &= -(0.0144) \quad \text{Exponent: } (-0.12)^2 = 0.0144 \\
 &= -0.0144 \quad \text{Negate.}
\end{align*}
\]

Note that we square first, then we negate second. Thus, if \(x = -0.12\), then \(-x^2 = -0.0144\).

Answer: \(-0.0016\)

EXAMPLE 8. Given \(x = -0.3\), evaluate \(1.2x^2 - 3.4x\).

Solution: Following *Tips for Evaluating Algebraic Expressions*, first replace all occurrences of variable \(x\) in the expression \(1.2x^2 - 3.4x\) with open parentheses. Next, substitute \(-0.3\) for \(x\) in the open parentheses, then simplify.

\[
\begin{align*}
1.2x^2 - 3.4x &= 1.2\left( \ ight)^2 - 3.4\left( \ ight) \quad \text{Replace } x \text{ with parentheses.} \\
 &= 1.2(-0.3)^2 - 3.4(-0.3) \quad \text{Substitute } -0.3 \text{ for } x. \\
 &= 1.2(0.09) - 3.4(-0.3) \quad \text{Exponent: } (-0.3)^2 = 0.09. \\
 &= 0.108 - (-1.02) \quad \text{Multiply: } 1.2(0.09) = 0.108 \text{ and } 3.4(-0.3) = -1.02. \\
 &= 0.108 + 1.02 \quad \text{Add the opposite.} \\
 &= 1.128 \quad \text{Simplify.}
\end{align*}
\]

Thus, if \(x = -0.3\), then \(1.2x^2 - 3.4x = 1.128\). Answer: \(-0.3615\)

We saw earlier that we can change a fraction to a decimal by dividing.

EXAMPLE 9. Given \(x = \frac{2}{5}\), evaluate \(-3.2x + 5\).

Solution: Following *Tips for Evaluating Algebraic Expressions*, first replace all occurrences of variable \(x\) in the expression \(-3.2x + 5\) with open parentheses. Next, substitute \(\frac{2}{5}\) for \(x\) in the open parentheses.

\[
\begin{align*}
-3.2x + 5 &= -3.2\left( \ ight) + 5 \quad \text{Replace } x \text{ with open parentheses.} \\
 &= -3.2\left( \frac{2}{5} \right) + 5 \quad \text{Substitute } \frac{2}{5} \text{ for } x.
\end{align*}
\]

Given \(y = -0.15\), evaluate:

\[-1.4y^2 + 2.2y\]

Answer: \(-0.3615\)

You Try It!
CHAPTER 1. THE ARITHMETIC OF NUMBERS

One approach is to change $2/5$ to a decimal by dividing the numerator by the denominator. Thus, $2/5 = 0.4$.

\[
\begin{align*}
\text{Replace } 2/5 \text{ with } 0.4. \\
= -3.2(0.4) + 5 \\
= -1.28 + 5 \\
= 3.72
\end{align*}
\]

Multiply: $-3.2(0.4) = -1.28$.

Add: $-1.28 + 5 = 3.72$.

Answer: 8.725

Thus, if $x = 2/5$, then $-3.2x + 5 = 3.72$.

As we saw in Example 2, we can easily change a terminating decimal into a fraction by placing the number (without the decimal point) over the proper power of ten. The choice of the power of ten should match the number of digits to the right of the decimal point. For example:

\[
\begin{align*}
0.411 &= \frac{411}{1000} \\
3.11 &= \frac{311}{100} \\
15.1111 &= \frac{151111}{10000}
\end{align*}
\]

Note that the number of zeros in each denominator matches the number of digits to the right of the decimal point.

You Try It!

**EXAMPLE 10.** Given $y = -0.25$, evaluate $\frac{-3}{5}y + 4$.

**Solution:** Following *Tips for Evaluating Algebraic Expressions*, first replace all occurrences of variable $y$ in the expression $-(3/5)y + 4$ with open parentheses. Next, substitute $-0.25$ for $y$ in the open parentheses.

\[
\begin{align*}
\frac{-3}{5}y + 4 &= \frac{-3}{5} \left( -0.25 \right) + 4 \\
&= \frac{-3}{5} \left( \frac{-1}{4} \right) + 4
\end{align*}
\]

Replace $y$ with open parentheses.

Substitute $-0.25$ for $y$.

Place 25 over 100 to determine that $-0.25 = -25/100$, or after reduction, $-0.25 = -1/4$.

\[
\begin{align*}
&= \frac{-3}{5} \left( \frac{-1}{4} \right) + 4 \\
&= \frac{3}{20} + 4 \\
&= \frac{3}{20} + \frac{80}{20} \\
&= \frac{83}{20}
\end{align*}
\]

Replace $-0.25$ with $-1/4$.

Multiply: $\frac{-3}{5} \left( \frac{-1}{4} \right) = \frac{3}{20}$.

Make equivalent fractions with LCD.

Add.

Answer: $133/25$

Thus, if $y = -0.25$, then $-(3/5)y + 4 = 83/20$. 

□
1.4. DECIMAL NOTATION

Rounding Using the Graphing Calculator

Here is the algorithm for rounding a decimal number to a particular place.

**Rules for rounding.** To round a number to a particular place, follow these steps:

1. Mark the place you wish to round to. The digit in this place is called the *rounding digit*.

2. Mark the digit in the place to the immediate right of the rounding digit. This is called the *test digit*.

   a) If the test digit is greater than or equal to 5, add 1 to the rounding digit, then replace all digits to the right of the rounding digit with zeros. Trailing zeros to the right of the decimal point may be deleted.

   b) If the test digit is less than 5, keep the rounding digit the same, then replace all digits to the right of the rounding digit with zeros. Trailing zeros to the right of the decimal point may be deleted.

**EXAMPLE 11.** Use your graphing calculator to evaluate \(125x^3 - 17.5x + 44.8\) at \(x = -3.13\). Round your answer to the nearest tenth.

**Solution.** First, store \(-3.13\) in the variable X with the following keystrokes.

\[
(-) \quad 3 \quad \cdot \quad 1 \quad 3 \quad \text{STO} \quad \text{⊿} \quad X,T,\theta,n \quad \text{ENTER}
\]

The result is shown in the first image in Figure 1.15. Next, enter the expression \(125x^3 - 17.5x + 44.8\) with the following keystrokes.

\[
(-) \quad 1 \quad 2 \quad 5 \quad \times \quad X,T,\theta,n \quad \wedge \quad 3 \quad - \quad 1 \quad 7 \quad \cdot \quad 5
\]

\[
\times \quad X,T,\theta,n \quad + \quad 4 \quad 4 \quad \cdot \quad 8 \quad \text{ENTER}
\]

The result is shown in the second image in Figure 1.15.

Thus, the answer is approximately \(-3733.462125\). We now need to round this answer to the nearest tenth. Mark the rounding digit in the tenths place and the test digit to its immediate right.

Evaluate \(x^3 - 3x\) at \(x = -1.012\). Round to the nearest hundredth.
Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then replace all digits to the right of the rounding digit with zeros.

\[ -3733.462125 \approx -3733.500000 \]

Delete the trailing zeros from end of the fractional part of a decimal. This does not change our answer’s value.

\[ -3733.462125 \approx -3733.5 \]

Therefore, if \( x = -3.13 \), then to the nearest tenth:

\[ 125x^3 - 17.5x + 44.8 \approx -3733.5 \]

Answer: 2.0

\[ -3.13 \times 3 - 17.5 \times 3 + 44.8 \approx -3733.5 \]
In Exercises 1-33, simplify the given expression.

1. $-2.835 + (-8.759)$
2. $-5.2 + (-2)$
3. $19.5 - (-1.6)$
4. $9.174 - (-7.7)$
5. $-2 - 0.49$
6. $-50.86 - 9$
7. $(-1.2)(-0.05)$
8. $(-7.9)(0.9)$
9. $-0.13 + 23.49$
10. $-30.82 + 75.93$
11. $16.4 + (-41.205)$
12. $-7.8 + 3.5$
13. $-0.4508 ÷ 0.49$
14. $0.2378 ÷ (-0.29)$
15. $(-1.42)(-3.6)$
16. $(-8.64)(4.6)$
17. $2.184 ÷ (-0.24)$
18. $7.395 ÷ (-0.87)$
19. $(-7.1)(-4.9)$
20. $(5.8)(-1.9)$
21. $7.41 ÷ (-9.5)$
22. $-1.911 ÷ 4.9$
23. $-24.08 ÷ 2.8$
24. $61.42 ÷ (-8.3)$
25. $(-4.04)(-0.6)$
26. $(-5.43)(0.09)$
27. $-7.2 - (-7)$
28. $-2.761 - (-1.5)$
29. $(46.9)(-0.1)$
30. $(-98.9)(-0.01)$
31. $(86.6)(-1.9)$
32. $(-20.5)(8.1)$

In Exercises 33-60, simplify the given expression.

33. $-4.3 - (-6.1)(-2.74)$
34. $-1.4 - 1.9(3.36)$
35. $-3.49 + | - 6.9 - (-15.7)|$
36. $1.3 + | - 13.22 - 8.79|$
37. $|18.9 - 1.55| - | - 16.1 - (-17.04)|$
38. $| - 17.5 - 16.4| - | - 15.58 - (-4.5)|$
39. $8.2 - (-3.1)^3$
40. $-8.4 - (-6.8)^3$
41. $5.7 - (-8.6)(1.1)^2$
42. $4.8 - 6.3(6.4)^2$
43. $(5.67)(6.8) - (1.8)^2$
44. $(-8.7)(8.3) - (-1.7)^2$
45. $9.6 + (-10.05 - 13.16)$
46. $-4.2 + (17.1 - 14.46)$
47. $8.1 + 3.7(5.77)$
48. $8.1 + 2.3(-5.53)$
### CHAPTER 1. THE ARITHMETIC OF NUMBERS

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<tr>
<td><strong>49.</strong> $7.5 + 34.5 / (-1.6 + 8.5)$</td>
<td><strong>55.</strong> $-4.37 -</td>
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<tr>
<td><strong>50.</strong> $-8.8 + 0.3 / (-7.2 + 7.3)$</td>
<td><strong>56.</strong> $4.1 -</td>
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<tr>
<td><strong>51.</strong> $(8.0 + 2.2) / 5.1 - 4.6$</td>
<td><strong>57.</strong> $7.06 - (-1.1 - 4.41)$</td>
</tr>
<tr>
<td><strong>52.</strong> $(35.3 + 1.8) / 5.3 - 5.4$</td>
<td><strong>58.</strong> $7.74 - (0.9 - 7.37)$</td>
</tr>
<tr>
<td><strong>53.</strong> $-18.24 -</td>
<td>-18.5 - 19.7</td>
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<td><strong>54.</strong> $16.8 -</td>
<td>4.58 - 17.14</td>
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<tr>
<td><strong>61.</strong> Evaluate $a - b^2$ at $a = -2.9$ and $b = -5.4$.</td>
<td><strong>68.</strong> Evaluate $</td>
</tr>
<tr>
<td><strong>62.</strong> Evaluate $a - b^3$ at $a = -8.3$ and $b = -6.9$.</td>
<td><strong>69.</strong> Evaluate $a + b / (c + d)$ at $a = 4.7$, $b = 54.4$, $c = 1.7$, and $d = 5.1$.</td>
</tr>
<tr>
<td><strong>63.</strong> Evaluate $a +</td>
<td>b - c</td>
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<tr>
<td><strong>64.</strong> Evaluate $a -</td>
<td>b - c</td>
</tr>
<tr>
<td><strong>65.</strong> Evaluate $a - bc$ at $a = 4.3$, $b = 8.5$, and $c = 1.73$.</td>
<td><strong>72.</strong> Evaluate $a + (b - c)$ at $a = 12.6$, $b = -13.42$, and $c = -15.09$.</td>
</tr>
<tr>
<td><strong>66.</strong> Evaluate $a + bc$ at $a = 4.1$, $b = 3.1$, and $c = -7.03$.</td>
<td><strong>73.</strong> Evaluate $a -</td>
</tr>
<tr>
<td><strong>67.</strong> Evaluate $a - (b - c)$ at $a = -17.6$, $b = -17.9$, and $c = -19.07$.</td>
<td><strong>74.</strong> Evaluate $a - bc^2$ at $a = -3.32$, $b = -5.4$, and $c = -8.5$.</td>
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### Additional Calculations

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<tr>
<td><strong>75.</strong> Use your graphing calculator to evaluate $3.5 - 1.7x$ at $x = 1.25$ Round your answer to the nearest tenth.</td>
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<tr>
<td><strong>76.</strong> Use your graphing calculator to evaluate $2.35x - 1.7$ at $x = -12.23$ Round your answer to the nearest tenth.</td>
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<tr>
<td><strong>77.</strong> Use your graphing calculator to evaluate $1.7x^2 - 3.2x + 4.5$ at $x = 2.86$ Round your answer to the nearest hundredth.</td>
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<tr>
<td><strong>78.</strong> Use your graphing calculator to evaluate $19.5 - 4.4x - 1.2x^2$ at $x = -1.23$ Round your answer to the nearest hundredth.</td>
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<tr>
<td><strong>79.</strong> Use your graphing calculator to evaluate $-18.6 + 4.4x^2 - 3.2x^3$ at $x = 1.27$ Round your answer to the nearest thousandth.</td>
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</tr>
<tr>
<td><strong>80.</strong> Use your graphing calculator to evaluate $-4.4x^3 - 7.2x - 18.2$ at $x = 2.29$ Round your answer to the nearest thousandth.</td>
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<td>1.</td>
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<td>5.</td>
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<td>7.</td>
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<td>9.</td>
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<td>−21.014</td>
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<td>35.</td>
<td>5.31</td>
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<td>37.</td>
<td>16.41</td>
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<td>39.</td>
<td>37.991</td>
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1.5 Algebraic Expressions

The associative property of multiplication is valid for all numbers.

**Associative Property of Multiplication.** Let \( a, b, \) and \( c \) be any numbers. Then:

\[
a \cdot (b \cdot c) = (a \cdot b) \cdot c
\]

The associative property of multiplication is useful in a number of situations.

**You Try It!**

**EXAMPLE 1.** Simplify: \(-3(4y)\)

**Solution:** Currently, the grouping \(-3(4y)\) demands that we first multiply 4 and \(y\). However, we can use the associative property of multiplication to regroup, first multiplying \(-3\) and 4.

\[
-3(4y) = (-3 \cdot 4)y \quad \text{The associative property of multiplication.}
= -12y \quad \text{Multiply: } -3 \cdot 4 = -12
\]

Answer: \(6x\)

Thus, \(-3(4y) = -12y\). □

Let’s look at another example.

**You Try It!**

**EXAMPLE 2.** Simplify: \(-2(-4xy)\)

**Solution:** Currently, the grouping \(-2(-4xy)\) demands that we first multiply \(-4\) and \(xy\). However, we can use the associative property of multiplication to regroup, first multiplying \(-2\) and \(-4\).

\[
-2(-4xy) = (-2 \cdot (-4))xy \quad \text{The associative property of multiplication.}
= 8xy \quad \text{Multiply: } -2 \cdot (-4) = 8
\]

Answer: \(24u^2\)

Thus, \(-2(-4xy) = 8xy\). □

In practice, we can move quicker if we perform the regrouping mentally, then simply write down the answer. For example:

\[
-2(-4t) = 8t \quad \text{and} \quad 2(-5z^2) = -10z^2 \quad \text{and} \quad -3(4u^3) = -12u^3
\]
1.5. ALGEBRAIC EXPRESSIONS

The Distributive Property

We now discuss a property that couples addition and multiplication. Consider the expression $2 \cdot (3 + 5)$. The Rules Guiding Order of Operations require that we first simplify the expression inside the parentheses.

$$2 \cdot (3 + 5) = 2 \cdot 8 \quad \text{Add: } 3 + 5 = 8$$
$$= 16 \quad \text{Multiply: } 2 \cdot 8 = 16$$

Alternatively, we can instead distribute the 2 times each term in the parentheses. That is, we will first multiply the 3 by 2, then multiply the 5 by 2. Then we add the results.

$$2 \cdot (3 + 5) = 2 \cdot 3 + 2 \cdot 5 \quad \text{Distribute the 2.}$$
$$= 6 + 10 \quad \text{Multiply: } 2 \cdot 3 = 6 \text{ and } 2 \cdot 5 = 10$$
$$= 16 \quad \text{Add: } 6 + 10 = 16$$

Note that both methods produce the same result, namely 16. This example demonstrates an extremely important property of numbers called the distributive property.

The Distributive Property. Let $a$, $b$, and $c$ be any numbers. Then:

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

That is, multiplication is distributive with respect to addition.

EXAMPLE 3. Use the distributive property to expand $2(3x + 7)$. Expand: $5(2y + 7)$

Solution: First distribute the 2 times each term in the parentheses. Then simplify.

$$2(3x + 7) = 2(3x) + 2(7) \quad \text{Use the distributive property.}$$
$$= 6x + 14 \quad \text{Multiply: } 2(3x) = 6x \text{ and } 2(7) = 14$$

Thus, $2(3x + 7) = 6x + 14$. 

Answer: $10y + 35$

Multiplication is also distributive with respect to subtraction.

EXAMPLE 4. Use the distributive property to expand $-2(5y - 6)$. Expand: $-3(2z - 7)$
CHAPTER 1. THE ARITHMETIC OF NUMBERS

Solution: Change to addition by adding the opposite, then apply the distributive property.

\[-2(5y - 6) = -2(5y + (-6))\]

Add the opposite.

\[= -2(5y) + (-2)(-6)\]

Use the distributive property.

\[= -10y + 12\]

Multiply: \(-2(5y) = -10y\) and \((-2)(-6) = 12\)

Thus, \(-2(5y - 6) = -10y + 12\).

Answer: \(-6z + 21\)

Speeding Things Up a Bit

In Example 4, we changed the subtraction to addition, applied the distributive property, then several steps later we were finished. However, if you understand that subtraction is really the same as adding the opposite, and if you are willing to do a few steps in your head, you should be able to simply write down the answer immediately following the given problem.

If you look at the expression \(-2(5y - 6)\) from Example 4 again, only this time think “multiply \(-2\) times \(5y\), then multiply \(-2\) times \(-6\), then the result is immediate.

\[-2(5y - 6) = -10y + 12\]

Let’s try this “speeding it up” technique in a couple more examples.

You Try It!

EXAMPLE 5. Use the distributive property to expand \(-3(-2x + 5y - 12)\).

Solution: To distribute the \(-3\), we simply think as follows: “\(-3(-2x) = 6x\), \(-3(5y) = -15y\), and \(-3(-12) = 36\).” This sort of thinking allows us to write down the answer immediately without any additional steps.

\[-3(-2x + 5y - 12) = 6x - 15y + 36\]

Answer: \(6a - 9b + 21\)

You Try It!

EXAMPLE 6. Use the distributive property to expand \(-5(-2a - 5b + 8)\).

Solution: To distribute the \(-5\), we simply think as follows: “\(-5(-2a) = 10a\), \(-5(-5b) = 25b\), and \(-5(8) = -40\).” This sort of thinking allows us to write down the answer immediately without any additional steps.

\[-5(-2a - 5b + 8) = 10a + 25b - 40\]

Answer: \(4x + 8y + 28\)
1.5. ALGEBRAIC EXPRESSIONS

Distributing a Negative Sign

Recall that negating a number is equivalent to multiplying the number by \(-1\).

**Multiplicative Property of Minus One.** If \(a\) is any number, then:

\[ (-1)a = -a \]

This means that if we negate an expression, it is equivalent to multiplying the expression by \(-1\).

**EXAMPLE 7.** Expand \(- (7x - 8y - 10)\).

**Solution:** First, negating is equivalent to multiplying by \(-1\). Then we can change subtraction to addition by “adding the opposite” and use the distributive property to finish the expansion.

\[
- (7x - 8y - 10) = -1(7x - 8y - 10) \\
= -1(7x + (-8y) + (-10)) \\
= -1(7x) + (-1)(-8y) + (-1)(-10) \\
= -7x + 8y + 10
\]

Thus, \(- (7x - 8y - 10) = -7x + 8y + 10.\)

While being mathematically precise, the technique of Example 7 can be simplified by noting that negating an expression surrounded by parentheses simply changes the sign of each term inside the parentheses to the opposite sign.

Once we understand this, we can simply “distribute the minus sign” and write:

\[-(7x - 8y - 10) = -7x + 8y + 10\]

In similar fashion,

\[-(-3a + 5b - c) = 3a - 5b + c\]

and

\[-(-3x - 8y + 11) = 3x + 8y - 11.\]
Combining Like Terms

We can use the distributive property to distribute a number times a sum.

\[ a(b + c) = ab + ac \]

However, the distributive property can also be used in reverse, to “unmultiply” or factor an expression. Thus, we can start with the expression \( ab + ac \) and “factor out” the common factor \( a \) as follows:

\[ ab + ac = a(b + c) \]

You can also factor out the common factor on the right.

\[ ac + bc = (a + b)c \]

We can use this latter technique to combine like terms.

---

**You Try It!**

**EXAMPLE 8.** Simplify: \( 7x + 5x \)

**Solution:** Use the distributive property to factor out the common factor \( x \) from each term, then simplify the result.

\[
7x + 5x = (7 + 5)x \\
= 12x
\]

Factor out an \( x \) using the distributive property. Simplify: \( 7 + 5 = 12 \)

Answer: 11

Thus, \( 7x + 5x = 12x \).

---

**You Try It!**

**EXAMPLE 9.** Simplify: \( -8a^2 + 5a^2 \)

**Solution:** Use the distributive property to factor out the common factor \( a^2 \) from each term, then simplify the result.

\[
-8a^2 + 5a^2 = (-8 + 5)a^2 \\
= -3a^2
\]

Factor out an \( a^2 \) using the distributive property. Simplify: \( -8 + 5 = -3 \)

Answer: \( 4z^3 \)

Thus, \( -8a^2 + 5a^2 = -3a^2 \).

---

Examples 8 and 9 combine what are known as “like terms.” Examples 8 and 9 also suggest a possible shortcut for combining like terms.
Like Terms. Two terms are called like terms if they have identical variable parts, which means that the terms must contain the same variables raised to the same exponents.

For example, $2x^2y$ and $11x^2y$ are like terms because they contain identical variables raised to the same exponents. On the other hand, $-3st^2$ and $4s^2t$ are not like terms. They contain the same variables, but the variables are not raised to the same exponents.

Consider the like terms $2x^2y$ and $11x^2y$. The numbers 2 and 11 are called the coefficients of the like terms. We can use the distributive property to combine these like terms as we did in Examples 8 and 9, factoring out the common factor $x^2y$.

$$2x^2y + 11x^2y = (2 + 11)x^2y$$
$$= 13x^2y$$

However, a much quicker approach is simply to add the coefficients of the like terms, keeping the same variable part. That is, $2 + 11 = 13$, so:

$$2x^2y + 11x^2y = 13x^2y$$

This is the procedure we will follow from now on.

**You Try It!**

**EXAMPLE 10.** Simplify: $-8w^2 + 17w^2$

**Solution:** These are like terms. If we add the coefficients $-8$ and $17$, we get 9. Thus:

$$-8w^2 + 17w^2 = 9w^2$$

Add the coefficients and repeat the variable part.

Answer: $-11w^2$

**EXAMPLE 11.** Simplify: $-4uv - 9uv$

**Solution:** These are like terms. If we add $-4$ and $-9$, we get $-13$. Thus:

$$-4uv - 9uv = -13uv$$

Add the coefficients and repeat the variable part.

Answer: $-11uv$

**EXAMPLE 12.** Simplify: $-3x^2y + 2xy^2$

Simplify: $5ab + 11bc$
Solution: These are not like terms. They do not have the same variable parts. They do have the same variables, but the variables are not raised to the same exponents. Consequently, this expression is already simplified as much as possible.

\[-3x^2y + 2xy^2\]  Unlike terms. Already simplified.

Answer: \(5ab + 11bc\)

Sometimes we have more than just a single pair of like terms. In that case, we want to group together the like terms and combine them.

**You Try It!**

**EXAMPLE 13.** Simplify: \(-8u - 4v - 12u + 9v\)

**Solution:** Use the associative and commutative property of addition to change the order and regroup, then combine line terms.

\[-8u - 4v - 12u + 9v = (-8u - 12u) + (-4v + 9v)\]  Reorder and regroup.

\[= -20u + 5v\]  Combine like terms.

Note that \(-8u - 12u = -20u\) and \(-4v + 9v = 5v\).

**Alternate solution.** You may skip the reordering and regrouping step if you wish, simply combining like terms mentally. That is, it is entirely possible to order your work as follows:

\[-8u - 4v - 12u + 9v = -20u + 5v\]  Combine like terms.

Answer: \(-11z^2 - 5z\)

In Example 13, the “Alternate solution” allows us to move more quickly and will be the technique we follow from here on, grouping and combining terms mentally.

**Order of Operations**

Now that we know how to combine like terms, let’s tackle some more complicated expressions that require the Rules Guiding Order of Operations.

**Rules Guiding Order of Operations.** When evaluating expressions, proceed in the following order.

1. Evaluate expressions contained in grouping symbols first. If grouping symbols are nested, evaluate the expression in the innermost pair of grouping symbols first.
2. Evaluate all exponents that appear in the expression.
3. Perform all multiplications and divisions in the order that they appear in the expression, moving left to right.
4. Perform all additions and subtractions in the order that they appear in the expression, moving left to right.

EXAMPLE 14. Simplify: \(4(-3a + 2b) - 3(4a - 5b)\)

**Solution:** Use the distributive property to distribute the 4 and the -3, then combine like terms.

\[
4(-3a + 2b) - 3(4a - 5b) = -12a + 8b - 12a + 15b = -24a + 23b
\]

Note that \(-12a - 12a = -24a\) and \(8b + 15b = 23b\).

Answer: \(4x - 15\)

EXAMPLE 15. Simplify: \(-2(3x - 4y) - (5x - 2y)\)

**Solution:** Use the distributive property to multiply -2 times \(3x - 4y\), then distribute the minus sign times each term of the expression \(5x - 2y\). After that, combine like terms.

\[
-2(3x - 4y) - (5x - 2y) = -6x + 8y - 5x + 2y = -11x + 10y
\]

Note that \(-6x - 5x = -11x\) and \(8y + 2y = 10y\).

Answer: \(-4u + 2v\)

EXAMPLE 16. Simplify: \(-2(x^2y - 3xy^2) - 4(-x^2y + 3xy^2)\)

**Solution:** Use the distributive property to multiply -2 times \(x^2y - 3xy^2\) and -4 times \(-x^2y + 3xy^2\). After that, combine like terms.

\[
-2(x^2y - 3xy^2) - 4(-x^2y + 3xy^2) = -2x^2y + 6xy^2 + 4x^2y - 12xy^2 = 2x^2y - 6xy^2
\]

Note that \(-2x^2y + 4x^2y = 2x^2y\) and \(6xy^2 - 12xy^2 = -6xy^2\).

Answer: \(5u^2v - 12uv^2\)
When grouping symbols are nested, evaluate the expression inside the innermost pair of grouping symbols first.

**You Try It!**

**EXAMPLE 17.** Simplify: \(-2x - 2(-2x - 2[-2x - 2])\)

Solution: Inside the parentheses, we have the expression \(-2x - 2[-2x - 2]\). The Rules Guiding Order of Operations dictate that we should multiply first, expanding \(-2[-2x - 2]\) and combining like terms.

\[
-2x - 2(-2x - 2[-2x - 2]) = -2x - 2(-2x + 4x + 2)
= -2x - 2(2x + 2)
\]

In the remaining expression, we again multiply first, expanding \(-2(2x + 2)\) and combining like terms.

\[
= -2x - 4x - 4
= -6x - 4
\]

Answer: \(-5x - 8\)
1.5. ALGEBRAIC EXPRESSIONS

Exercises

In Exercises 1-6, use the associative property of multiplication to simplify the expression. Note: You must show the regrouping step using the associative property on your homework.

1. \(-3(6a)\)  
2. \(-10(2y)\)  
3. \(-9(6ab)\)  
4. \(8(5xy)\)  
5. \(-7(3x^2)\)  
6. \(-6(8z)\)

In Exercises 7-18, use the distributive property to expand the given expression.

7. \(4(3x - 7y)\)  
8. \(-4(5a + 2b)\)  
9. \(-6(-y + 9)\)  
10. \(5(-9w + 6)\)  
11. \(-9(s + 9)\)  
12. \(6(-10y + 3)\)  
13. \(-(-3u - 6v + 8)\)  
14. \(-(3u - 3v - 9)\)  
15. \(-8(4u^2 - 6u^2)\)  
16. \(-5(8x - 9y)\)  
17. \(-(7u + 10v + 8)\)  
18. \(-(7u - 8v - 5)\)

In Exercises 19-26, combine like terms by first using the distributive property to factor out the common variable part, and then simplifying. Note: You must show the factoring step on your homework.

19. \(-19x + 17x - 17x\)  
20. \(11n - 3n - 18n\)  
21. \(14x^3 - 10x^3\)  
22. \(-11y^3 - 6y^3\)  
23. \(9y^2x + 13y^2x - 3y^2x\)  
24. \(4x^3 - 8x^3 + 16x^3\)  
25. \(15m + 14m\)  
26. \(19q + 5q\)

In Exercises 27-38, simplify each of the following expressions by rearranging and combining like terms mentally. Note: This means write down the problem, then write down the answer. No work.

27. \(9 - 17m - m + 7\)  
28. \(-11 + 20x + 16x - 14\)  
29. \(-6y^2 - 3x^3 + 4y^2 + 3x^3\)  
30. \(14y^3 - 11y^2x + 11y^3 + 10y^2x\)  
31. \(-5m - 16 + 5 - 20m\)  
32. \(-18q + 12 - 8 - 19q\)  
33. \(-16x^2y + 7y^3 - 12y^3 - 12x^2y\)  
34. \(10x^3 + 4y^3 - 13y^3 - 14x^3\)
CHAPTER 1. THE ARITHMETIC OF NUMBERS

35. \(-14r + 16 - 7r - 17\)
36. \(-9s - 5 - 10s + 15\)

37. \(14 - 16y - 10 - 13y\)
38. \(18 + 10x + 3 - 18x\)

In Exercises 39-58, use the distributive property to expand the expression, then combine like terms mentally.

39. \(3 - (5y + 1)\)
40. \(5 - (10q + 3)\)
41. \(-(9y^2 + 2x^2) - 8(5y^2 - 6x^2)\)
42. \(-8(-8y^2 + 4x^3) - 7(3y^2 + x^3)\)
43. \(2(10 - 6p) + 10(-2p + 5)\)
44. \(2(3 - 7x) + (7x + 9)\)
45. \(4(-10n + 5) - 7(7n - 9)\)
46. \(3(-9n + 10) + 6(-7n + 8)\)
47. \(-4x - 4 - (10x - 5)\)
48. \(8y + 9 - (-8y + 8)\)

49. \(-7 - (5 + 3x)\)
50. \(10 - (6 - 4m)\)
51. \(-8(-5y - 8) - 7(-2 + 9y)\)
52. \(6(-3s + 7) - (4 - 2s)\)
53. \(4(-7y^2 - 9x^2) - 6(-5x^2y - 5y^2)\)
54. \(-6(x^3 + 3y^2x) + 8(-y^2x - 9x^3)\)
55. \(6s - 7 - (2 - 4s)\)
56. \(4x - 9 - (-6 + 5x)\)
57. \(9(9 - 10r) + (-8 - 2r)\)
58. \(-7(6 + 2p) + 5(5 - 5p)\)

In Exercises 59-64, use the distributive property to simplify the given expression.

59. \(-7x + 7(2x - 5[8x + 5])\)
60. \(-9x + 2(5x + 6[-8x - 3])\)
61. \(6x - 4(-3x + 2[5x - 7])\)

62. \(2x + 4(5x - 7[8x + 9])\)
63. \(-8x - 5(2x - 3[-4x + 9])\)
64. \(8x + 6(3x + 7[-9x + 5])\)

<table>
<thead>
<tr>
<th>Answers</th>
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<tbody>
<tr>
<td>1. (-18a)</td>
</tr>
<tr>
<td>3. (-54ab)</td>
</tr>
<tr>
<td>5. (-21x^2)</td>
</tr>
<tr>
<td>7. (12x - 28y)</td>
</tr>
<tr>
<td>9. (6y - 54)</td>
</tr>
</tbody>
</table>
21. $4x^3$
23. $10y^2x$
25. $29m$
27. $16 - 18m$
29. $-2y^2$
31. $-25m - 11$
33. $-28x^2y - 5y^3$
35. $-21r - 1$
37. $4 - 29y$
39. $2 + 5y$
41. $-49y^2 + 46x^2$
43. $70 - 32p$
45. $-89n + 83$
47. $-14x + 1$
49. $-12 - 3x$
51. $-23y + 78$
53. $2y^2 - 6x^2y$
55. $10s - 9$
57. $73 - 92r$
59. $-273x - 175$
61. $-22x + 56$
63. $-78x + 135$
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