Elementary Algebra Textbook

Second Edition

Chapter 7

Department of Mathematics
College of the Redwoods

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Chapter 7

Rational Expressions

Our ability to communicate and record ideas is enhanced by concise mathematical language and notation. More importantly, our ability to conceive new ideas is broadened by notation that the mind can effectively use to organize and recognize patterns. Exponential notation is a simple example of such notation that expands our thinking. The modern notation we use started seriously in 1636 and 1637 with James Humes and Rene Descartes, after centuries of mathematicians flirting with various approaches. Descartes may have been the first to use today’s radical symbol, combining the check symbol with the bar over the top in 1637. Scientific notation, as we define it today, is a relatively new notation, first appearing in the mid-1900’s. One may wonder where the next significant notational breakthrough will originate. Maybe it will be your contribution.
CHAPTER 7. RATIONAL EXPRESSIONS

7.1 Negative Exponents

We begin with a seemingly silly but powerful definition on what it means to raise a number to a power of \(-1\).

**Raising to a Power of \(-1\).** To raise an object to a power of \(-1\), simply invert the object (turn it upside down).

More formally, inverting a number is known as taking its *reciprocal*.

**You Try It!**

**EXAMPLE 1.** Simplify each of the following expressions:

a) \(4^{-1}\)  
b) \(\left(\frac{2}{3}\right)^{-1}\)  
c) \(\left(-\frac{3}{5}\right)^{-1}\)

**Solution:** In each case, we simply invert the given number.

a) \(4^{-1} = \frac{1}{4}\)  
b) \(\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}\)  
c) \(\left(-\frac{3}{5}\right)^{-1} = -\frac{5}{3}\)

**Solution:** In each case, we simply invert the given number.

a) \(4^{-1} = \frac{1}{4}\)

b) \((-5)^{-1} = -\frac{1}{5}\)

c) \(\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}\)

d) \(\left(-\frac{3}{5}\right)^{-1} = -\frac{5}{3}\)

Answer: \(\frac{4}{7}\)
7.1. NEGATIVE EXPONENTS

You might be asking “Why does raising to the power of minus one invert the number?” To answer this question, recall the product of a number and its reciprocal is one. For example,

\[ 4 \cdot \frac{1}{4} = 1. \] (7.1)

Next, consider what happens when we multiply \( 4^1 \) and \( 4^{-1} \). If we apply the usual law of exponents (assuming they work for both positive and negative exponents), we would add the exponents \((1 + (-1)) = 0\).

\[ 4^1 \cdot 4^{-1} = 4^0 \] (7.2)

However, because \( 4^1 = 4 \) and \( 4^0 = 1 \), this last equation is equivalent to:

\[ 4 \cdot 4^{-1} = 1 \] (7.3)

When you compare Equation 7.1 and 7.3, it is clear that \( 4^{-1} \) and \( \frac{1}{4} \) are both reciprocals of the number 4. Because reciprocals are unique, \( 4^{-1} = \frac{1}{4} \).

In similar fashion, one can discover the meaning of \( a^{-n} \). Start with the fact that multiplying reciprocals yields an answer of one.

\[ a^n \cdot \frac{1}{a^n} = 1 \] (7.4)

If we multiply \( a^n \) and \( a^{-n} \), we add the exponents as follows.

\[ a^n \cdot a^{-n} = a^0 \]

Providing \( a \neq 0 \), then \( a^0 = 1 \), so we can write

\[ a^n \cdot a^{-n} = 1 \] (7.5)

Comparing Equations 7.4 and 7.5, we note that both \( 1/a^n \) and \( a^{-n} \) are reciprocals of \( a^n \). Because every number has a unique reciprocal, \( a^{-n} \) and \( 1/a^n \) are equal.

**Raising to a negative integer.** Provided \( a \neq 0 \),

\[ a^{-n} = \frac{1}{a^n}. \]
EXAMPLE 2. Simplify each of the following expressions:

a) \(2^{-3}\)  
b) \((-5)^{-2}\)  
c) \((-4)^{-3}\)

Solution: In each example, we use the property \(a^{-n} = 1/a^n\) to simplify the given expression.

\[
a) \quad 2^{-3} = \frac{1}{2^3} = \frac{1}{8} \\
b) \quad (-5)^{-2} = \frac{1}{(-5)^2} = \frac{1}{25} \\
c) \quad (-4)^{-3} = \frac{1}{(-4)^3} = -\frac{1}{64}
\]

Answer: \(\frac{1}{9}\)

In Raising to a Negative Integer on page 482, we’ll address how you can perform each of the above computations mentally.

Laws of Exponents

In the arguments demonstrating that \(4^{-1} = 1/4\) and \(a^{-n} = 1/a^n\), we appealed to one of the laws of exponents learned in Chapter 5, Section 5. Fortunately, the laws of exponents work exactly the same whether the exponents are positive or negative integers.

Laws of Exponents. If \(m\) and \(n\) are integers, then:

1. \(a^m a^n = a^{m+n}\)
2. \(a^m / a^n = a^{m-n}\)
3. \((a^m)^n = a^{mn}\)
4. \((ab)^n = a^n b^n\)
5. \((a/b)^n = a^n / b^n\)

EXAMPLE 3. Simplify each of the following expressions:

a) \(y^5 y^{-7}\)  
b) \(2^{-2} \cdot 2^{-3}\)  
c) \(x^{-4} x^6\)

Solution: In each case, we use the first law of exponents \((a^m a^n = a^{m+n})\). Because we are multiplying like bases, we repeat the base and add the exponents.
7.1. **NEGATIVE EXPONENTS**

a) \( y^5 y^{-7} = y^{5+(-7)} = y^{-2} \)
b) \( 2^{-2} \cdot 2^{-3} = 2^{-2+(-3)} = 2^{-5} \)
c) \( x^{-4} x^6 = x^{-4+6} = x^2 \)

**Answer:** \( t^4 \)

---

**EXAMPLE 4.** Simplify each of the following expressions:

\[
\begin{align*}
\text{a) } \frac{x^4}{x^7} & = x^{4-7} = x^{-3} \\
\text{b) } \frac{3^{-4}}{3^5} & = 3^{-4-5} = 3^{-9} \\
\text{c) } \frac{z^{-3}}{z^{-5}} & = z^{-3-(-5)} = z^{-3+5} = z^2
\end{align*}
\]

**Solution:** In each case, we use the second law of exponents \( (a^m/a^n = a^{m-n}) \). Because we are dividing like bases, we repeat the base and subtract the exponents. Recall that subtraction means “add the opposite.”

\[
\begin{align*}
\text{a) } \frac{x^4}{x^7} & = x^{4-7} = x^{-3} \\
\text{b) } \frac{3^{-4}}{3^5} & = 3^{-4-5} = 3^{-9} \\
\text{c) } \frac{z^{-3}}{z^{-5}} & = z^{-3-(-5)} = z^{-3+5} = z^2
\end{align*}
\]

**Answer:** \( y^{-4} \)

---

**EXAMPLE 5.** Simplify each of the following expressions:

\[
\begin{align*}
\text{a) } (5^{-2})^3 & = 5^{(-2)(3)} = 5^{-6} \\
\text{b) } (a^{-3})^{-4} & = a^{(-3)(-4)} = a^{12} \\
\text{c) } (w^2)^{-7} & = w^{2(-7)} = w^{-14}
\end{align*}
\]

**Solution:** In each case, we are using the third law of exponents \( ((a^m)^n = a^{mn}) \). Because we are raising a power to another power, we repeat the base and multiply the exponents.

\[
\begin{align*}
\text{a) } (5^{-2})^3 & = 5^{(-2)(3)} = 5^{-6} \\
\text{b) } (a^{-3})^{-4} & = a^{(-3)(-4)} = a^{12} \\
\text{c) } (w^2)^{-7} & = w^{2(-7)} = w^{-14}
\end{align*}
\]

**Answer:** \( z^{-10} \)

---

**You Try It!**
Raising to a Negative Integer

We know what happens when you raise a number to \(-1\), you invert the number or turn it upside down. But what happens when you raise a number to a negative integer other than negative one?

As an example, consider the expression \(3^{-2}\). Using the third law of exponents \(((a^m)^n = a^{mn})\), we can write this expression in two equivalent forms.

1. Note that \(3^{-2}\) is equivalent to \((3^2)^{-1}\). They are equivalent because the third law of exponents instructs us to multiply the exponents when raising a power to another power. Finally, note that to evaluate \((3^2)^{-1}\), we first square, then invert the result.

\[
3^{-2} = (3^2)^{-1} \quad \text{Repeat base and multiply exponents.} \\
= 9^{-1} \quad \text{Simplify: } 3^2 = 9. \\
= \frac{1}{9} \quad \text{Simplify: } 9^{-1} = 1/9.
\]

2. Note that \(3^{-2}\) is also equivalent to \((3^{-1})^2\). They are equivalent because the third law of exponents instructs us to multiply the exponents when raising a power to another power. Finally, note that to evaluate \((3^{-1})^2\), we first invert, then square the result.

\[
3^{-2} = (3^{-1})^2 \quad \text{Repeat base and multiply exponents.} \\
= \left(\frac{1}{3}\right)^2 \quad \text{Simplify: } 3^{-1} = 1/3. \\
= \frac{1}{9} \quad \text{Simplify: } (1/3)^2 = 1/9.
\]

Using either technique, \(3^{-2} = 1/9\). You can either square and invert, or you can invert and square. In each case, the 2 means “square” and the minus sign means “invert,” and this example shows that it doesn’t matter which you do first.

You Try It!

EXAMPLE 6. Simplify each of the following expressions:

a) \(5^{-3}\)  
b) \((-4)^{-2}\)  
c) \(\left(\frac{3}{5}\right)^{-2}\)  
d) \(\left(-\frac{2}{3}\right)^{-3}\)

Solution:

a) We’ll cube then invert.

\[
5^{-3} = (5^3)^{-1} \quad \text{Repeat base, multiply exponents.} \\
= 125^{-1} \quad \text{Simplify: } 5^3 = 125. \\
= \frac{1}{125} \quad \text{Invert: } 125^{-1} = 1/125.
\]
7.1. **NEGATIVE EXPONENTS**

Note that the three means “cube” and the minus sign means “invert,” so it is possible to do all of this work mentally: cube 5 to get 125, then invert to get 1/125.

b) We’ll square then invert.

\[
(-4)^{-2} = ((-4)^2)^{-1} \quad \text{Repeat base, multiply exponents.}
\]

\[
= 16^{-1} \quad \text{Simplify: } (-4)^2 = 16.
\]

\[
= \frac{1}{16} \quad \text{Invert: } 16^{-1} = 1/16.
\]

Note that the two means “square” and the minus sign means “invert,” so it is possible to do all of this work mentally: square −4 to get 16, then invert to get 1/16.

c) Again, we’ll square then invert.

\[
\left(\frac{3}{5}\right)^{-2} = \left(\left(\frac{3}{5}\right)^2\right)^{-1} \quad \text{Repeat base, multiply exponents.}
\]

\[
= \left(\frac{9}{25}\right)^{-1} \quad \text{Simplify: } (3/5)^2 = 9/25.
\]

\[
= \frac{25}{9} \quad \text{Invert: } (9/25)^{-1} = 25/9.
\]

Note that the two means “square” and the minus sign means “invert,” so it is possible to do all of this work mentally: square 3/5 to get 9/25, then invert to get 25/9.

d) This time we’ll cube then invert.

\[
\left(-\frac{2}{3}\right)^{-3} = \left(\left(-\frac{2}{3}\right)^3\right)^{-1} \quad \text{Repeat base, multiply exponents.}
\]

\[
= \left(-\frac{8}{27}\right)^{-1} \quad \text{Simplify: } (-2/3)^3 = -8/27.
\]

\[
= -\frac{27}{8} \quad \text{Invert: } (-8/27)^{-1} = -27/8.
\]

Note that the three means “cube” and the minus sign means “invert,” so it is possible to do all of this work mentally: cube −2/3 to get −8/27, then invert to get −27/8.

**Answer:** \(\frac{64}{125}\)
Applying the Laws of Exponents

In this section we’ll simplify a few more complicated expressions using the laws of exponents.

EXAMPLE 7. Simplify: \((2x^{-2}y^3)(-3x^5y^{-6})\)

Solution: All the operators involved are multiplication, so the commutative and associative properties of multiplication allow us to change the order and grouping. We’ll show this regrouping here, but this step can be done mentally.

\[(2x^{-2}y^3)(-3x^5y^{-6}) = [(2)(-3)](x^{-2}x^5)(y^3y^{-6})\]

When multiplying, we repeat the base and add the exponents.

\[= -6x^{-2+5}y^{3+(-6)}\]
\[= -6x^3y^{-3}\]

In the solution above, we’ve probably shown way too much work. It’s far easier to perform all of these steps mentally, multiplying the 2 and the \(-3\), then repeating bases and adding exponents, as in:

\[(2x^{-2}y^3)(-3x^5y^{-6}) = -6x^3y^{-3}\]

Answer: \(10x^2y^{-3}\)

EXAMPLE 8. Simplify: \(\frac{6x^{-2}y^5}{9x^3y^{-2}}\)

Solution: The simplest approach is to first write the expression as a product.

\[\frac{6x^{-2}y^5}{9x^3y^{-2}} = \frac{6}{9} \cdot \frac{x^{-2}}{x^3} \cdot \frac{y^5}{y^{-2}}\]

Reduce \(6/9\) to lowest terms. Because we are dividing like bases, we repeat the base and subtract the exponents.

\[= \frac{2}{3}x^{-2-3}y^{5-(-2)}\]
\[= \frac{2}{3}x^{-5}y^7\]
7.1. NEGATIVE EXPONENTS

In the solution above, we’ve probably shown way too much work. It’s far easier to imagine writing the expression as a product, reducing 6/9, then repeating bases and subtracting exponents, as in:

\[
\frac{6x^{-2}y^5}{9x^3y^{-2}} = \frac{2}{3}x^{-5}y^7
\]

Answer: \(\frac{5}{2}x^5y^{-6}\)

EXAMPLE 9. Simplify: \((2x^{-2}y^4)^{-3}\)

**Solution:** The fourth law of exponents \(((ab)^n = a^n b^n)\) says that when you raise a product to a power, you must raise each factor to that power. So we begin by raising each factor to the minus three power.

\[(2x^{-2}y^4)^{-3} = 2^{-3}(x^{-2})^{-3}(y^4)^{-3}\]

To raise two to the minus three, we must cube two and invert: \(2^{-3} = 1/8\). Secondly, raising a power to a power requires that we repeat the base and multiply exponents.

\[= \frac{1}{8}x^{(-2)(-3)}y^{(4)(-3)}\]

\[= \frac{1}{8}x^6y^{-12}\]

In the solution above, we’ve probably shown way too much work. It’s far easier to imagine writing the expression as a product, reducing 2/8, then repeating bases and subtracting exponents, as in

\[(2x^{-2}y^4)^{-3} = \frac{1}{8}x^6y^{-12}\]

Answer: \(\frac{1}{9}e^{-8}y^6\)

Clearing Negative Exponents

Often, we’re asked to provide a final answer that is free of negative exponents. It is common to hear the instruction “no negative exponents in the final answer.” Let’s explore a couple of techniques that allow us to clear our answer of negative exponents.
EXAMPLE 10. Consider the expression:

\[
\frac{x^2}{y^{-3}}
\]

Simplify so that the resulting equivalent expression contains no negative exponents.

Solution: Raising \( y \) to the \(-3\) means we have to cube and invert, so \( y^{-3} = 1/y^3 \).

\[
\frac{x^2}{y^{-3}} = \frac{x^2}{1/y^3}
\]

To divide \( x^2 \) by \( 1/y^3 \), we invert and multiply.

\[
= x^2 \div \frac{1}{y^3}
= \frac{x^2}{1} \cdot \frac{y^3}{1}
= x^2y^3
\]

Alternate approach: An alternate approach takes advantage of the laws of exponents. We begin by multiplying numerator and denominator by \( y^3 \).

\[
\frac{x^2}{y^{-3}} = \frac{x^2}{y^{-3}} \cdot \frac{y^3}{y^3}
= \frac{x^2y^3}{y^0}
= x^2y^3
\]

Answer: \( y^5x^2 \)

In the last step, note how we used the fact that \( y^0 = 1 \).

EXAMPLE 11. Consider the expression:

\[
\frac{x^{-3}y^2}{3z^{-4}}
\]

Simplify so that the resulting equivalent expression contains no negative exponents.

\[
\frac{2x^2y^{-2}}{z^3}
\]
Solution: Again, we can remove all the negative exponents by taking reciprocals. In this case \( y^{-2} = 1/y^2 \) (square and invert).

\[
\frac{2x^2 y^{-2}}{z^3} = \frac{2x^2 \cdot 1}{y^2 z^3}
\]

\[
= \frac{2x^2}{y^2 z^3}
\]

To divide \( 2x^2/y^2 \) by \( z^3 \), we invert and multiply.

\[
= \frac{2x^2}{y^2} \div z^3
\]

\[
= \frac{2x^2}{y^2} \cdot \frac{1}{z^3}
\]

\[
= \frac{2x^2}{y^2 z^3}
\]

Alternate approach: An alternate approach again takes advantage of the laws of exponents. We begin by multiplying numerator and denominator by \( y^2 \).

\[
\frac{2x^2 y^{-2}}{z^3} = \frac{2x^2 y^{-2}}{z^3} \cdot \frac{y^2}{y^2}
\]

\[
= \frac{2x^2 y^0}{y^2 z^3}
\]

\[
= \frac{2x^2}{y^2 z^3}
\]

In the last step, note how we used the fact that \( y^0 = 1 \).

Answer: \( \frac{y^2 z^4}{3x^3} \)
### Exercises

In Exercises 1-8, simplify the given expression.

1. \( \left( \frac{1}{7} \right)^{-1} \)  
2. \( \left( \frac{-3}{5} \right)^{-1} \)  
3. \( \left( \frac{-8}{9} \right)^{-1} \)  
4. \( \left( \frac{-3}{2} \right)^{-1} \)  
5. \( (18)^{-1} \)  
6. \( (-11)^{-1} \)  
7. \( (16)^{-1} \)  
8. \( (7)^{-1} \)  

In Exercises 9-16, simplify the given expression.

9. \( a^{-9}a^3 \)  
10. \( x^{-5}x^{-5} \)  
11. \( b^{-9}b^8 \)  
12. \( v^{-7}v^{-2} \)  
13. \( 2^9 \cdot 2^{-4} \)  
14. \( 2^2 \cdot 2^{-7} \)  
15. \( 9^6 \cdot 9^{-5} \)  
16. \( 9^7 \cdot 9^{-5} \)  

In Exercises 17-24, simplify the given expression.

17. \( \frac{2^6}{2^{-8}} \)  
18. \( \frac{6^8}{6^{-1}} \)  
19. \( \frac{z^{-1}}{z^9} \)  
20. \( \frac{w^{-4}}{w^3} \)  
21. \( \frac{w^{-9}}{w^{-1}} \)  
22. \( \frac{r^5}{r^{-1}} \)  
23. \( \frac{7^{-3}}{7^{-1}} \)  
24. \( \frac{6^{-8}}{6^9} \)  

In Exercises 25-32, simplify the given expression.

25. \( (t^{-1})^4 \)  
26. \( (a^8)^{-7} \)  
27. \( (6^{-6})^7 \)  
28. \( (2^{-7})^{-7} \)
7.1. NEGATIVE EXPONENTS

29. \((z^{-9})^{-9}\) \hspace{1cm} 31. \((3^{-2})^3\)
30. \((e^6)^{-2}\) \hspace{1cm} 32. \((8^{-1})^6\)

In Exercises 37–36, simplify the given expression.

33. \(4^{-3}\) \hspace{1cm} 38. \(\left(\frac{1}{3}\right)^{-3}\)
34. \(5^{-2}\) \hspace{1cm} 39. \(\left(\frac{1}{2}\right)^{-5}\)
35. \(2^{-4}\) \hspace{1cm} 40. \(\left(\frac{1}{2}\right)^{-4}\)
36. \((-3)^{-4}\)
37. \(\left(\frac{1}{2}\right)^{-5}\)

In Exercises 41–56, simplify the given expression.

41. \((4u^{-6}v^{-9}) (5u^8v^{-8})\)
42. \((6a^{-9}c^{-6}) (-8a^8c^5)\)
43. \((6x^{-6}y^{-5}) (-4x^4y^{-2})\)
44. \((5v^{-3}w^{-8}) (8v^{-9}w^5)\)
45. \(-6x^7z^9\)
46. \(2u^{-2}v^6\)
47. \(-6a^9c^6\)
48. \(-4u^{-4}w^4\)
49. \((2v^{-2}w^4)^{-5}\)
50. \((3s^{-6}t^5)^{-4}\)
51. \((3x^{-1}y^7)^4\)
52. \((-4b^{-8}c^{-4})^3\)
53. \((2x^6z^{-7})^5\)
54. \((-4v^4w^{-9})^3\)
55. \((2a^{-4}c^8)^{-4}\)
56. \((11b^9c^{-1})^{-2}\)

In Exercises 57–76, clear all negative exponents from the given expression.

57. \(\frac{x^3y^{-2}}{z^3}\)
58. \(\frac{x^4y^{-9}}{z^7}\)
59. \(\frac{r^0s^{-2}}{t^3}\)
60. \(\frac{u^5v^{-3}}{w^2}\)
61. \(\frac{x^3}{y^{-8}z^5}\)
62. \(\frac{x^9}{y^{-4}z^3}\)
63. \( \frac{u^9}{v^{-4}w^1} \)
64. \( \frac{a^{7}}{b^{8}c^{6}} \)
65. \( (7x^{-1})(-7x^{-1}) \)
66. \( (3a^{-8})(-7a^{-7}) \)
67. \( (8a^{-8})(7a^{-7}) \)
68. \( (-7u^{3})(-8u^{-6}) \)
69. \( \frac{4x^{-9}}{8u^{3}} \)
70. \( \frac{2t^{-8}}{-6t^{9}} \)
71. \( \frac{6x^{2}}{-4c^{3}} \)
72. \( \frac{6v^{-9}}{-8v^{-4}} \)
73. \( (-3s^{9})^{-4} \)
74. \( (-3s^{8})^{-4} \)
75. \( (2y^{4})^{-5} \)
76. \( (2u^{4})^{-5} \)

### Answers

1. 7
2. \( 9 \)
3. \( \frac{9}{8} \)
4. \( \frac{1}{18} \)
5. \( \frac{1}{16} \)
6. \( a^{-6} \)
7. \( b^{-1} \)
8. \( 2^{5} \)
9. \( 9^{-11} \)
10. \( 2^{14} \)
11. \( z^{-10} \)
12. \( w^{-16} \)
13. \( 7^{-2} \)
14. \( t^{-4} \)
15. \( 2^{14} \)
16. \( \frac{1}{32} \)
17. \( v^{10}w^{-20} \)
18. \( 81x^{-4}y^{28} \)
19. \( 32x^{30}z^{-35} \)
7.1. NEGATIVE EXPONENTS

55. \( \frac{1}{16} a^{16} e^{-32} \)

57. \( \frac{x^5}{y^2 z^3} \)

59. \( \frac{r^9}{s^2 t^3} \)

61. \( \frac{x^3 y^8}{z^5} \)

63. \( \frac{u^3 v^4}{w^7} \)

65. \( \frac{-49}{x^2} \)

67. \( \frac{56}{a^{16}} \)

69. \( \frac{1}{2x^{12}} \)

71. \( \frac{-3}{2c^5} \)

73. \( \frac{1}{81s^{36}} \)

75. \( \frac{1}{32y^{20}} \)
7.2 Scientific Notation

We begin this section by examining powers of ten.

\[10^1 = 10\]
\[10^2 = 10 \cdot 10 = 100\]
\[10^3 = 10 \cdot 10 \cdot 10 = 1,000\]
\[10^4 = 10 \cdot 10 \cdot 10 \cdot 10 = 10,000\]

Note that the answer for \(10^3\) is a one followed by three zeros. The answer for \(10^4\) is a one followed by four zeros. Do you see the pattern?

**Nonnegative powers of ten.** In the expression \(10^n\), the exponent matches the number of zeros in the answer. Hence, \(10^n\) will be a 1 followed by \(n\) zeros.

**You Try It!**

**EXAMPLE 1.** Simplify: \(10^9\).

**Solution.** \(10^9\) should be a 1 followed by 9 zeros.

\[10^9 = 1,000,000,000\]

Answer: 1,000,000

Next, let’s examine negative powers of ten.

\[10^{-1} = \frac{1}{10} = 0.1\]
\[10^{-2} = \frac{1}{100} = 0.01\]
\[10^{-3} = \frac{1}{1000} = 0.001\]
\[10^{-4} = \frac{1}{10000} = 0.0001\]

Note that the answer for \(10^{-3}\) has three decimal places and the answer for \(10^{-4}\) contains four decimal places.

**Negative powers of ten.** In the expression \(10^{-n}\), the exponent matches the number of decimal places in the answer. Hence, \(10^{-n}\) will have \(n\) decimal places, the first \(n - 1\) of which are zeros and the digit in the \(n\)th decimal place is a 1.
EXAMPLE 2. Simplify: $10^{-7}$.

**Solution.** $10^{-7}$ should have seven decimal places, the first six of which are zeros, and the digit in the seventh decimal place is a 1.

$$10^{-7} = 0.000001$$

Answer: $0.00001$

---

### Multiplying Decimal Numbers by Powers of Ten

Let’s multiply $1.234567$ by $10^3$, or equivalently, by 1,000.

$$
\begin{array}{c}
1.234567 \\
\times 1000 \\
\hline
1234.567000
\end{array}
$$

The sum total of digits to the right of the decimal point in $1.234567$ and $1000$ is 6. Therefore, we place the decimal point in the product so that there are six digits to the right of the decimal point.

However, the trailing zeros may be removed without changing the value of the product. That is, $1.234567$ times 1000 is 1234.567. Note that the decimal point in the product is three places further to the right than in the original factor. This observation leads to the following result.

**Multiplying by a nonnegative power of ten.** Multiplying a decimal number by $10^n$, where $n = 0, 1, 2, 3, \ldots$, will move the decimal point $n$ places to the right.

---


**Solution.** Multiplying by $10^2$ will move the decimal point two places to the right. Thus:

$$325.6783 \times 10^2 = 32,567.83$$

Answer: $23,578.89$

---

EXAMPLE 4. Simplify: $1.25 \times 10^5$.

**Simplify:** $2.35 \times 10^4$
CHAPTER 7. RATIONAL EXPRESSIONS

Solution. Multiplying by $10^5$ will move the decimal point two places to the right. In this case, we need to add zeros at the end of the number to accomplish moving the decimal 5 places to the right.

$$1.25 \times 10^5 = 125,000$$

Answer: 23,500

Let’s multiply 453.9 by $10^{-2}$, or equivalently, by 0.01.

$$
\begin{array}{c}
453.9 \\
\times 0.01 \\
\hline
4.539
\end{array}
$$

The sum total of digits to the right of the decimal point in 453.9 and 0.01 is 3. Therefore, we place the decimal point in the product so that there are 3 digits to the right of the decimal point. That is, $453.9 \times 10^{-2} = 4.539$. Note that the decimal point in the product is two places further to the left than in the original factor. This observation leads to the following result.

Multiplying by a negative power of ten. Multiplying a decimal number by $10^{-n}$, where $n = 1, 2, 3, \ldots$, will move the decimal point $n$ places to the left.

You Try It!

EXAMPLE 5. Simplify: $14,567.8 \times 10^{-3}$.

Solution. Multiplying by $10^{-3}$ will move the decimal point three places to the left. Thus:

$$14,567.8 \times 10^{-3} = 14.5678$$

Answer: 385.42

You Try It!


Solution. Multiplying by $10^{-4}$ will move the decimal point four places to the left. In this case, we need to add some leading zeros at the beginning of the number to accomplish moving the decimal 4 places to the left.

$$4.3 \times 10^{-4} = 0.00043$$

Note also the leading zero before the decimal point. Although .00043 is an equivalent number, the form 0.00043 is preferred in mathematics and science.
7.2. SCIENTIFIC NOTATION

Scientific Notation Form

We start by defining the form of a number called *scientific notation*.

**Scientific notation.** A number having the form
\[ a \times 10^b, \]
where \( b \) is an integer and \( 1 \leq |a| < 10 \), is said to be in *scientific notation*.

The requirement \( 1 \leq |a| < 10 \) says that the magnitude of \( a \) must be at least 1 and less than 10.

- The number \( 12.34 \times 10^{-4} \) is **not** in scientific notation because \( |12.34| = 12.34 \) is larger than 10.
- The number \( -0.95 \times 10^3 \) is **not** in scientific notation because \( |-0.95| = 0.95 \) is less than 1.
- The number \( 7.58 \times 10^{-12} \) is in scientific notation because \( |7.58| = 7.58 \) is greater than or equal to 1 and less than 10.
- The number \( -1.0 \times 10^{15} \) is in scientific notation because \( |-1.0| = 1.0 \) is greater than or equal to 1 and less than 10.

After contemplating these examples, it follows that a number in scientific notation should have exactly one of the digits 1, 2, 3, \ldots, 9 before the decimal point. Exactly one, no more, no less. Thus, each of the following numbers is in scientific notation.

\[ 4.7 \times 10^8, \quad -3.764 \times 10^{-1}, \quad 3.2 \times 10^0, \quad \text{and} \quad -1.25 \times 10^{-22} \]

**Placing a Number in Scientific Notation**

To place a number into scientific notation, we need to move the decimal point so that exactly one of the digits 1, 2, 3, \ldots, 9 remains to the left of the decimal point, then multiply by the appropriate power of 10 so that the result is equivalent to the original number.

**EXAMPLE 7.** Place the number 1,234 in scientific notation.

**Solution.** Move the decimal point three places to the left so that it is positioned just after the 1. To make this new number equal to 1,234, multiply by \( 10^3 \). Thus:

\[ 1,234 = 1.234 \times 10^3 \]
Check: Multiplying by $10^3$ moves the decimal three places to the right, so:

$$1.234 \times 10^3 = 1,234$$

Answer: $5.4321 \times 10^4$

This is the original number, so our scientific notation form is correct.

---

**EXAMPLE 8.** Place the number 0.000025 in scientific notation.

**Solution.** Move the decimal point five places to the right so that it is positioned just after the 2. To make this new number equal to 0.000025, multiply by $10^{-5}$. Thus:

$$0.000025 = 2.5 \times 10^{-5}$$

Check: Multiplying by $10^{-5}$ moves the decimal five places to the left, so:

$$2.5 \times 10^{-5} = 0.000025$$

Answer: $1.75 \times 10^{-2}$

This is the original number, so our scientific notation form is correct.

---

**EXAMPLE 9.** Place the number $34.5 \times 10^{-11}$ in scientific notation.

**Solution.** First, move the decimal point one place to the left so that it is positioned just after the three. To make this new form equal to 34.5, multiply by $10^1$.

$$34.5 \times 10^{-11} = 3.45 \times 10^1 \times 10^{-11}$$

Now, repeat the base 10 and add the exponents.

$$= 3.45 \times 10^{-10}$$

Answer: $7.5698 \times 10^{-3}$

---

**EXAMPLE 10.** Place the number $0.00093 \times 10^{12}$ in scientific notation.

**Solution.** First, move the decimal point four places to the right so that it is positioned just after the nine. To make this new form equal to 0.00093, multiply by $10^{-4}$.

$$0.00093 \times 10^{12} = 9.3 \times 10^{-4} \times 10^{12}$$
Now, repeat the base 10 and add the exponents.

\[ = 9.3 \times 10^8 \]

**Scientific Notation and the Graphing Calculator**

The TI-84 graphing calculator has a special button for entering numbers in scientific notation. Locate the “comma” key just about the number 7 key on the calculator’s keyboard (see Figure 7.1). Just above the “comma” key, printed on the calculator’s case is the symbol EE. It’s in the same color as the 2nd key, so you’ll have to use the 2nd key to access this symbol.

![Figure 7.1: Locate the EE label above the “comma” key on the keyboard.](image)

We know that \(2.3 \times 10^2 = 230\). Let’s see if the calculator gives the same interpretation.

1. Enter 2.3.

2. Press the 2nd key, then the comma key. This will put E on the calculator view screen.

3. Enter a 2.

4. Press ENTER.

The result of these steps is shown in the first image in Figure 7.2. Note that the calculator interprets \(2.3 \text{E}2\) as \(2.3 \times 10^2\) and gives the correct answer, 230. You can continue entering numbers in scientific notation (see the middle image in Figure 7.2). However, at some point the numbers become too large and the calculator responds by outputting the numbers in scientific notation. You can also force your calculator to display numbers in scientific notation in all situations, by first pressing the MODE key, then selecting SCI on the first
CHAPTER 7. RATIONAL EXPRESSIONS

Figure 7.2: Entering numbers in scientific notation.

line and pressing the ENTER key (see the third image in Figure 7.2). You can return your calculator to “normal” mode by selecting NORMAL and pressing the ENTER key.

You Try It!

EXAMPLE 11. Use the graphing calculator to simplify:

\[(3.42 \times 10^6)(5.86 \times 10^{-9})\]

Solution. First, note that we can approximate \((2.35 \times 10^{-12})(3.25 \times 10^{-4})\) by taking the product of 2 and 3 and adding the powers of ten.

\[
(2.35 \times 10^{-12})(3.25 \times 10^{-4}) \\
\approx (2 \times 10^{-12})(3 \times 10^{-4}) \\
\approx 6 \times 10^{-16}
\]

Approximate: 2.35 \approx 2 and 3.25 \approx 3.

The graphing calculator will provide an accurate answer. Enter 2.35E-12, press the “times” button, then enter 3.25E-4 and press the ENTER button. Be sure to use the “negate” button and not the “subtract” button to produce the minus sign. The result is shown in Figure 7.3.

Figure 7.3: Computing \((2.35 \times 10^{-12})(3.25 \times 10^{-4})\).

Answer: \(2.00412 \times 10^{-2}\)

Thus, \((2.35 \times 10^{-12})(3.25 \times 10^{-4}) = 7.6375 \times 10^{-16}\). Note that this is fairly close to our estimate of \(6 \times 10^{-16}\).
7.2. SCIENTIFIC NOTATION

**Reporting your answer on your homework.** After computing the answer to Example 11 on your calculator, write the following on your homework:

\[(2.35 \times 10^{-12})(3.25 \times 10^{-4}) = 7.6375 \times 10^{-16}\]

Do **not** write 7.6375E-16.

---

**EXAMPLE 12.** Use the graphing calculator to simplify:

\[
\frac{6.1 \times 10^{-3}}{(2.7 \times 10^{4})(1.1 \times 10^{8})}
\]

**Solution.** Again, it is not difficult to produce an approximate answer.

\[
\begin{align*}
\frac{6.1 \times 10^{-3}}{(2.7 \times 10^{4})(1.1 \times 10^{8})} & \approx \frac{6 \times 10^{-3}}{(3 \times 10^{4})(1 \times 10^{8})} \\
& \approx \frac{6 \times 10^{-3}}{3 \times 10^{12}} \\
& \approx 2 \times 10^{-15}
\end{align*}
\]

Let’s get a precise answer with our calculator. Enter the numerator as 6.1E-3, then press the “division” button. Remember that we must surround the denominator with parentheses. So press the open parentheses key, then enter 2.7E4. Press the “times” key, then enter 1.1E8. Press the close parentheses key and press the ENTER button. The result is shown in Figure 7.4.

**Figure 7.4:** Computing 6.1 \times 10^{-3}/(2.7 \times 10^{4} \times 1.1 \times 10^{8}).

Thus, \(6.1 \times 10^{-3}/(2.7 \times 10^{4} \times 1.1 \times 10^{8}) = 2.05387205 \times 10^{-15}\). Note that this is fairly close to our estimate of \(2 \times 10^{-15}\).

**Answer:** 5.8126537 \times 10^{-3}

---

**You Try It!**

Use the graphing calculator to simplify:

\[
\frac{2.6 \times 10^{4}}{(7.1 \times 10^{-2})(6.3 \times 10^{7})}
\]
EXAMPLE 13. Isaac Newton’s universal law of gravitation is defined by the formula

\[ F = \frac{GmM}{r^2} \]

where \( F \) is the force of attraction between two objects having mass \( m \) and \( M \), \( r \) is the distance between the two objects, and \( G \) is Newton’s gravitational constant defined by:

\[ G = 6.67428 \times 10^{-11} \text{ N(m/kg)}^2 \]

Given that the mass of the moon is \( 7.3477 \times 10^{22} \) kilograms (kg), the mass of the earth is \( 5.9736 \times 10^{24} \) kilograms (kg), and the average distance between the moon and the earth is \( 3.84403 \times 10^8 \) meters (m), find the force of attraction between the earth and the moon (in newtons (N)).

**Solution.** Plug the given numbers intoNewton’s universal law of gravitation.

\[ F = \frac{(6.673 \times 10^{-11})(7.3477 \times 10^{22})(5.9736 \times 10^{24})}{(3.84403 \times 10^8)^2} \]

Enter the expression into your calculator (see Figure 7.5) as:

\[(6.673E-11*7.3477E22*5.9736E24)/(3.84403E8)^2\]

Figure 7.5: Computing force of attraction between earth and the moon.

Hence, the force of attraction between the earth and the moon is approximately \( 1.98 \times 10^{20} \) newtons (N).

\[ \approx 1.20 \times 10^9 \text{ N} \]

Answer: \( \approx 1.20 \times 10^9 \text{ N} \)
7.2. SCIENTIFIC NOTATION

One-year’s time. The speed of light is 186,000 miles per second. How many miles from the earth is Alpha Centauri?

**Solution:** Because the speed of light is measured in miles per second, let’s first compute the number of seconds in 4.37 years. Because there are 365 days in a year, 24 hours in a day, 60 minutes in an hour, and 60 seconds in a minute, we can write:

\[
4.37 \text{ yr} = 4.37 \text{ yr} \times 365 \frac{\text{day}}{\text{yr}} \times 24 \frac{\text{hr}}{\text{day}} \times 60 \frac{\text{min}}{\text{hr}} \times 60 \frac{s}{\text{min}}
\]

\[
= 4.37 \text{ yr} \times 365 \frac{\text{day}}{\text{yr}} \times 24 \frac{\text{hr}}{\text{day}} \times 60 \frac{\text{min}}{\text{hr}} \times 60 \frac{s}{\text{min}}
\]

Note how the units cancel, indicating that the final answer is in seconds. With our calculator mode set to scientific notation (see the image on the right in Figure 7.2), we multiply the numbers to get the result shown in Figure 7.6. Rounding, the number of seconds in 4.37 years is approximately \(1.38 \times 10^8\) seconds.

Next, we compute the distance the light travels in 4.37 years. Using the fact that the distance traveled equals the speed times the time traveled, we have:

\[
\text{Distance} = \text{Speed} \times \text{Time}
\]

\[
= 1.86 \times 10^5 \frac{\text{mi}}{s} \times 1.38 \times 10^8 \text{s}
\]

\[
= 1.86 \times 10^5 \frac{\text{mi}}{s} \times 1.38 \times 10^8 \text{s}
\]

Note how the units cancel, indicating that our answer is in miles. Again, with our calculator set in scientific notation mode, we compute the product of \(1.86 \times 10^5\) and \(1.38 \times 10^8\). The result is shown in the image on the right in Figure 7.6.

Thus, the star Alpha Centauri is approximately \(2.5668 \times 10^{13}\) miles from the earth, or

\[
2.5668 \times 10^{13} \text{ miles} \approx 25,668,000,000,000 \text{ miles},
\]

pronounced “twenty-five quadrillion, six hundred sixty-eight trillion miles.”

**Answer:**
\[
\approx 5.2425 \times 10^{13} \text{ miles}
\]
### Exercises

In Exercises 1-8, write each of the following in decimal format.

1. $10^{-4}$
2. $10^{-13}$
3. $10^{-8}$
4. $10^{-9}$
5. $10^8$
6. $10^{14}$
7. $10^7$
8. $10^9$

In Exercises 9-16, write each of the following in decimal format.

9. $6506399.9 \times 10^{-4}$
10. $19548.4 \times 10^{-2}$
11. $3959.430928 \times 10^2$
12. $976.841866 \times 10^4$
13. $440906.28 \times 10^{-4}$
14. $9147437.4 \times 10^{-4}$
15. $849.855115 \times 10^4$
16. $492.4414 \times 10^3$

In Exercises 17-24, convert each of the given numbers into scientific notation.

17. 390000
18. 0.0004902
19. 0.202
20. 3231
21. 0.81
22. 83400
23. 0.0007264
24. 0.00395

In Exercises 25-32, convert each of the given expressions into scientific notation.

25. $0.04264 \times 10^{-4}$
26. $0.0019 \times 10^{-1}$
27. $130000 \times 10^3$
28. $738 \times 10^{-1}$
29. $30.04 \times 10^5$
30. $76000 \times 10^{-1}$
31. $0.011 \times 10^1$
32. $496000 \times 10^{-3}$
In Exercises 33-38, each of the following numbers are examples of numbers reported on the graphing calculator in scientific notation. Express each in plain decimal notation.

**33.** 1.134E-1
**34.** 1.370E-4
**35.** 1.556E-2
**36.** 1.802E4
**37.** 1.748E-4
**38.** 1.402E0

In Exercises 39-42, first, use the technique of Example 11 to approximate the given product without the use of a calculator. Next, use the MODE button to set you calculator in SCI and FLOAT mode, then enter the given product using scientific notation. When reporting your answer, report all digits shown in your calculator view screen.

**39.** (2.5 × 10^{-1})(1.6 × 10^{-7})
**40.** (2.91 × 10^{-1})(2.81 × 10^{-4})
**41.** (1.4 × 10^{7})(1.8 × 10^{-4})
**42.** (7.48 × 10^{7})(1.19 × 10^{6})

In Exercises 43-46, first, use the technique of Example 12 to approximate the given quotient without the use of a calculator. Next, push the MODE button, then highlight SCI mode and press ENTER. Move your cursor to the same row containing the FLOAT command, then highlight the number 2 and press ENTER. This will round your answers to two decimal places. Press 2nd MODE to quit the MODE menu. With these settings, enter the given expression using scientific notation. When entering your answer, report all digits shown in the viewing window.

**43.** 3.2 × 10^{-5} ÷ 2.5 × 10^{-7}
**44.** 6.47 × 10^{-5} ÷ 1.79 × 10^{8}
**45.** 5.9 × 10^{3} ÷ 2.3 × 10^{8}
**46.** 8.81 × 10^{-9} ÷ 3.06 × 10^{-1}

47. Overall the combined weight of biological material – animals, plants, insects, crops, bacteria, and so on – has been estimated to be at about 75 billion tons or 6.8 × 10^{13} kg (http://en.wikipedia.org/wiki/Nature). If the Earth has mass of 5.9736 × 10^{24} kg, what is the percent of the Earth’s mass that is made up of biomass?

48. The Guinness World Record for the longest handmade noodle was set on March 20, 2011. The 1,704-meter-long stretch of noodle was displayed during a noodle-making activity at a square in Southwest China’s Yunnan province. Meigan estimates that the average width of the noodle (it’s diameter) to be the same as her index finger or 1.5 cm. Using the volume formula for a cylinder ($V = \pi r^2h$) estimate the volume of the noodle in cubic centimeters.
49. Assume there are $1.43 \times 10^6$ miles of paved road in the United States. If you could travel at an average of 65 miles per hour nonstop, how many days would it take you to travel over all of the paved roads in the USA? How many years?

50. The population of the USA in mid-2011 was estimated to be $3.12 \times 10^8$ people and the world population at that time to be about $7.012 \times 10^9$ people. What percentage of the world population live in the USA?

<table>
<thead>
<tr>
<th>Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $0.0001$</td>
</tr>
<tr>
<td>3. $0.00000001$</td>
</tr>
<tr>
<td>5. $100,000,000$</td>
</tr>
<tr>
<td>7. $10,000,000$</td>
</tr>
<tr>
<td>9. $650.639999$</td>
</tr>
<tr>
<td>11. $395943.0928$</td>
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<tr>
<td>13. $44.090628$</td>
</tr>
<tr>
<td>15. $8498551.15$</td>
</tr>
<tr>
<td>17. $3.9 \times 10^5$</td>
</tr>
<tr>
<td>19. $2.02 \times 10^{-1}$</td>
</tr>
<tr>
<td>21. $8.1 \times 10^{-1}$</td>
</tr>
<tr>
<td>23. $7.264 \times 10^{-4}$</td>
</tr>
<tr>
<td>25. $4.264 \times 10^{-6}$</td>
</tr>
<tr>
<td>27. $1.3 \times 10^8$</td>
</tr>
<tr>
<td>29. $3.004 \times 10^6$</td>
</tr>
<tr>
<td>31. $1.1 \times 10^{-1}$</td>
</tr>
<tr>
<td>33. $0.1134$</td>
</tr>
<tr>
<td>35. $0.01556$</td>
</tr>
<tr>
<td>37. $0.0001748$</td>
</tr>
<tr>
<td>39. $4 \times 10^{-8}$</td>
</tr>
<tr>
<td>41. $2.52 \times 10^3$</td>
</tr>
<tr>
<td>43. $1.28 \times 10^2$</td>
</tr>
<tr>
<td>45. $2.57 \times 10^{-2}$</td>
</tr>
<tr>
<td>47. $1.14 \times 10^{-11}$</td>
</tr>
<tr>
<td>49. 916.7 days, 2.5 yr</td>
</tr>
</tbody>
</table>
7.3 Simplifying Rational Expressions

Any time you divide a polynomial by a second polynomial, you form what is known as a rational expression.

**Rational expression.** The expression

\[
\frac{p(x)}{q(x)}
\]

where \( p(x) \) and \( q(x) \) are polynomials, is called a rational expression.

For example, each of the following is a rational expression.

a) \( \frac{x + 2}{3x} \)

b) \( \frac{x + 3}{x^2 - 2x - 4} \)

c) \( \frac{2x}{3y^2} \)

In example a), the rational expression is composed of a binomial over a monomial. Example b) is constructed by dividing a binomial by a trinomial. Example c) is composed of a monomial over a monomial, the type of rational expression that will gain the most attention in this section.

**Multiplying and Dividing Rational Expressions**

We will concentrate on rational expressions with monomial numerators and denominators. Recall that to form the product of two rational numbers, we simply multiply numerators and denominators. The same technique is used to multiply any two rational expressions.

**Multiplying rational expressions.** Given \( \frac{a}{b} \) and \( \frac{c}{d} \), their product is defined as:

\[
\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}
\]

Remember, you need only multiply numerators and denominators. For example:

\[
\frac{x}{3} \cdot \frac{2}{y} = \frac{2x}{3y}, \quad \frac{2a}{3b^2} \cdot \frac{5a}{9b^3} = \frac{10a^2}{27b^5}, \quad \text{and} \quad \frac{x}{2y} \cdot \left(-\frac{3x}{4y^2}\right) = -\frac{3x^2}{8y^3}
\]

Of course, as the next example shows, sometimes you also need to reduce your answer to lowest terms.

**EXAMPLE 1.** Simplify: \( \frac{2}{x} \cdot \frac{x^2}{4} \)

You Try It!

**EXAMPLE 1.** Simplify: \( \frac{9}{x^2} \cdot \frac{x}{6} \)
Solution: Multiply numerators and denominators.

\[
\frac{2}{x} \cdot \frac{x^2}{4} = \frac{2x^5}{4x^3}
\]

Now, there are several different ways you can reduce this answer to lowest terms, two of which are shown below.

You can factor numerator and denominator, then cancel common factors.

\[
\frac{2x^5}{4x^3} = \frac{2 \cdot x \cdot x \cdot x \cdot x \cdot x}{2 \cdot 2 \cdot x \cdot x \cdot x} = \frac{2 \cdot x \cdot x}{2 \cdot 2 \cdot x} = \frac{x^2}{2}
\]

Or you can write the answer as a product, repeat the base and subtract exponents.

\[
\frac{2x^5}{4x^3} = \frac{2 \cdot x^5}{4 \cdot x^3} = \frac{1}{2} \cdot x^{5-3} = \frac{1}{2} \cdot x^2
\]

Answer: \(\frac{3}{2x}\)

As dividing by 2 is the same as multiplying by \(1/2\), these answers are equivalent. Also, note that the right-hand method is more efficient.

Recall that when dividing fractions, we invert the second fraction and multiply.

Dividing rational expressions. Given \(a/b\) and \(c/d\), their quotient is defined as:

\[
\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}
\]

You Try It!

EXAMPLE 2. Simplify: \(\frac{x^2}{y} \div \frac{x^4}{2y^2}\)

Solution: Invert, then multiply.

\[
\frac{x^2}{y} \div \frac{x^4}{2y^2} = \frac{x^2}{y} \cdot \frac{2y^2}{x^4} = \frac{2x^2y^2}{x^4y}
\]

Now, there are several different ways you can reduce this answer to lowest terms, two of which are shown below.
You can factor numerator and denominator, then cancel common factors.

\[
\frac{2x^2 y^2}{x^4 y} = \frac{2 \cdot x \cdot x \cdot y \cdot y}{x \cdot x \cdot x \cdot y} = \frac{2 \cdot x \cdot x \cdot y \cdot y}{x \cdot x \cdot x \cdot y} = \frac{2y}{x^2}
\]

Or you can write the answer as a product, repeat the base and subtract exponents.

\[
\frac{2x^2 y^2}{x^4 y} = \frac{2 \cdot x^2 \cdot y^2}{x^4 y^1} = \frac{2y}{x^2}
\]

In the last step, \(x^{-2}\) is the same as \(1/x^2\), then we multiply numerators and denominators.

Note that the right-hand method is more efficient. Answer: \(\frac{12}{x^2 y}\)

### Adding and Subtracting Rational Expressions

First, recall the rules for adding or subtracting fractions that have a “common” denominator.

**Adding rational expressions.** Given \(a/c\) and \(b/c\), their sum is defined as:

\[
\frac{a}{c} + \frac{b}{c} = \frac{a + b}{c}
\]

That is, add the numerators and place the result over the common denominator.

The following examples each share a common denominator. We add the numerators, then place the result over the common denominator.

\[
\frac{5}{7} + \frac{1}{7} = \frac{6}{7}, \quad \frac{2}{x} + \frac{3}{x} = \frac{5}{x}, \quad \text{and} \quad \frac{x}{y} + \frac{3y}{y} = \frac{x + 3y}{y}
\]

**You Try It!**

**EXAMPLE 3.** Simplify: \(\frac{3x}{xy} + \frac{2y}{xy}\)

**Solution:** Add the numerators, placing the result over the common denominator.

\[
\frac{3x}{xy} + \frac{2y}{xy} = \frac{3x + 2y}{xy}
\]

**Answer:** \(\frac{3x + 2y}{xy}\)

Simplify: \(\frac{4x}{x^2 y} + \frac{5y^2}{x^2 y}\)

**Answer:** \(\frac{4x + 5y^2}{x^2 y}\)
Subtracting rational expressions. Given \( a/c \) and \( b/c \), their difference is defined as:

\[
\frac{a}{c} - \frac{b}{c} = \frac{a - b}{c}
\]

That is, subtract the numerators and place the result over the common denominator.

The following examples each share a common denominator. We subtract the numerators, then place the result over the common denominator.

\[
\frac{7}{9} - \frac{5}{9} = \frac{2}{9}, \quad \frac{5a}{b} - \frac{3a}{b} = \frac{2a}{b}, \quad \text{and} \quad \frac{3x}{xy} - \frac{5y}{xy} = \frac{3x - 5y}{xy}
\]

As the next example shows, sometimes you may have to reduce your answer to lowest terms.

You Try It!

EXAMPLE 4. Simplify: \( \frac{5xy}{2z} - \frac{3xy}{2z} \)

Solution: Subtract the numerators, placing the result over the common denominator.

\[
\frac{5xy}{2z} - \frac{3xy}{2z} = \frac{5xy - 3xy}{2z} = \frac{2xy}{2z}
\]

To reduce to lowest terms, divide both numerator and denominator by 2.

Answer: \( \frac{2x}{yz^2} \)

The Least Common Denominator

When adding or subtracting, if the rational expressions do not share a common denominator, you must first make equivalent fractions with a common denominator.

Least common denominator. If the fractions \( a/b \) and \( c/d \) do not share a common denominator, then the least common denominator for \( b \) and \( d \) is defined as the smallest number (or expression) divisible by both \( b \) and \( d \). In symbols, \( \text{LCD}(b, d) \) represents the least common denominator of \( b \) and \( d \).
EXAMPLE 5. Simplify: \( \frac{x}{6} + \frac{2x}{9} \)

Solution: The smallest number divisible by both 6 and 9 is 18; i.e., LCD(6, 9) = 18. We must first make equivalent fractions with a common denominator of 18.

\[
\frac{x}{6} + \frac{2x}{9} = \frac{x}{6} \cdot \frac{3}{3} + \frac{2x}{9} \cdot \frac{2}{2}
\]

\[
= \frac{3x}{18} + \frac{4x}{18}
\]

We can now add the numerators and put the result over the common denominator.

\[
= \frac{7x}{18}
\]

Answer: \( \frac{29x}{24} \)

EXAMPLE 6. Simplify: \( \frac{y}{8x} - \frac{y}{12x} \)

Solution: The smallest expression divisible by both 8x and 12x is 24x; i.e., LCD(8x, 12x) = 24x. We must first make equivalent fractions with a common denominator of 24x, then place the difference of the numerators over the common denominator.

\[
\frac{y}{8x} - \frac{y}{12x} = \frac{y}{8x} \cdot \frac{3}{3} - \frac{y}{12x} \cdot \frac{2}{2}
\]

\[
= \frac{3y}{24x} - \frac{2y}{24x}
\]

\[
= \frac{y}{24x}
\]

Answer: \( \frac{x}{40y} \)

In Example 5, it was not difficult to imagine the smallest number divisible by both 6 and 9. A similar statement might apply to Example 6. This is not the case in all situations.

EXAMPLE 7. Simplify: \( \frac{5y}{72} - \frac{y}{108} \)

Simplify: \( \frac{3x}{40} \)
Solution: In this example, it is not easy to conjure up the smallest number divisible by both 72 and 108. As we shall see, prime factorization will come to the rescue.

\[
\begin{align*}
72 &= 2^3 \cdot 3^2 \\
108 &= 2^2 \cdot 3^3 \\
\text{LCD} &= 2^3 \cdot 3^3
\end{align*}
\]

Thus, \(72 = 2^3 \cdot 3^2\) and \(108 = 2^2 \cdot 3^3\).

Procedure for finding the least common denominator (LCD). To find the least common denominator for two or more fractions, proceed as follows:

1. Prime factor each denominator, putting your answers in exponential form.
2. To determine the LCD, write down each factor that appears in your prime factorizations to the highest power that it appears.

Following the procedure above, we list the prime factorization of each denominator in exponential form. The highest power of 2 that appears is \(2^3\). The highest power of 3 that appears is \(3^3\).

\[
\begin{align*}
72 &= 2^3 \cdot 3^2 & \text{Prime factor 72.} \\
108 &= 2^2 \cdot 3^3 & \text{Prime factor 108.} \\
\text{LCD} &= 2^3 \cdot 3^3 & \text{Highest power of 2 is } 2^3. \\
& & \text{Highest power of 3 is } 3^3.
\end{align*}
\]

Therefore, the LCD is \(2^3 \cdot 3^3 = 8 \cdot 27\) or 216. Hence:

\[
\begin{align*}
\frac{5y}{72} - \frac{y}{108} &= \frac{5y}{72} \cdot \frac{3}{3} - \frac{y}{108} \cdot \frac{2}{2} \\
&= \frac{15y}{216} - \frac{2y}{216} \\
&= \frac{13y}{216}
\end{align*}
\]

Make equivalent fractions.
Simplify.
Subtract numerators.

Answer: \(\frac{43x}{360}\)
### 7.3. SIMPLIFYING RATIONAL EXPRESSIONS

**EXAMPLE 8.** Simplify: \( \frac{7}{15xy^2} - \frac{11}{20x^2} \)

**Solution:** Prime factor each denominator, placing the results in exponential form.

\[
15xy^2 = 3 \cdot 5 \cdot x \cdot y^2 \\
20x^2 = 2^2 \cdot 5 \cdot x^2
\]

To find the LCD, list each factor that appears to the highest power that it appears.

\[
\text{LCD} = 2^2 \cdot 3 \cdot 5 \cdot x^2 \cdot y^2
\]

Simplify.

\[
\text{LCD} = 60x^2y^2
\]

After making equivalent fractions, place the difference of the numerators over this common denominator.

\[
\frac{7}{15xy^2} - \frac{11}{20x^2} = \frac{7 \cdot 4x}{15xy^2} - \frac{11 \cdot 3y^2}{20x^2} \\
= \frac{28x}{60x^2y^2} - \frac{33y^2}{60x^2y^2} \\
= \frac{28x - 33y^2}{60x^2y^2}
\]

**Answer:** \( \frac{55 + 21x^2}{90x^2y} \)

---

**Dividing a Polynomial by a Monomial**

We know that multiplication is distributive with respect to addition; that is, \( a(b + c) = ab + ac \). We use this property to perform multiplications such as:

\[
x^2(2x^2 - 3x - 8) = 2x^4 - 3x^3 - 8x^2
\]

However, it is also true that division is distributive with respect to addition.

**Distributive property for division.** If \( a, b, \) and \( c \) are any numbers, then:

\[
\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}
\]
For example, note that
\[ \frac{4 + 6}{2} = \frac{4}{2} + \frac{6}{2}. \]

This form of the distributive property can be used to divide a polynomial by a monomial.

**EXAMPLE 9.** Divide \( x^2 - 2x - 3 \) by \( x^2 \).

**Solution:** We use the distributive property, dividing each term by \( x^2 \).

\[
\frac{x^2 - 2x - 3}{x^2} = \frac{x^2}{x^2} - \frac{2x}{x^2} - \frac{3}{x^2}
\]

Now we reduce each term of the last result to lowest terms, canceling common factors.

\[ = 1 - \frac{2}{x} - \frac{3}{x^2} \]

Answer: \( 3x + \frac{8}{3} - \frac{2}{x} \)

**EXAMPLE 10.** Divide \( 2x^3 - 3x + 12 \) by \( 6x^3 \).

**Solution:** We use the distributive property, dividing each term by \( 6x^3 \).

\[
\frac{2x^3 - 3x + 12}{6x^3} = \frac{2x^3}{6x^3} - \frac{3x}{6x^3} + \frac{12}{6x^3}
\]

Now we reduce each term of the last result to lowest terms, canceling common factors.

\[ = \frac{1}{3} - \frac{1}{2x^2} + \frac{2}{x^3} \]

Answer: \( \frac{2}{x^2} + \frac{3}{x^3} - \frac{9}{x^4} \)
7.3. SIMPLIFYING RATIONAL EXPRESSIONS

Exercises

In Exercises 1-8, simplify each of the given expressions.

1. \[ \frac{12}{s^2} \cdot \frac{s^5}{9} \]
2. \[ \frac{6}{x^4} \cdot \frac{x^2}{10} \]
3. \[ \frac{12}{v^3} \cdot \frac{v^4}{10} \]
4. \[ \frac{10}{t^4} \cdot \frac{t^5}{12} \]
5. \[ \frac{s^5}{t^3} \div \frac{9s^2}{t^2} \]
6. \[ \frac{s^2}{t^2} \div \frac{6s^4}{t^4} \]
7. \[ \frac{b^4}{c^4} \div \frac{9b^2}{c^2} \]
8. \[ \frac{b^5}{c^4} \div \frac{8b^2}{c^2} \]

In Exercises 9-14, simplify each of the given expressions.

9. \[ -\frac{10s}{18} + \frac{19s}{18} \]
10. \[ -\frac{14y}{2} + \frac{10y}{2} \]
11. \[ \frac{5}{9c} - \frac{17}{9c} \]
12. \[ \frac{19}{14r} - \frac{17}{14r} \]
13. \[ -\frac{8x}{15yz} - \frac{16x}{15yz} \]
14. \[ -\frac{17a}{20bc} - \frac{9a}{20bc} \]

In Exercises 15-20, simplify each of the given expressions.

15. \[ \frac{9z}{10} + \frac{5z}{2} \]
16. \[ \frac{7u}{2} + \frac{11u}{6} \]
17. \[ \frac{3}{10w} - \frac{4}{5w} \]
18. \[ \frac{9}{10v} - \frac{7}{2v} \]
19. \[ -\frac{8r}{5st} - \frac{9r}{10st} \]
20. \[ -\frac{7x}{6yz} - \frac{3x}{2yz} \]

In Exercises 21-32, simplify each of the given expressions.

21. \[ \frac{11}{18rs^2} + \frac{5}{24r^2s} \]
22. \[ \frac{5}{12uw^2} + \frac{13}{54u^2w} \]
23. \[ \frac{5}{24rs^2} + \frac{17}{36r^2s} \]
24. \[ \frac{13}{54vw^2} + \frac{19}{24v^2w} \]
In Exercises 33-48, use the distributive property to divide each term in the numerator by the term in the denominator.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Expression</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>(\frac{6v + 12}{3})</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>(\frac{28u + 36}{4})</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>(\frac{25u + 45}{5})</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>(\frac{16x + 4}{2})</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>(\frac{2s - 4}{s})</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>(\frac{7r - 8}{r})</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>(\frac{3r - 5}{r})</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>(\frac{4u - 2}{u})</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>(\frac{3x^2 - 8x - 9}{x^2})</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>(\frac{6b^2 - 5b - 8}{b^2})</td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>(\frac{2x^2 - 3x - 6}{x^2})</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>(\frac{6u^2 - 5u - 2}{u^2})</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>(\frac{12t^2 + 2t - 16}{12t^2})</td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>(\frac{18b^2 + 9b - 15}{18b^2})</td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>(\frac{4s^2 + 2s - 10}{4s^2})</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>(\frac{10w^2 + 12w - 2}{10w^2})</td>
<td></td>
</tr>
</tbody>
</table>
13. \( \frac{8x}{5yz} \)
15. \( \frac{17z}{5} \)
17. \( -\frac{1}{2v} \)
19. \( -\frac{5r}{2st} \)
21. \( \frac{44r + 15s}{72r^2s^2} \)
23. \( \frac{15r + 34s}{72r^2s^2} \)
25. \( \frac{28z^3 + 33y^3}{144y^3z^3} \)
27. \( \frac{15w^3 + 52v^3}{144v^3w^3} \)
29. \( \frac{44z - 45x}{200xyz} \)
31. \( \frac{76c - 85a}{200abc} \)
33. \( 2v + 4 \)
35. \( 5u + 9 \)
37. \( 2 - \frac{4}{s} \)
39. \( 3 - \frac{5}{r} \)
41. \( 3 - \frac{8}{x} - \frac{9}{x^2} \)
43. \( 2 - \frac{3}{x} - \frac{6}{x^2} \)
45. \( 1 + \frac{1}{6t} - \frac{4}{3t^2} \)
47. \( 1 + \frac{1}{2s} - \frac{5}{2s^2} \)
7.4 Solving Rational Equations

In Section 3 of Chapter 2, we showed that the most efficient way to solve an equation containing fractions was to first clear the fractions by multiplying both sides of the equation by the least common denominator. For example, given the equation

\[ \frac{1}{2}x + \frac{1}{3} = \frac{1}{4}, \]

we would first clear the fractions by multiplying both sides by 12.

\[ 12 \left( \frac{1}{2}x + \frac{1}{3} \right) = 12 \cdot \frac{1}{4} \]

\[ 6x + 4 = 3 \]

This procedure works equally well when the denominators contain a variable.

You Try It!

EXAMPLE 1. Solve for \( x \): \( 1 - \frac{2}{x} = \frac{3}{x^2} \)

Solution: The common denominator is \( x^2 \). We begin by clearing fractions, multiplying both sides of the equation by \( x^2 \).

\[ 1 - \frac{2}{x} = \frac{3}{x^2} \quad \text{Original equation.} \]

\[ x^2 \left( 1 - \frac{2}{x} \right) = \left[ \frac{3}{x^2} \right] x^2 \quad \text{Multiply both sides by } x^2. \]

Now we use the distributive property.

\[ x^2 \left[ 1 \right] - x^2 \left[ \frac{2}{x} \right] = \left[ \frac{3}{x^2} \right] x^2 \quad \text{Distribute } x^2. \]

Now we cancel common factors and simplify.

\[ x^2 - 2x = 3 \quad \text{Cancel. Simplify.} \]

The resulting equation is nonlinear (\( x \) is raised to a power larger than 1). Make one side zero, then factor.

\[ x^2 - 2x - 3 = 0 \quad \text{Nonlinear. Make one side zero.} \]

\[ (x - 3)(x + 1) = 0 \quad \text{Factor.} \]
7.4. SOLVING RATIONAL EQUATIONS

Use the zero product property to complete the solution. Either the first factor is zero or the second factor is zero.

\[ x - 3 = 0 \quad \text{or} \quad x + 1 = 0 \]

\[ x = 3 \quad x = -1 \]

Hence, the solutions are \( x = -1 \) and \( x = 3 \).

**Check.** Substitute \(-1\) for \( x \), then 3 for \( x \) in the original equation and simplify.

\[
\begin{align*}
1 - \frac{2}{x} & = \frac{3}{x^2} \\
1 - \frac{2}{(-1)} & = \frac{3}{(-1)^2} \\
1 + 2 & = 3 \\
3 & = 3
\end{align*}
\]

Note that both result in true statements, showing that both \( x = -1 \) and \( x = 3 \) check in the original equation. Answer: 2, 4

---

**EXAMPLE 2.** Solve for \( x \): \( 6 - \frac{22}{x^2} = \frac{29}{x} \)

**Solution:** The common denominator is \( x^2 \).

\[
\begin{align*}
x^2 \left( 6 - \frac{22}{x^2} \right) & = \left( \frac{29}{x} \right) x^2 \\
x^2 \cdot 6 - 22 & = 29 x \\
6x^2 - 22 & = 29 x
\end{align*}
\]

This last equation is nonlinear. Make one side zero.

\[ 6x^2 - 29x - 22 = 0 \]

The integer pair 4 and \(-33\) has product \( ac = -132 \) and sum \( b = -29 \). Break up the middle term into a sum using this pair, then factor by grouping.

\[
\begin{align*}
6x^2 + 4x - 33x - 22 & = 0 \\
2x(3x + 2) - 11(3x + 2) & = 0 \\
(2x - 11)(3x + 2) & = 0
\end{align*}
\]
Finally, use the zero product property to write:

\[
\begin{align*}
2x - 11 &= 0 \quad \text{or} \quad 3x + 2 = 0 \\
2x &= 11 \quad \text{or} \quad 3x = -2 \\
x &= \frac{11}{2} \quad \text{or} \quad x = -\frac{2}{3}
\end{align*}
\]

**Check:** Let’s check these solutions with our calculators. Enter 11/2, push the STO▶ button, push the X,T,θ,n button and the ENTER key (see the calculator screen on the left in Figure 7.7). Next, enter the left-hand side of the equation as \(6-\frac{22}{x^2}\) and press ENTER. Enter the right-hand side of the equation as \(\frac{29}{x}\) and press ENTER. The results are the same (see the calculator screen on the left in Figure 7.7). This verifies that 11/2 is a solution of \(6-\frac{22}{x^2} = \frac{29}{x}\).

The calculator screen on the right in Figure 7.7 shows a similar check of the solution \(x = -\frac{2}{3}\).

| 11/2⇒X | 5.5 |
| 6-22/X^2 | 5.272727273 |
| 29/X | 5.272727273 |

| -2/3⇒X | -1.666666667 |
| 6-22/X^2 | -43.5 |
| 29/X | -43.5 |

Figure 7.7: Checking the solutions of \(6-\frac{22}{x^2} = \frac{29}{x}\).

Answer: \(-1/4, -7/2\)

---

**Solving Rational Equations with the Graphing Calculator**

Let’s use the graphing calculator to solve an equation containing rational expressions.

**You Try It!**

Solve the equation

\[
2 + \frac{5}{x} = \frac{12}{x^2}
\]

both algebraically and graphically, then compare your solutions.

**EXAMPLE 3.** Consider the following equation:

\[
2 - \frac{9}{x} = \frac{5}{x^2}
\]

Solve the equation algebraically, then solve the equation graphically using your graphing calculator. Compare your solutions.
7.4. SOLVING RATIONAL EQUATIONS

Algebraic solution: First, an algebraic approach. Multiply both sides of the equation by the common denominator \( x^2 \).

\[
\begin{align*}
2 - \frac{9}{x} &= \frac{5}{x^2} & \text{Original equation.} \\
\left( \frac{2}{x} - \frac{9}{x} \right) x^2 &= \left( \frac{5}{x^2} \right) x^2 & \text{Multiply both sides by } x^2. \\
x^2 \left( \frac{2}{x} - \frac{9}{x} \right) &= \left( \frac{5}{x^2} \right) x^2 & \text{Distribute } x^2. \\
2x^2 - 9x &= 5 & \text{Cancel and simplify.}
\end{align*}
\]

The last equation is nonlinear. Make one side zero.

\[
2x^2 - 9x - 5 = 0 & \quad \text{Make one side zero.}
\]

The integer pair \(-10\) and \(1\) have product equaling \(ac = -10\) and sum equaling \(b = -9\). Break up the middle term using this pair, then factor by grouping.

\[
\begin{align*}
2x^2 - 10x + x - 5 &= 0 & \quad -10x + x = -9x. \\
2x(x - 5) + 1(x - 5) &= 0 & \quad \text{Factor by grouping.} \\
(2x + 1)(x - 5) &= 0 & \quad \text{Factor out } x - 5.
\end{align*}
\]

Now use the zero product property to write:

\[
\begin{align*}
2x + 1 &= 0 & \quad \text{or} & \quad x - 5 &= 0 \\
2x &= -1 & \quad x &= 5 \\
x &= -\frac{1}{2}
\end{align*}
\]

Hence, the solutions are \(x = -1/2\) and \(x = 5\).

Graphical solution: We could load each side of the equation separately, then use the intersect utility to find where the graphs intersect. However, in this case, it’s a bit easier to make one side of the equation zero, draw a single graph, then note where the graph crosses the \(x\)-axis.

\[
\begin{align*}
2 - \frac{9}{x} &= \frac{5}{x^2} & \text{Original equation.} \\
2 - \frac{9}{x} - \frac{5}{x^2} &= 0 & \text{Make one side zero.}
\end{align*}
\]

Load the left-hand side of the equation into \(Y1\) as \(2-9/X-5/X^2\) (see the image on the left in Figure 7.8), then select 6:ZStandard from the ZOOM menu to produce the image at the right in Figure 7.8.

Next, the solutions of

\[
2 - \frac{9}{x} - \frac{5}{x^2} = 0
\]
are found by noting where the graph of \( y = 2 - \frac{9}{x} - \frac{5}{x^2} \) cross the \( x \)-axis. Select 2:zero from the CALC menu. Use the arrow keys to move the cursor to the left of the first \( x \)-intercept, then press ENTER to set the “Left bound.” Next, move the cursor to the right of the first \( x \)-intercept, then press ENTER to set the “Right bound.” Finally, leave the cursor where it is and press ENTER to set your “Guess.” The calculator responds with the result shown in the figure on the left in Figure 7.9.

Repeat the zero-finding procedure to capture the coordinates of the second \( x \)-intercept (see the image on the right in Figure 7.9).

**Reporting the solution on your homework:** Duplicate the image in your calculator’s viewing window on your homework page. Use a ruler to draw all lines, but freehand any curves.

- Label the horizontal and vertical axes with \( x \) and \( y \), respectively (see Figure 7.10).
- Place your WINDOW parameters at the end of each axis (see Figure 7.10).
- Label the graph with its equation (see Figure 7.10).
- Drop dashed vertical lines through each \( x \)-intercept. Shade and label the \( x \)-values of the points where the dashed vertical line crosses the \( x \)-axis. These are the solutions of the equation \( 2 - \frac{9}{x} - \frac{5}{x^2} = 0 \) (see Figure 7.10).
7.4. SOLVING RATIONAL EQUATIONS

Figure 7.10: Reporting your graphical solution on your homework.

Thus, the calculator is reporting that the solutions of \( 2 - \frac{9}{x} - \frac{5}{x^2} = 0 \) are
\( x = -0.5 \) and \( x = 5 \), which match the algebraic solutions \( x = -\frac{1}{2} \) and \( x = 5 \).

Numerical Applications

Let’s apply what we’ve learned to an application.

**EXAMPLE 4.** The sum of a number and its reciprocal is 41/20. Find the number.

**Solution:** In the solution, we address each step of the *Requirements for Word Problem Solutions*.

1. *Set up a variable dictionary.* Let \( x \) represent the unknown number.

2. *Set up an equation.* If the unknown number is \( x \), then its reciprocal is \( \frac{1}{x} \). Thus, the “sum of a number and its reciprocal is 41/20” becomes:

\[
x + \frac{1}{x} = \frac{41}{20}
\]

Answer: \(-4, 3/2\)
3. Solve the equation. Clear the fractions by multiplying both sides by 20x, the least common denominator.

\[ \frac{x}{x} + \frac{1}{x} = \frac{41}{20} \quad \text{Model equation.} \]

\[ 20x \left[ \frac{x}{x} + \frac{1}{x} \right] = \frac{41}{20} \cdot 20x \quad \text{Multiply both sides by 20x.} \]

\[ 20x \cdot \frac{x}{x} + 20x \cdot \frac{1}{x} = \frac{41}{20} \cdot 20x \quad \text{Distribute 20x.} \]

\[ 20x^2 + 20 = 41x \quad \text{Cancel and simplify.} \]

The equation is nonlinear. Make one side zero.

\[ 20x^2 - 41x + 20 = 0 \quad \text{Make one side zero.} \]

The integer pair \(-16\) and \(-25\) has product \(ac = 400\) and sum \(b = -41\). Break up the middle term in the last equation into a sum of like terms using this pair, then factor by grouping.

\[ 20x^2 - 16x - 25x + 20 = 0 \quad \text{or} \quad -16x - 25x = -41x. \]

\[ 4x(5x - 4) - 5(5x - 4) = 0 \quad \text{Factor by grouping.} \]

\[ (4x - 5)(5x - 4) = 0 \quad \text{Factor out} \ 5x - 4. \]

We can now use the zero product property to write:

\[ 4x - 5 = 0 \quad \text{or} \quad 5x - 4 = 0 \]

\[ 4x = 5 \quad 5x = 4 \]

\[ x = \frac{5}{4} \quad x = \frac{4}{5} \]

4. Answer the question. There are two possible numbers, \(5/4\) and \(4/5\).

5. Look back. The sum of the unknown number and its reciprocal is supposed to equal \(41/20\). The answer \(5/4\) has reciprocal \(4/5\). Their sum is:

\[ \frac{5}{4} + \frac{4}{5} = \frac{16}{20} + \frac{25}{20} = \frac{41}{20} \]

Thus, \(5/4\) is a valid solution.

The second answer \(4/5\) has reciprocal \(5/4\), so it is clear that their sum is also \(41/20\). Hence, \(4/5\) is also a valid solution.

Answer: \(2/7, 7/2\)
7.4. SOLVING RATIONAL EQUATIONS

Exercises

In Exercises 1-8, solve the equation.

1. \( x = 11 + \frac{26}{x} \)
2. \( x = 7 + \frac{60}{x} \)
3. \( 1 - \frac{12}{x} = \frac{27}{x^2} \)
4. \( 1 + \frac{6}{x} = \frac{7}{x^2} \)
5. \( 1 - \frac{10}{x} = \frac{11}{x^2} \)
6. \( 1 - \frac{20}{x} = -\frac{96}{x^2} \)
7. \( x = 7 + \frac{44}{x} \)
8. \( x = 2 + \frac{99}{x} \)

In Exercises 9-16, solve the equation.

9. \( 12x = 97 - \frac{8}{x} \)
10. \( 7x = -19 - \frac{10}{x} \)
11. \( 20 + \frac{19}{x} = -\frac{3}{x^2} \)
12. \( 33 - \frac{8}{x} = \frac{1}{x^2} \)
13. \( 8x = 19 - \frac{11}{x} \)
14. \( 28x = 25 - \frac{3}{x} \)
15. \( 40 + \frac{6}{x} = \frac{1}{x^2} \)
16. \( 18 + \frac{11}{x} = -\frac{1}{x^2} \)

In Exercises 17-20, solve each equation algebraically, then use the calculator to check your solutions.

17. \( 36x = -13 - \frac{1}{x} \)
18. \( 9x = 43 + \frac{10}{x} \)
19. \( 14x = 9 - \frac{1}{x} \)
20. \( 3x = 16 - \frac{20}{x} \)

In Exercises 21-24, solve the equation algebraically, then solve the equation using the graphing calculator using the technique shown in Example 3. Report your solution using the Calculator Submission Guidelines demonstrated in Example 3.

21. \( 1 - \frac{1}{x} = \frac{12}{x^2} \)
22. \( 1 + \frac{11}{x} = -\frac{28}{x^2} \)
23. \( 2x = 3 + \frac{44}{x} \)
24. \( 2x = 9 - \frac{4}{x} \)
25. The sum of a number and its reciprocal is \(\frac{5}{2}\). Find the number.

26. The sum of a number and its reciprocal is \(\frac{65}{8}\). Find the number.

27. The sum of a number and 8 times its reciprocal is \(\frac{17}{3}\). Find all possible solutions.

28. The sum of a number and 4 times its reciprocal is \(\frac{17}{2}\). Find all possible solutions.

<table>
<thead>
<tr>
<th></th>
<th>Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(-2, 13)</td>
</tr>
<tr>
<td>3.</td>
<td>(3, 9)</td>
</tr>
<tr>
<td>5.</td>
<td>(11, -1)</td>
</tr>
<tr>
<td>7.</td>
<td>(-4, 11)</td>
</tr>
<tr>
<td>9.</td>
<td>(8, 1/12)</td>
</tr>
<tr>
<td>11.</td>
<td>(-3/4, -1/5)</td>
</tr>
<tr>
<td>13.</td>
<td>(11/8, 1)</td>
</tr>
<tr>
<td>15.</td>
<td>(-1/4, 1/10)</td>
</tr>
<tr>
<td>17.</td>
<td>(-1/9, -1/4)</td>
</tr>
<tr>
<td>19.</td>
<td>(1/2, 1/7)</td>
</tr>
<tr>
<td>21.</td>
<td>(-3, 4)</td>
</tr>
<tr>
<td>23.</td>
<td>(-4, 11/2)</td>
</tr>
<tr>
<td>25.</td>
<td>(2, 1/2)</td>
</tr>
<tr>
<td>27.</td>
<td>(3, \frac{8}{3})</td>
</tr>
</tbody>
</table>
7.5 DIRECT AND INVERSE VARIATION

7.5 Direct and Inverse Variation

We start with the definition of the phrase “is proportional to.”

**Proportional.** We say that \( y \) is proportional to \( x \) if and only if
\[
y = kx,
\]
where \( k \) is a constant called the constant of proportionality. The phrase “\( y \) varies directly as \( x \)” is an equivalent way of saying “\( y \) is proportional to \( x \).”

Here are a few examples that translate the phrase “is proportional to.”

- Given that \( d \) is proportional to \( t \), we write \( d = kt \), where \( k \) is a constant.
- Given that \( y \) is proportional to the cube of \( x \), we write \( y = kx^3 \), where \( k \) is a constant.
- Given that \( s \) is proportional to the square of \( t \), we write \( s = kt^2 \), where \( k \) is a constant.

We are not restricted to always using the letter \( k \) for our constant of proportionality.

**EXAMPLE 1.** Given that \( y \) is proportional to \( x \) and the fact that \( y = 12 \) when \( x = 5 \), determine the constant of proportionality, then determine the value of \( y \) when \( x = 10 \).

**Solution:** Given the fact the \( y \) is proportional to \( x \), we know immediately that
\[
y = kx,
\]
where \( k \) is the proportionality constant. Because we are given that \( y = 12 \) when \( x = 5 \), we can substitute 12 for \( y \) and 5 for \( x \) to determine \( k \).

\[
\begin{align*}
y &= kx \\
12 &= k(5) \\
\frac{12}{5} &= k
\end{align*}
\]

Next, substitute the constant of proportionality \( 12/5 \) for \( k \) in \( y = kx \), then substitute 10 for \( x \) to determine \( y \) when \( x = 10 \).

\[
\begin{align*}
y &= \frac{12}{5}x \\
y &= \frac{12}{5}(10) \\
y &= 24
\end{align*}
\]

Answer: 63
A ball is dropped from the edge of a cliff on a certain planet. The distance $s$ the ball falls is proportional to the square of the time $t$ that has passed since the ball’s release. If the ball falls 50 feet during the first 5 seconds, how far does the ball fall in 8 seconds?

**Solution:** Given the fact the $s$ is proportional to the square of $t$, we know immediately that

$$s = kt^2,$$

where $k$ is the proportionality constant. Because we are given that the ball falls 144 feet during the first 3 seconds, we can substitute 144 for $s$ and 3 for $t$ to determine the constant of proportionality.

\[
\begin{align*}
144 &= k(3)^2 \\
144 &= 9k \\
16 &= k
\end{align*}
\]

Next, substitute the constant of proportionality 16 for $k$ in $s = kt^2$, and then substitute 9 for $t$ to determine the distance fallen when $t = 9$ seconds.

\[
\begin{align*}
s &= 16t^2 \\
s &= 16(9)^2 \\
s &= 1296
\end{align*}
\]

Answer: 128 feet

Thus, the ball falls 1,296 feet during the first 9 seconds.

---

Tony and Paul are hanging weights on a spring in the physics lab. Each time a weight is hung, they measure the distance the spring stretches. They discover that the distance $y$ that the spring stretches is proportional to the weight hung on the spring (Hooke’s Law). If a 0.5 pound weight stretches the spring 3 inches, how far will a 0.75 pound weight stretch the spring?

**Solution:** Let $W$ represent the weight hung on the spring. Let $y$ represent the distance the spring stretches. We’re told that the distance $y$ the spring stretches is proportional to the amount of weight $W$ hung on the spring. Hence, we can write:

$$y = kW$$

$y$ is proportional to $W$. 

---
7.5. DIRECT AND INVERSE VARIATION

Substitute 3 for \( y \), 0.5 for \( W \), then solve for \( k \).

\[
3 = k(0.5) \quad \text{Substitute 3 for } y, \text{ 0.5 for } W.
\]

\[
\frac{3}{0.5} = k \quad \text{Divide both sides by 0.5.}
\]

\[
k = 6 \quad \text{Simplify.}
\]

Substitute 6 for \( k \) in \( y = kW \) to produce:

\[
y = 6W \quad \text{Substitute 6 for } k \text{ in } y = kW.
\]

To determine the distance the spring will stretch when 0.75 pounds are hung on the spring, substitute 0.75 for \( W \).

\[
y = 6(0.75) \quad \text{Substitute 0.75 for } W.
\]

\[
y = 4.5 \quad \text{Simplify.}
\]

Thus, the spring will stretch 4.5 inches. Answer: 8 inches

Inversely Proportional

In Examples 1, 2, and 3, where one quantity was proportional to a second quantity, you may have noticed that when one quantity increased, the second quantity also increased. Vice-versa, when one quantity decreased, the second quantity also decreased.

However, not all real-world situations follow this pattern. There are times when as one quantity increases, the related quantity decreases. For example, consider the situation where you increase the number of workers on a job and note that the time to finish the job decreases. This is an example of a quantity being inversely proportional to a second quantity.

**Inversely proportional.** We say the \( y \) is inversely proportional to \( x \) if and only if

\[
y = \frac{k}{x},
\]

where \( k \) is a constant called the constant of proportionality. The phrase “\( y \) varies inversely as \( x \)” is an equivalent way of saying “\( y \) in inversely proportional to \( x \).”

Here are a few examples that translate the phrase “is inversely proportional to.”

- Given that \( d \) is inversely proportional to \( t \), we write \( d = \frac{k}{t} \), where \( k \) is a constant.
CHAPTER 7. RATIONAL EXPRESSIONS

- Given that \( y \) is inversely proportional to the cube of \( x \), we write \( y = \frac{k}{x^3} \), where \( k \) is a constant.

- Given that \( s \) is inversely proportional to the square of \( t \), we write \( s = \frac{k}{t^2} \), where \( k \) is a constant.

We are not restricted to always using the letter \( k \) for our constant of proportionality.

---

**You Try It!**

*EXAMPLE 4.* Given that \( y \) is inversely proportional to \( x \) and the fact that \( y = 5 \) when \( x = 8 \), determine the value of \( y \) when \( x = 10 \).

**Solution:**

Given the fact the \( y \) is inversely proportional to \( x \), we know immediately that

\[
y = \frac{k}{x},
\]

where \( k \) is the proportionality constant. Because we are given that \( y = 4 \) when \( x = 2 \), we can substitute 4 for \( y \) and 2 for \( x \) to determine \( k \).

\[
y = \frac{k}{x} \quad \text{\( y \) is inversely proportional to \( x \).}
\]

\[
4 = \frac{k}{2} \quad \text{Substitute 4 for \( y \), 2 for \( x \).}
\]

\[
8 = k \quad \text{Multiply both sides by 2.}
\]

Substitute 8 for \( k \) in \( y = \frac{k}{x} \), then substitute 4 for \( x \) to determine \( y \) when \( x = 4 \).

\[
y = \frac{8}{x} \quad \text{Substitute 8 for \( k \).}
\]

\[
y = \frac{8}{4} \quad \text{Substitute 4 for \( x \).}
\]

\[
y = 2 \quad \text{Reduce.}
\]

Answer: 4

Note that as \( x \) increased from 2 to 4, \( y \) decreased from 4 to 2.

---

*EXAMPLE 5.* The intensity \( I \) of light is inversely proportional to the square of the distance \( d \) from the light source. If the light intensity 5 feet from the light source is 3 foot-candles, what is the intensity of the light 15 feet from the light source?

---
Solution: Given the fact that the intensity $I$ of the light is inversely proportional to the square of the distance $d$ from the light source, we know immediately that

$$I = \frac{k}{d^2},$$

where $k$ is the proportionality constant. Because we are given that the intensity is $I = 3$ foot-candles at $d = 5$ feet from the light source, we can substitute 3 for $I$ and 5 for $d$ to determine $k$.

$I = \frac{k}{d^2}$ \hspace{1cm} I is inversely proportional to $d^2$.

\[\begin{align*}
3 &= \frac{k}{5^2} \\
3 &= \frac{k}{25} \\
75 &= k
\end{align*}\]


Substitute 75 for $k$ in $I = k/d^2$, then substitute 15 for $d$ to determine $I$ when $d = 15$.

\[\begin{align*}
I &= \frac{75}{d^2} \\
I &= \frac{75}{15^2} \\
I &= \frac{75}{225} \\
I &= \frac{1}{3}
\end{align*}\]

Thus, the intensity of the light 15 feet from the light source is $1/3$ foot-candle.

Answer: $1/3$ foot-candle

---

**EXAMPLE 6.** Suppose that the price per person for a camping experience is inversely proportional to the number of people who sign up for the experience. If 10 people sign up, the price per person is $350. What will be the price per person if 50 people sign up?

Solution: Let $p$ represent the price per person and let $N$ be the number of people who sign up for the camping experience. Because we are told that the price per person is inversely proportional to the number of people who sign up for the camping experience, we can write:

$$p = \frac{k}{N}.$$
where \( k \) is the proportionality constant. Because we are given that the price per person is $350 when 10 people sign up, we can substitute 350 for \( p \) and 10 for \( N \) to determine \( k \).

\[
p = \frac{k}{N} \quad \text{\( p \) is inversely proportional to \( N \).}
\]

\[
350 = \frac{k}{10} \quad \text{Substitute 350 for \( p \), 10 for \( N \).}
\]

\[
3500 = k \quad \text{Multiply both sides by 10.}
\]

Substitute 3500 for \( k \) in \( p = k/N \), then substitute 50 for \( N \) to determine \( p \) when \( N = 50 \).

\[
p = \frac{3500}{N} \quad \text{Substitute 3500 for \( k \).}
\]

\[
p = \frac{3500}{50} \quad \text{Substitute 50 for \( N \).}
\]

\[
p = 70 \quad \text{Simplify.}
\]

Thus, the price per person is $70 if 50 people sign up for the camping experience.

Answer: $28
7.5. DIRECT AND INVERSE VARIATION

Exercises

1. Given that \( s \) is proportional to \( t \) and the fact that \( s = 632 \) when \( t = 79 \), determine the value of \( s \) when \( t = 50 \).

2. Given that \( s \) is proportional to \( t \) and the fact that \( s = 264 \) when \( t = 66 \), determine the value of \( s \) when \( t = 60 \).

3. Given that \( s \) is proportional to the cube of \( t \) and the fact that \( s = 1588867 \) when \( t = 61 \), determine the value of \( s \) when \( t = 63 \).

4. Given that \( d \) is proportional to the cube of \( t \) and the fact that \( d = 318028 \) when \( t = 43 \), determine the value of \( d \) when \( t = 76 \).

5. Given that \( q \) is proportional to the square of \( c \) and the fact that \( q = 13448 \) when \( c = 82 \), determine the value of \( q \) when \( c = 29 \).

6. Given that \( q \) is proportional to the square of \( c \) and the fact that \( q = 3125 \) when \( c = 25 \), determine the value of \( q \) when \( c = 87 \).

7. Given that \( y \) is proportional to the square of \( x \) and the fact that \( y = 14700 \) when \( x = 70 \), determine the value of \( y \) when \( x = 45 \).

8. Given that \( y \) is proportional to the square of \( x \) and the fact that \( y = 2028 \) when \( x = 26 \), determine the value of \( y \) when \( x = 79 \).

9. Given that \( F \) is proportional to the cube of \( x \) and the fact that \( F = 214375 \) when \( x = 35 \), determine the value of \( F \) when \( x = 36 \).

10. Given that \( d \) is proportional to the cube of \( t \) and the fact that \( d = 2465195 \) when \( t = 79 \), determine the value of \( d \) when \( t = 45 \).

11. Given that \( d \) is proportional to \( t \) and the fact that \( d = 496 \) when \( t = 62 \), determine the value of \( d \) when \( t = 60 \).

12. Given that \( d \) is proportional to \( t \) and the fact that \( d = 405 \) when \( t = 45 \), determine the value of \( d \) when \( t = 65 \).

13. Given that \( h \) is inversely proportional to \( x \) and the fact that \( h = 16 \) when \( x = 29 \), determine the value of \( h \) when \( x = 20 \).

14. Given that \( y \) is inversely proportional to \( x \) and the fact that \( y = 23 \) when \( x = 15 \), determine the value of \( y \) when \( x = 10 \).

15. Given that \( q \) is inversely proportional to the square of \( c \) and the fact that \( q = 11 \) when \( c = 9 \), determine the value of \( q \) when \( c = 3 \).

16. Given that \( s \) is inversely proportional to the square of \( t \) and the fact that \( s = 11 \) when \( t = 8 \), determine the value of \( s \) when \( t = 10 \).

17. Given that \( F \) is inversely proportional to \( x \) and the fact that \( F = 19 \) when \( x = 22 \), determine the value of \( F \) when \( x = 16 \).

18. Given that \( d \) is inversely proportional to \( t \) and the fact that \( d = 21 \) when \( t = 16 \), determine the value of \( d \) when \( t = 24 \).
19. Given that $y$ is inversely proportional to the square of $x$ and the fact that $y = 14$ when $x = 4$, determine the value of $y$ when $x = 10$.

20. Given that $d$ is inversely proportional to the square of $t$ and the fact that $d = 21$ when $t = 8$, determine the value of $d$ when $t = 12$.

21. Given that $d$ is inversely proportional to the cube of $t$ and the fact that $d = 18$ when $t = 2$, determine the value of $d$ when $t = 3$.

22. Given that $q$ is inversely proportional to the cube of $c$ and the fact that $q = 10$ when $c = 5$, determine the value of $q$ when $c = 6$.

23. Given that $q$ is inversely proportional to the cube of $c$ and the fact that $q = 16$ when $c = 8$, determine the value of $q$ when $c = 6$.

24. Given that $q$ is inversely proportional to the cube of $c$ and the fact that $q = 15$ when $c = 6$, determine the value of $q$ when $c = 2$.

25. Joe and Mary are hanging weights on a spring in the physics lab. Each time a weight is hung, they measure the distance the spring stretches. They discover that the distance that the spring stretches is proportional to the weight hung on the spring. If a 2 pound weight stretches the spring 16 inches, how far will a 5 pound weight stretch the spring?

26. Liz and Denzel are hanging weights on a spring in the physics lab. Each time a weight is hung, they measure the distance the spring stretches. They discover that the distance that the spring stretches is proportional to the weight hung on the spring. If a 5 pound weight stretches the spring 12.5 inches, how far will a 12 pound weight stretch the spring?

27. The intensity $I$ of light is inversely proportional to the square of the distance $d$ from the light source. If the light intensity 4 feet from the light source is 20 foot-candles, what is the intensity of the light 18 feet from the light source?

28. The intensity $I$ of light is inversely proportional to the square of the distance $d$ from the light source. If the light intensity 5 feet from the light source is 10 foot-candles, what is the intensity of the light 10 feet from the light source?

29. Suppose that the price per person for a camping experience is inversely proportional to the number of people who sign up for the experience. If 18 people sign up, the price per person is $204. What will be the price per person if 35 people sign up? Round your answer to the nearest dollar.

30. Suppose that the price per person for a camping experience is inversely proportional to the number of people who sign up for the experience. If 17 people sign up, the price per person is $213. What will be the price per person if 27 people sign up? Round your answer to the nearest dollar.
### 7.5. DIRECT AND INVERSE VARIATION

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<th></th>
<th></th>
<th>Answers</th>
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<tbody>
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<td>1.</td>
<td>400</td>
<td>17.</td>
<td>$\frac{209}{8}$</td>
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</tr>
<tr>
<td>3.</td>
<td>1750329</td>
<td>19.</td>
<td>$\frac{56}{25}$</td>
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<td>5.</td>
<td>1682</td>
<td>21.</td>
<td>$\frac{16}{3}$</td>
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<td>7.</td>
<td>6075</td>
<td>23.</td>
<td>$\frac{250}{27}$</td>
<td></td>
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<tr>
<td>9.</td>
<td>233280</td>
<td>25.</td>
<td>40 inches</td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>480</td>
<td>27.</td>
<td>1.0 foot-candles</td>
<td></td>
</tr>
<tr>
<td>13.</td>
<td>$\frac{116}{5}$</td>
<td>29.</td>
<td>$105$</td>
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<tr>
<td>15.</td>
<td>99</td>
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