Prealgebra Textbook

Second Edition

Chapter 1 Odd Solutions

Department of Mathematics
College of the Redwoods

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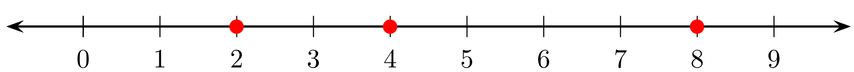
Contents

Chapter 1

The Whole Numbers

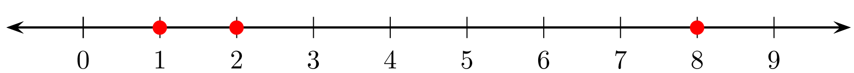
1.1 An Introduction to the Whole Numbers

1. Arrange the numbers 2, 8, and 4 on a number line.



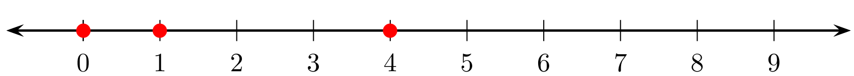
Thus, listing the numbers in order from smallest to largest, 2, 4, and 8.

3. Arrange the numbers 1, 8, and 2 on a number line.



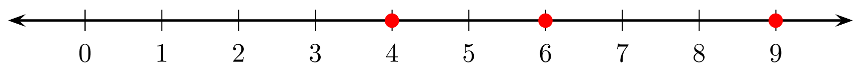
Thus, listing the numbers in order from smallest to largest, 1, 2, and 8.

5. Arrange the numbers 0, 4, and 1 on a number line.



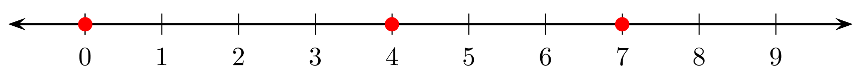
Thus, listing the numbers in order from smallest to largest, 0, 1, and 4.

7. Arrange the numbers 4, 9, and 6 on a number line.



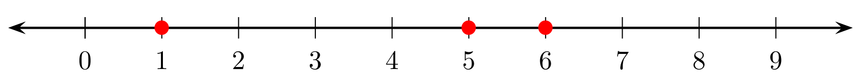
Thus, listing the numbers in order from smallest to largest, 4, 6, and 9.

9. Arrange the numbers 0, 7, and 4 on a number line.



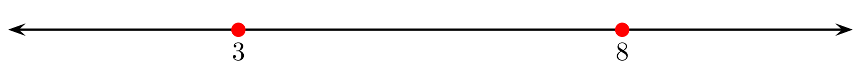
Thus, listing the numbers in order from smallest to largest, 0, 4, and 7.

11. Arrange the numbers 1, 6, and 5 on a number line.



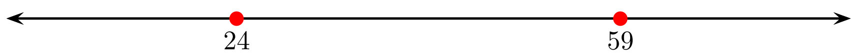
Thus, listing the numbers in order from smallest to largest, 1, 5, and 6.

13. On the number line, 3 lies to the left of 8.



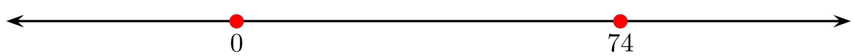
Therefore, $3 < 8$.

15. On the number line, 59 lies to the right of 24.



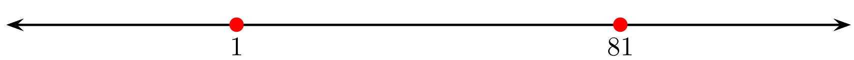
Therefore, $59 > 24$.

17. On the number line, 0 lies to the left of 74.



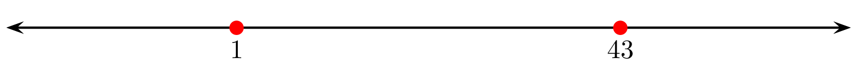
Therefore, $0 < 74$.

19. On the number line, 1 lies to the left of 81.



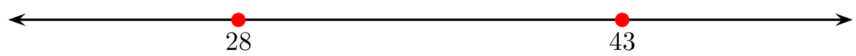
Therefore, $1 < 81$.

21. On the number line, 43 lies to the right of 1.



Therefore, $43 > 1$.

23. On the number line, 43 lies to the right of 28.



Therefore, $43 > 28$.

25. The thousands column is the fourth column from the right. In the number 2,054,867,372, the digit in the thousands column is 7.

27. The hundred thousands column is the sixth column from the right. In the number 8,311,900,272, the digit in the hundred thousands column is 9.

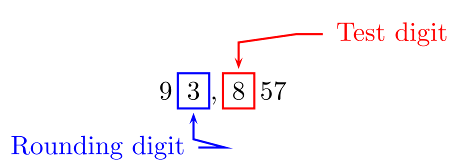
29. The hundred millions column is the ninth column from the right. In the number 9,482,616,000, the digit in the hundred millions column is 4.

31. The ten millions column is the eighth column from the right. In the number 5,840,596,473, the digit in the ten millions column is 4.

33. The hundred millions column is the ninth column from the right. In the number 5,577,422,501, the digit in the hundred millions column is 5.

35. The tens column is the second column from the right. In the number 2,461,717,362, the digit in the tens column is 6.

37. Rounding to the nearest thousand. Identify the rounding digit and the test digit.



Because the test digit is greater than or equal to five, add one to the rounding digit, then make each digit to the right of the rounding digit a zero. Thus,

$$93,857 \approx 94000.$$

39. Rounding to the nearest ten. Identify the rounding digit and the test digit.

Test digit

$$9{,}7\boxed{2}\boxed{5}$$

Rounding digit

Because the test digit is greater than or equal to five, add one to the rounding digit, then make each digit to the right of the rounding digit a zero. Thus,

$$9,725 \approx 9730.$$

41. Rounding to the nearest hundred. Identify the rounding digit and the test digit.

Test digit

$$58{,}\boxed{7}\boxed{3}9$$

Rounding digit

Because the test digit is less than five, leave the rounding digit alone, then make each digit to the right of the rounding digit a zero. Thus,

$$58,739 \approx 58700.$$

43. Rounding to the nearest ten. Identify the rounding digit and the test digit.

Test digit

$$2{,}3\boxed{5}\boxed{8}$$

Rounding digit

Because the test digit is greater than or equal to five, add one to the rounding digit, then make each digit to the right of the rounding digit a zero. Thus,

$$2,358 \approx 2360.$$

45. Rounding to the nearest thousand. Identify the rounding digit and the test digit.

Test digit

$$3\boxed{9}{,}\boxed{7}56$$

Rounding digit

Because the test digit is greater than or equal to five, add one to the rounding digit, then make each digit to the right of the rounding digit a zero. Thus,

$$39,756 \approx 40000.$$

47. Rounding to the nearest ten. Identify the rounding digit and the test digit.

Test digit

$$5{,}8\,\boxed{9}\,\boxed{4}$$

Rounding digit

Because the test digit is less than five, leave the rounding digit the same, then make each digit to the right of the rounding digit a zero. Thus,

$$5,894 \approx 5890.$$

49. Rounding to the nearest hundred. Identify the rounding digit and the test digit.

Test digit

$$56,\,\boxed{1}\,\boxed{2}\,3$$

Rounding digit

Because the test digit is less than five, leave the rounding digit alone, then make each digit to the right of the rounding digit a zero. Thus,

$$56,123 \approx 56100.$$

51. Rounding to the nearest ten. Identify the rounding digit and the test digit.

Test digit

$$5{,}4\,\boxed{8}\,\boxed{3}$$

Rounding digit

Because the test digit is less than five, leave the rounding digit the same, then make each digit to the right of the rounding digit a zero. Thus,

$$5,483 \approx 5480.$$

53. Mark the rounding and test digits.

Test digit

$$304,\boxed{0}\,\boxed{5}\,9{,}724$$

Rounding digit

The test digit is greater than or equal to 5. The "Rules for Rounding" require that we add 1 to the rounding digit, then make all digits to the right of the rounding digit zeros. Thus, rounded to the nearest hundred thousand,

$$304,059,724 \approx 304,100,000.$$

55. Mark the rounding and test digits.

Test digit

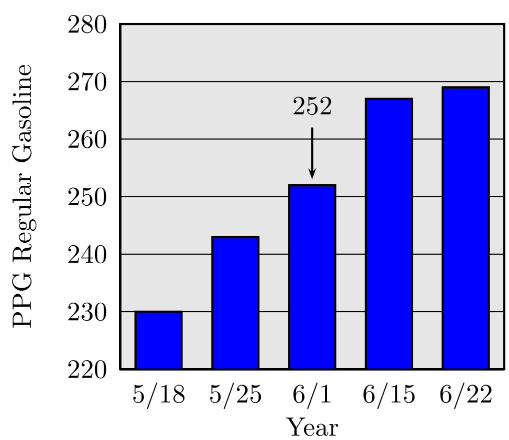
$$1\,\boxed{2}\,\boxed{9}\,{,}000$$

Rounding digit

The test digit is greater than or equal to 5. The "Rules for Rounding" require that we add 1 to the rounding digit, then make all digits to the right of the rounding digit zeros. Thus, rounded to the nearest ten thousand,

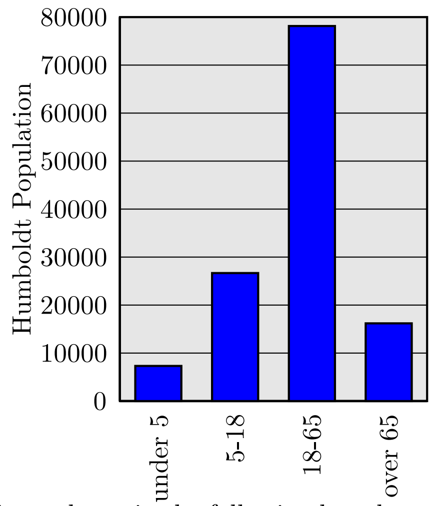
$$129,000 \approx 130,000.$$

57. We note the week of June 1, or 6/15, in the bar chart.



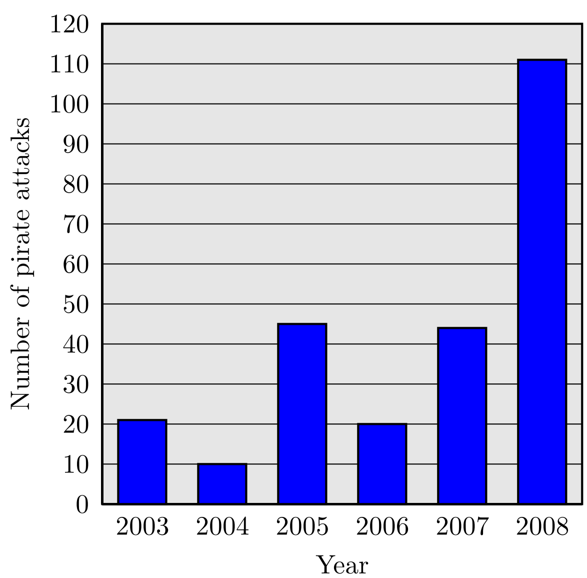
It appears that on June 1, that is, 6/15, a gallon of regular gasoline cost approximately 252 cents.

59.

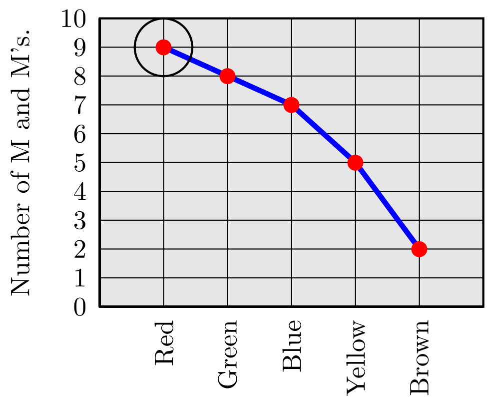


61. a) Note that the first column in the following bar chart represents the year 2003. Estimate the height of this bar to be approximately 21 according to the provided vertical scale at the left of the bar chart. Hence, the number of pirate attacks in 2003 was approximately 21.

b) In like manner, the last vertical bar represents the year 2008. The height of this bar is approximately 111. Thus, the number of pirate attacks in 2008 was approximately 111.

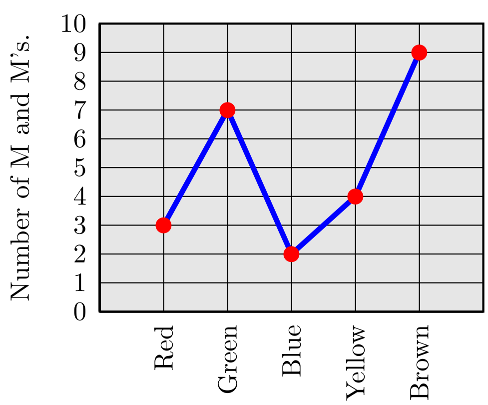


63. We've circled the point that indicates the number of red M and M's in the bowl.



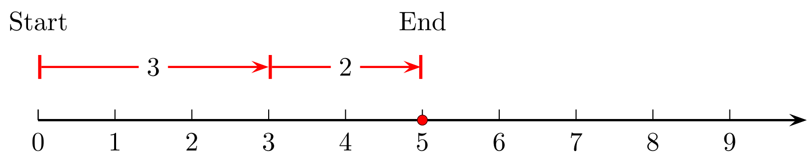
It appears that there were 9 red M and M's in the bowl.

65. Here is the line plot.



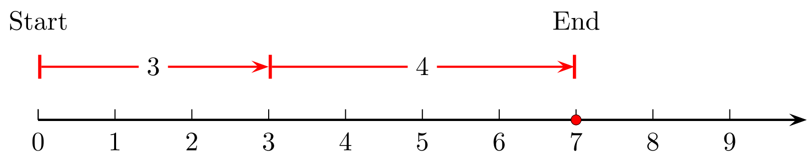
1.2 Adding and Subtracting Whole Numbers

1. Start at the number 0, then draw an arrow 3 units to the right, as shown below. Draw a second arrow of length 2, starting at the end of the first arrow. Mark the sum as a shaded dot on the number line.



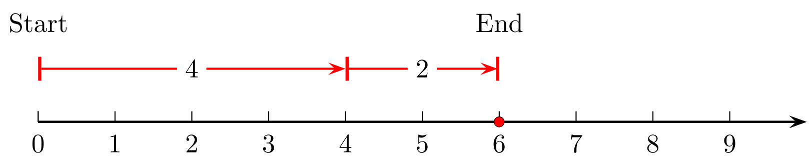
The number line shows that $3 + 2 = 5$.

3. Start at the number 0, then draw an arrow 3 units to the right, as shown below. Draw a second arrow of length 4, starting at the end of the first arrow. Mark the sum as a shaded dot on the number line.



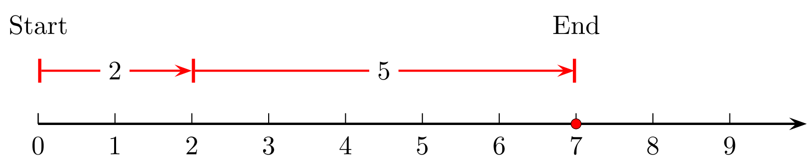
The number line shows that $3 + 4 = 7$.

5. Start at the number 0, then draw an arrow 4 units to the right, as shown below. Draw a second arrow of length 2, starting at the end of the first arrow. Mark the sum as a shaded dot on the number line.



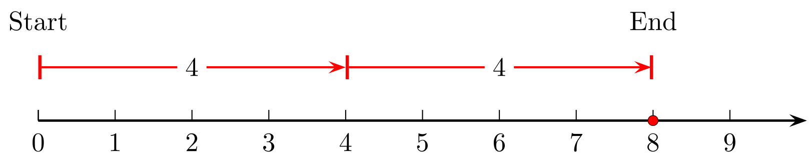
The number line shows that $4 + 2 = 6$.

7. Start at the number 0, then draw an arrow 2 units to the right, as shown below. Draw a second arrow of length 5, starting at the end of the first arrow. Mark the sum as a shaded dot on the number line.



The number line shows that $2 + 5 = 7$.

9. Start at the number 0, then draw an arrow 4 units to the right, as shown below. Draw a second arrow of length 4, starting at the end of the first arrow. Mark the sum as a shaded dot on the number line.



The number line shows that $4 + 4 = 8$.

11. Because the given identity

$$28 + 0 = 28$$

has the form

$$a + 0 = a,$$

this is an example of the additive identity property of addition.

13. Because the given identity

$$24 + 0 = 24$$

has the form

$$a + 0 = a,$$

this is an example of the additive identity property of addition.

15. Because the given identity

$$(51 + 66) + 88 = 51 + (66 + 88)$$

has the form

$$(a + b) + c = a + (b + c),$$

this is an example of the associative property of addition.

17. Because the given identity

$$64 + 39 = 39 + 64$$

has the form

$$a + b = b + a,$$

this is an example of the commutative property of addition.

19. Because the given identity

$$(70 + 27) + 52 = 70 + (27 + 52)$$

has the form

$$(a + b) + c = a + (b + c),$$

this is an example of the associative property of addition.

21. Because the given identity

$$79 + 0 = 79$$

has the form

$$a + 0 = a,$$

this is an example of the additive identity property of addition.

23. Because the given identity

$$10 + 94 = 94 + 10$$

has the form

$$a + b = b + a,$$

this is an example of the commutative property of addition.

25. Because the given identity

$$47 + 26 = 26 + 47$$

has the form

$$a + b = b + a,$$

this is an example of the commutative property of addition.

27. Because the given identity

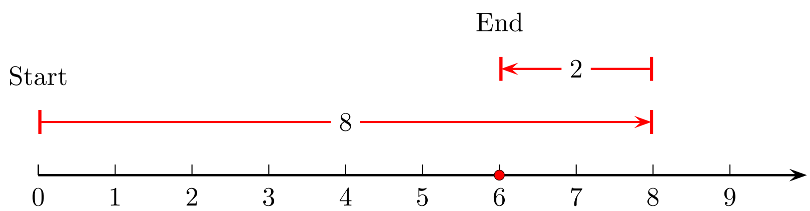
$$(61 + 53) + 29 = 61 + (53 + 29)$$

has the form

$$(a + b) + c = a + (b + c),$$

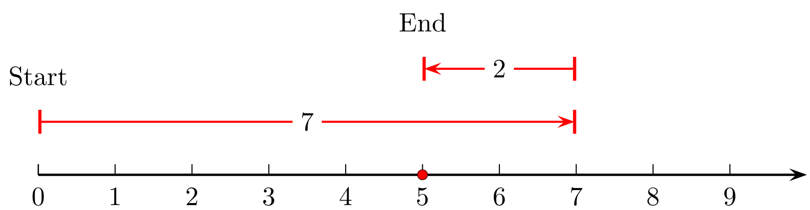
this is an example of the associative property of addition.

29. Start at the number 0, then draw an arrow 8 units to the right, as shown below. Draw a second arrow of length 2, starting at the end of the first arrow, but pointing to the left. Mark the difference as a shaded dot on the number line.



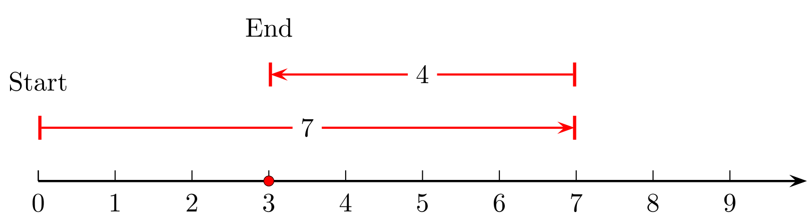
The number line shows that $8 - 2 = 6$.

31. Start at the number 0, then draw an arrow 7 units to the right, as shown below. Draw a second arrow of length 2, starting at the end of the first arrow, but pointing to the left. Mark the difference as a shaded dot on the number line.



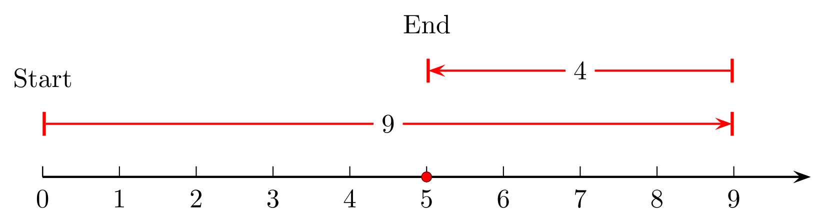
The number line shows that $7 - 2 = 5$.

33. Start at the number 0, then draw an arrow 7 units to the right, as shown below. Draw a second arrow of length 4, starting at the end of the first arrow, but pointing to the left. Mark the difference as a shaded dot on the number line.



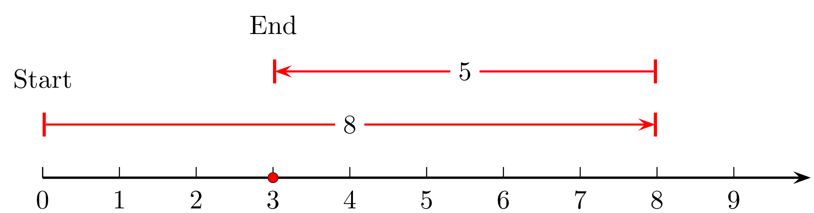
The number line shows that $7 - 4 = 3$.

35. Start at the number 0, then draw an arrow 9 units to the right, as shown below. Draw a second arrow of length 4, starting at the end of the first arrow, but pointing to the left. Mark the difference as a shaded dot on the number line.



The number line shows that $9 - 4 = 5$.

37. Start at the number 0, then draw an arrow 8 units to the right, as shown below. Draw a second arrow of length 5, starting at the end of the first arrow, but pointing to the left. Mark the difference as a shaded dot on the number line.



The number line shows that $8 - 5 = 3$.

39. Addition has no priority over subtraction; vice-versa, subtraction has no priority over addition. It is important that you perform additions and subtractions in the order that they appear, as you move from left to right through the expression.

$$\begin{aligned} 16 - 8 + 2 &= 8 + 2 && \text{Subtact: } 16 - 8 = 8. \\ &= 10 && \text{Add: } 8 + 2 = 10. \end{aligned}$$

41. Addition has no priority over subtraction; vice-versa, subtraction has no priority over addition. It is important that you perform additions and subtractions in the order that they appear, as you move from left to right through the expression.

$$\begin{aligned} 20 - 5 + 14 &= 15 + 14 && \text{Subtact: } 20 - 5 = 15. \\ &= 29 && \text{Add: } 15 + 14 = 29. \end{aligned}$$

43. Addition has no priority over subtraction; vice-versa, subtraction has no priority over addition. It is important that you perform additions and subtractions in the order that they appear, as you move from left to right through the expression.

$$\begin{aligned} 15 - 2 + 5 &= 13 + 5 && \text{Subtact: } 15 - 2 = 13. \\ &= 18 && \text{Add: } 13 + 5 = 18. \end{aligned}$$

45. Addition has no priority over subtraction; vice-versa, subtraction has no priority over addition. It is important that you perform additions and subtractions in the order that they appear, as you move from left to right through the expression.

$$\begin{aligned} 12 - 5 + 4 &= 7 + 4 && \text{Subtact: } 12 - 5 = 7. \\ &= 11 && \text{Add: } 7 + 4 = 11. \end{aligned}$$

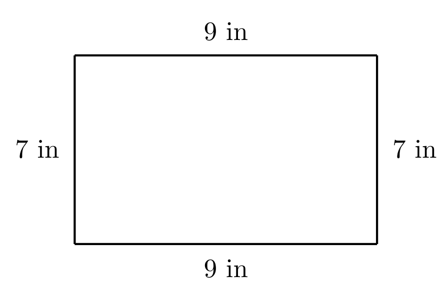
47. Addition has no priority over subtraction; vice-versa, subtraction has no priority over addition. It is important that you perform additions and subtractions in the order that they appear, as you move from left to right through the expression.

$$\begin{aligned} 12 - 6 + 4 &= 6 + 4 && \text{Subtact: } 12 - 6 = 6. \\ &= 10 && \text{Add: } 6 + 4 = 10. \end{aligned}$$

49. Addition has no priority over subtraction; vice-versa, subtraction has no priority over addition. It is important that you perform additions and subtractions in the order that they appear, as you move from left to right through the expression.

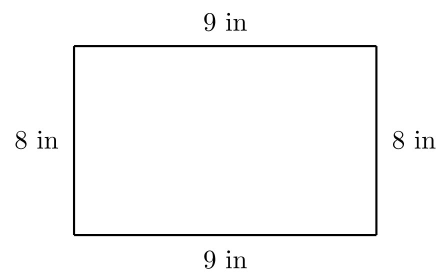
$$\begin{aligned} 15 - 5 + 8 &= 10 + 8 && \text{Subtact: } 15 - 5 = 10. \\ &= 18 && \text{Add: } 10 + 8 = 18. \end{aligned}$$

51. A rectangle is a quadrilateral (4 sides) with four right angles. The opposite sides of the rectangle are equal.



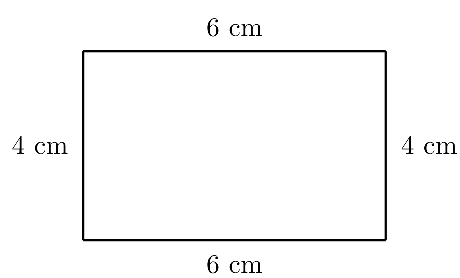
To find the perimeter of the rectangle, add up the sides. Therefore, the perimeter is $P = 7 + 9 + 7 + 9 = 32$ in.

53. A rectangle is a quadrilateral (4 sides) with four right angles. The opposite sides of the rectangle are equal.



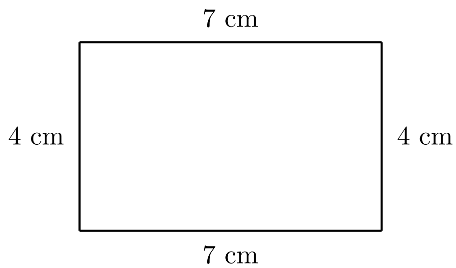
To find the perimeter of the rectangle, add up the sides. Therefore, the perimeter is $P = 8 + 9 + 8 + 9 = 34$ in.

55. A rectangle is a quadrilateral (4 sides) with four right angles. The opposite sides of the rectangle are equal.



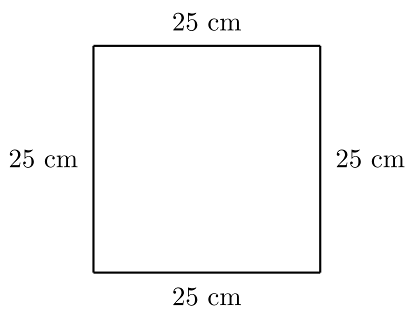
To find the perimeter of the rectangle, add up the sides. Therefore, the perimeter is $P = 4 + 6 + 4 + 6 = 20$ cm.

57. A rectangle is a quadrilateral (4 sides) with four right angles. The opposite sides of the rectangle are equal.



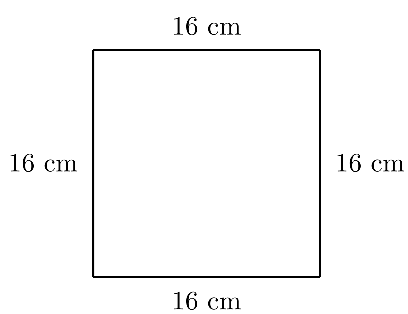
To find the perimeter of the rectangle, add up the sides. Therefore, the perimeter is $P = 4 + 7 + 4 + 7 = 22$ cm.

59. A square is a rectangle with 4 equal sides. Thus, each side of the square measures 25 cm.



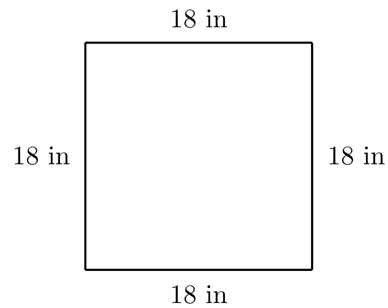
To find the perimeter of the square, add up the sides. Therefore, the perimeter is $P = 25 + 25 + 25 + 25 = 100$ cm.

61. A square is a rectangle with 4 equal sides. Thus, each side of the square measures 16 cm.



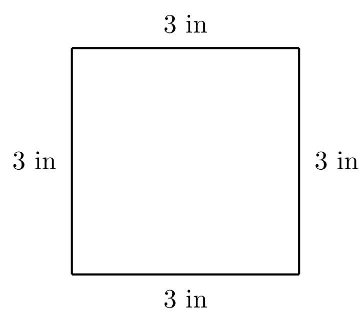
To find the perimeter of the square, add up the sides. Therefore, the perimeter is $P = 16 + 16 + 16 + 16 = 64$ cm.

63. A square is a rectangle with 4 equal sides. Thus, each side of the square measures 18 in.



To find the perimeter of the square, add up the sides. Therefore, the perimeter is $P = 18 + 18 + 18 + 18 = 72$ in.

65. A square is a rectangle with 4 equal sides. Thus, each side of the square measures 3 in.



To find the perimeter of the square, add up the sides. Therefore, the perimeter is $P = 3 + 3 + 3 + 3 = 12$ in.

67. Align the numbers vertically, then add, starting at the furthest column to the right.

$$\begin{array}{r} 3005 \\ +5217 \\ \hline 8222 \end{array}$$

Thus, $3005 + 5217 = 8222$.

69. Align the numbers vertically, then add, starting at the furthest column to the right.

$$\begin{array}{r} 575 \\ 354 \\ +759 \\ \hline 1688 \end{array}$$

Thus, $575 + 354 + 759 = 1688$.

71. Align the numbers vertically, then add, starting at the furthest column to the right. Add the grouped terms first.

$$\begin{array}{r} 520 \\ +575 \\ \hline 1095 \end{array}$$

and

$$\begin{array}{r} 472 \\ +1095 \\ \hline 1567 \end{array}$$

Thus, $472 + (520 + 575) = 1567$.

73. Align the numbers vertically, then add, starting at the furthest column to the right. Add the grouped terms first.

$$\begin{array}{r} 764 \\ +690 \\ \hline 1454 \end{array}$$

and

$$\begin{array}{r} 274 \\ +1454 \\ \hline 1728 \end{array}$$

Thus, $274 + (764 + 690) = 1728$.

75. Align the numbers vertically, then add, starting at the furthest column to the right.

$$\begin{array}{r} 8583 \\ +592 \\ \hline 9175 \end{array}$$

Thus, $8583 + 592 = 9175$.

77. Align the numbers vertically, then add, starting at the furthest column to the right.

$$\begin{array}{r} 899 \\ 528 \\ +116 \\ \hline 1543 \end{array}$$

Thus, $899 + 528 + 116 = 1543$.

79. Align the numbers vertically, then add, starting at the furthest column to the right. Add the grouped terms first.

$$\begin{array}{r} 466 \\ +744 \\ \hline 1210 \end{array}$$

and

$$\begin{array}{r} 1210 \\ +517 \\ \hline 1727 \end{array}$$

Thus, $(466 + 744) + 517 = 1727$.

81. Align the numbers vertically, then add, starting at the furthest column to the right.

$$\begin{array}{r} 563 \\ 298 \\ 611 \\ +828 \\ \hline 2300 \end{array}$$

Thus, $563 + 298 + 611 + 828 = 2300$.

83. Align the numbers vertically, then add, starting at the furthest column to the right.

$$\begin{array}{r} 607 \\ 29 \\ 270 \\ \underline{+245} \\ 1151 \end{array}$$

Thus, $607 + 29 + 270 + 245 = 1151$.

85. Align the numbers vertically, then add, starting at the furthest column to the right. Add the grouped terms first.

$$\begin{array}{r} 86 \\ \underline{+557} \\ 643 \end{array}$$

and

$$\begin{array}{r} 643 \\ \underline{+80} \\ 723 \end{array}$$

Thus, $(86 + 557) + 80 = 723$.

87. Align the numbers vertically, then subtract, starting at the furthest column to the right.

$$\begin{array}{r} 3493 \\ \underline{-2034} \\ 1459 \end{array}$$

and

$$\begin{array}{r} 1459 \\ \underline{-227} \\ 1232 \end{array}$$

Thus, $3493 - 2034 - 227 = 1232$.

89. Align the numbers vertically, then subtract, starting at the furthest column to the right.

$$\begin{array}{r} 8338 \\ -7366 \\ \hline 972 \end{array}$$

Thus, $8338 - 7366 = 972$.

91. Align the numbers vertically, then subtract, starting at the furthest column to the right.

$$\begin{array}{r} 2974 \\ -2374 \\ \hline 600 \end{array}$$

Thus, $2974 - 2374 = 600$.

93. Align the numbers vertically, then subtract, starting at the furthest column to the right. Subtract the grouped terms first.

$$\begin{array}{r} 777 \\ -241 \\ \hline 536 \end{array}$$

and

$$\begin{array}{r} 3838 \\ -536 \\ \hline 3302 \end{array}$$

Thus, $3838 - (777 - 241) = 3302$.

95. Align the numbers vertically, then subtract, starting at the furthest column to the right.

$$\begin{array}{r} 5846 \\ -541 \\ \hline 5305 \end{array}$$

and

$$\begin{array}{r} 5305 \\ -4577 \\ \hline 728 \end{array}$$

Thus, $5846 - 541 - 4577 = 728$.

97. Align the numbers vertically, then subtract, starting at the furthest column to the right. Subtract the grouped terms first.

$$\begin{array}{r} 2882 \\ \underline{-614} \\ 2268 \end{array}$$

and

$$\begin{array}{r} 3084 \\ \underline{-2268} \\ 816 \end{array}$$

Thus, $3084 - (2882 - 614) = 816$.

99. Align the numbers vertically, then subtract, starting at the furthest column to the right. Subtract the grouped terms first.

$$\begin{array}{r} 1265 \\ \underline{-251} \\ 1014 \end{array}$$

and

$$\begin{array}{r} 2103 \\ \underline{-1014} \\ 1089 \end{array}$$

Thus, $2103 - (1265 - 251) = 1089$.

101. Align the numbers vertically, then subtract, starting at the furthest column to the right.

$$\begin{array}{r} 9764 \\ \underline{-4837} \\ 4927 \end{array}$$

and

$$\begin{array}{r} 4927 \\ \underline{-150} \\ 4777 \end{array}$$

Thus, $9764 - 4837 - 150 = 4777$.

103. Align the numbers vertically, then subtract, starting at the furthest column to the right.

$$\begin{array}{r} 7095 \\ \underline{-226} \\ 6869 \end{array}$$

Thus, $7095 - 226 = 6869$.

105. To find the total water subsidy, add the numbers 79 and 439. Your answer will be in millions of dollars.

$$79 + 439 = 518$$

Therefore, the total water subsidy is \$518 million.

107. To find how much more the R16 model is, subtract 2,279 from 3,017.

$$3,017 - 2,279 = 738$$

Therefore, the R16 model costs \$738 more.

109. To find the difference between aphelion and perihelion, subtract 147 from 152. Your answer will be in millions of kilometers.

$$152 - 147 = 5$$

Therefore, aphelion is 5 million kilometers further than perihelion.

111. To find how many degrees cooler the sunspots are, subtract 6,300 from 10000. Your answer will be in degrees Fahrenheit.

$$10,000 - 6,300 = 3,700$$

Therefore, sunspots are 3,700 degrees cooler than the surrounding surface of the sun.

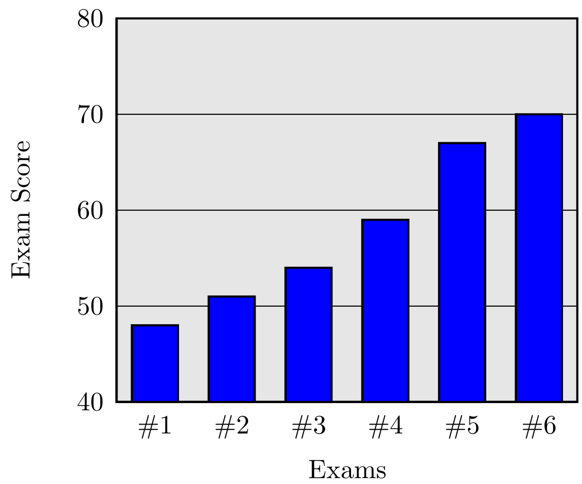
113. Add the values for each region.

$$1650 + 450 + 250 + 400 + 500 + 350 = 3600$$

There are an estimated 3600 wild tigers worldwide.

115.

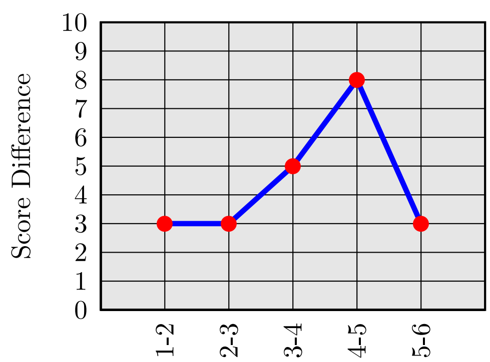
a) Bar chart.



b) Differences between consecutive exams are shown in the following table.

Exam	Difference
1-2	3
2-3	3
3-4	5
4-5	8
5-6	3

The following plot is a line plot of differences between scores on consecutive examinations.



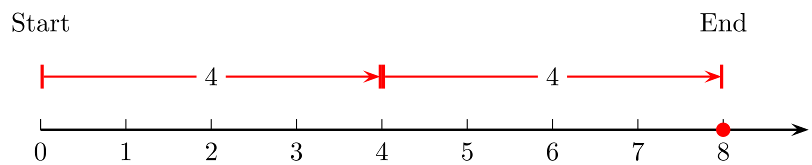
The largest improvement was between Exam #4 and Exam #5, where Emily improved by 8 points.

1.3 Multiplication and Division of Whole Numbers

1. Multiplication is equivalent to repeated addition. In this case,

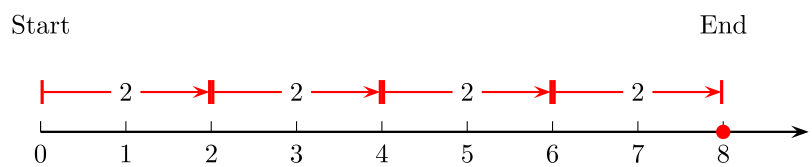
$$2 \cdot 4 = \underbrace{4 + 4}_{\text{2 times}}$$

Starting at zero, draw 2 arrows of length 4, connected tail to arrowhead and pointing to the right, as shown in the following figure.



Therefore, $2 \cdot 4 = 8$.

3. Starting at zero, draw 4 arrows of length 2, connected tail to arrowhead and pointing to the right, as shown in the following figure.



5. Because the identity

$$9 \cdot 8 = 8 \cdot 9$$

has the form

$$a \cdot b = b \cdot a,$$

this identity is an example of the commutative property of multiplication.

7. Because the identity

$$8 \cdot (5 \cdot 6) = (8 \cdot 5) \cdot 6$$

has the form

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

this identity is an example of the associative property of multiplication.

9. Because the identity

$$6 \cdot 2 = 2 \cdot 6$$

has the form

$$a \cdot b = b \cdot a,$$

this identity is an example of the commutative property of multiplication.

11. Because the identity

$$3 \cdot (5 \cdot 9) = (3 \cdot 5) \cdot 9$$

has the form

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

this identity is an example of the associative property of multiplication.

13. Because the identity

$$21 \cdot 1 = 21$$

has the form

$$a \cdot 1 = a,$$

this identity is an example of the multiplicative identity property.

15. Because the identity

$$13 \cdot 1 = 13$$

has the form

$$a \cdot 1 = a,$$

this identity is an example of the multiplicative identity property.

17. Use vertical format:

$$\begin{array}{r} 78 \\ \underline{\times\ 3} \\ 234 \end{array}$$

Thus, $78 \cdot 3 = 234$.

19. Use vertical format:

$$\begin{array}{r} 907 \\ \times\ \ 6 \\ \hline 5442 \end{array}$$

Thus, $907 \cdot 6 = 5442$.

21. Use vertical format:

$$\begin{array}{r} 128 \\ \times\ \ 30 \\ \hline 3840 \end{array}$$

Thus, $128 \cdot 30 = 3840$.

23. Use vertical format:

$$\begin{array}{r} 799 \\ \times\ \ 60 \\ \hline 47940 \end{array}$$

Thus, $799 \cdot 60 = 47940$.

25. Use vertical format:

$$\begin{array}{r} 14 \\ \times\ 70 \\ \hline 980 \end{array}$$

Thus, $14 \cdot 70 = 980$.

27. Use vertical format:

$$\begin{array}{r} 34 \\ \times\ 90 \\ \hline 3060 \end{array}$$

Thus, $34 \cdot 90 = 3060$.

29. Use vertical format:

$$\begin{array}{r} 237 \\ \times\ \ 54 \\ \hline 948 \\ 1185 \\ \hline 12798 \end{array}$$

Thus, $237 \cdot 54 = 12798$.

31. Use vertical format:

$$\begin{array}{r} 691 \\ \times\ \ 12 \\ \hline 1382 \\ 691 \\ \hline 8292 \end{array}$$

Thus, $691 \cdot 12 = 8292$.

33. Use vertical format:

$$\begin{array}{r} 955 \\ \times\ \ 89 \\ \hline 8595 \\ 7640 \\ \hline 84995 \end{array}$$

Thus, $955 \cdot 89 = 84995$.

35. Use vertical format:

$$\begin{array}{r} 266 \\ \times\ \ 61 \\ \hline 266 \\ 1596 \\ \hline 16226 \end{array}$$

Thus, $266 \cdot 61 = 16226$.

37. Use vertical format:

$$\begin{array}{r} 365 \\ \times\ 73 \\ \hline 1095 \\ 2555 \\ \hline 26645 \end{array}$$

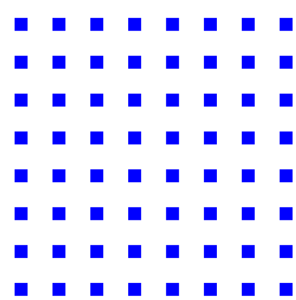
Thus, $365 \cdot 73 = 26645$.

39. Use vertical format:

$$\begin{array}{r} 955 \\ \times\ 57 \\ \hline 6685 \\ 4775 \\ \hline 54435 \end{array}$$

Thus, $955 \cdot 57 = 54435$.

41. Note that there are 8 rows and 8 columns in the array.

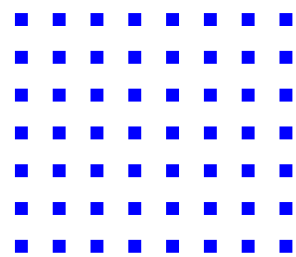


Multiply the number of rows by the number of columns to determine the number of objects in the array.

$$\begin{aligned} \text{Number of Objects} &= \text{Number of Rows} \cdot \text{Number of Columns} \\ &= 8 \cdot 8 \\ &= 64 \end{aligned}$$

Therefore, there are 64 objects in the array.

43. Note that there are 7 rows and 8 columns in the array.



Multiply the number of rows by the number of columns to determine the number of objects in the array.

$$\begin{aligned}\text{Number of Objects} &= \text{Number of Rows} \cdot \text{Number of Columns}\\ &= 7 \cdot 8\\ &= 56\end{aligned}$$

Therefore, there are 56 objects in the array.

45. The formula for the area of a rectangle is

$$A = LW,$$

where L is the length and W is the width of the rectangle, respectively. Substitute $L = 50\,\text{in}$ and $W = 25\,\text{in}$ and simplify.

$$\begin{aligned}A &= (50\,\text{in})(25\,\text{in}) && \text{Substitute } L = 50\,\text{in and } W = 25\,\text{in.}\\ &= 1250\,\text{in}^2 && \text{Multiply: } 50 \cdot 25 = 1250.\end{aligned}$$

Note also that $\text{in} \cdot \text{in} = \text{in}^2$. Hence, the area of the rectangle is $A = 1250$ square inches.

47. The formula for the area of a rectangle is

$$A = LW,$$

where L is the length and W is the width of the rectangle, respectively. Substitute $L = 47\,\text{in}$ and $W = 13\,\text{in}$ and simplify.

$$\begin{aligned}A &= (47\,\text{in})(13\,\text{in}) && \text{Substitute } L = 47\,\text{in and } W = 13\,\text{in.}\\ &= 611\,\text{in}^2 && \text{Multiply: } 47 \cdot 13 = 611.\end{aligned}$$

Note also that $\text{in} \cdot \text{in} = \text{in}^2$. Hence, the area of the rectangle is $A = 611$ square inches.

49. To find the perimeter, find the sum of the four sides of the rectangle. Hence, the perimeter of the rectangle having length L and width W is

$$P = L + W + L + W.$$

Susbstitute $L = 25\,\mathrm{in}$ and $W = 16\,\mathrm{in}$ and simplify.

$$\begin{aligned} P &= (25\,\mathrm{in}) + (16\,\mathrm{in}) + (25\,\mathrm{in}) + (16\,\mathrm{in}) && L = 25\,\mathrm{in} \text{ and } W = 16\,\mathrm{in}. \\ &= 82\,\mathrm{in} && \text{Add: } 25 + 16 + 25 + 16 = 82. \end{aligned}$$

Hence the perimeter is $P = 82$ inches.

51. To find the perimeter, find the sum of the four sides of the rectangle. Hence, the perimeter of the rectangle having length L and width W is

$$P = L + W + L + W.$$

Susbstitute $L = 30\,\mathrm{in}$ and $W = 28\,\mathrm{in}$ and simplify.

$$\begin{aligned} P &= (30\,\mathrm{in}) + (28\,\mathrm{in}) + (30\,\mathrm{in}) + (28\,\mathrm{in}) && L = 30\,\mathrm{in} \text{ and } W = 28\,\mathrm{in}. \\ &= 116\,\mathrm{in} && \text{Add: } 30 + 28 + 30 + 28 = 116. \end{aligned}$$

Hence the perimeter is $P = 116$ inches.

53.

$$19 \cdot 50 = 950 \text{ cents} = \$9.50$$

55.

$$47 \cdot 15 = 705 \text{ dollars}$$

57.

$$24 \cdot 12 \cdot 12 = 3456 \text{ eggs}$$

59.

$$5000 \cdot 4 = 20000 \text{ kilograms}$$

61. The expressions

$$\frac{30}{5}, \quad 30 \div 5, \quad 5\overline{)30}$$

are identical. The expression

$$5 \div 30$$

differs from the remaining three.

63. The expressions

$$\frac{8}{2}, \quad 8 \div 2, \quad 2\overline{)8}$$

are identical. The expression

$$8\overline{)2}$$

differs from the remaining three.

65. The expressions

$$\frac{14}{2}, \quad 14 \div 2, \quad 2\overline{)14}$$

are identical. The expression

$$14\overline{)2}$$

differs from the remaining three.

67. The expressions

$$\frac{24}{3}, \quad 24 \div 3, \quad 3\overline{)24}$$

are identical. The expression

$$3 \div 24$$

differs from the remaining three.

69. When a is a nonzero whole number, $0 \div a = 0$. There are zero groups of 11 in zero. Hence,

$$0 \div 11 = 0.$$

71. Division by zero is undefined. Hence,

$$17 \div 0$$

is undefined.

73. The Multiplication by Zero property says that $a \cdot 0 = 0$ for any whole number a. Hence,

$$10 \cdot 0 = 0.$$

75. Division by zero is undefined. Hence,

$$\frac{7}{0}$$

is undefined.

77. When zero is divided by a nonzero number, the answer is zero. Hence:

$$16\overline{)0}^{\;0}$$

79. When zero is divided by a nonzero whole number, the answer is zero. There are zero groups of 24 in zero. Hence,

$$\frac{0}{24} = 0.$$

81. Division by zero is undefined. Hence,

$$0\overline{)0}$$

is undefined.

83. By long division,

$$\begin{array}{r} 64 \\ 44\overline{)2816} \\ \underline{264} \\ 176 \\ \underline{176} \\ 0 \end{array}$$

Thus, $\dfrac{2816}{44} = 64.$

85. By long division,

$$\begin{array}{r} 27 \\ 83\overline{)2241} \\ \underline{166} \\ 581 \\ \underline{581} \\ 0 \end{array}$$

Thus, $\dfrac{2241}{83} = 27.$

87. By long division,

$$
\begin{array}{r}
44 \\
73\overline{)3212} \\
\underline{292} \\
292 \\
\underline{292} \\
0
\end{array}
$$

Thus, $\dfrac{3212}{73} = 44$.

89. By long division,

$$
\begin{array}{r}
89 \\
98\overline{)8722} \\
\underline{784} \\
882 \\
\underline{882} \\
0
\end{array}
$$

Thus, $\dfrac{8722}{98} = 89$.

91. By long division,

$$
\begin{array}{r}
15 \\
96\overline{)1440} \\
\underline{96} \\
470 \\
\underline{480} \\
0
\end{array}
$$

Thus, $\dfrac{1440}{96} = 15$.

93. By long division,

$$
\begin{array}{r}
95 \\
85\overline{)8075} \\
\underline{765} \\
425 \\
\underline{425} \\
0
\end{array}
$$

Thus, $\dfrac{8075}{85} = 95$.

95. By long division,

$$
\begin{array}{r}
193\\
92\overline{)17756}\\
\underline{92}\\
855\\
\underline{828}\\
276\\
\underline{276}\\
0
\end{array}
$$

Thus, $\dfrac{17756}{92} = 193.$

97. By long division,

$$
\begin{array}{r}
629\\
19\overline{)11951}\\
\underline{114}\\
55\\
\underline{38}\\
171\\
\underline{171}\\
0
\end{array}
$$

Thus, $\dfrac{11951}{19} = 629.$

99. By long division,

$$
\begin{array}{r}
564\\
32\overline{)18048}\\
\underline{160}\\
204\\
\underline{192}\\
128\\
\underline{128}\\
0
\end{array}
$$

Thus, $\dfrac{18048}{32} = 564.$

101. By long division,

$$
\begin{array}{r}
937\\
31\overline{)29047}\\
\underline{279}\\
114\\
\underline{93}\\
217\\
\underline{217}\\
0
\end{array}
$$

Thus, $\dfrac{29047}{31} = 937$.

103. By long division,

$$
\begin{array}{r}
426\\
53\overline{)22578}\\
\underline{212}\\
137\\
\underline{106}\\
318\\
\underline{318}\\
0
\end{array}
$$

Thus, $\dfrac{22578}{53} = 426$.

105. By long division,

$$
\begin{array}{r}
921\\
14\overline{)12894}\\
\underline{126}\\
29\\
\underline{28}\\
14\\
\underline{14}\\
0
\end{array}
$$

Thus, $\dfrac{12894}{14} = 921$.

107. $\dfrac{132}{6} = 22$, so 22 blocks are required.

109. 38 divided by 5 is 7, with a remainder of 3. Therefore, 8 trips are required.

111. The street is $5280 \cdot 4 = 21120$ feet long. 21120 divided by 145 is 145, with a remainder of 95. Therefore, 145 lights are required along the interior of the street. Counting the ends, 147 lights are required.

113. $\dfrac{292}{4} = 73$, so 73 blocks are required.

115. 32 divided by 3 is 10, with a remainder of 2. Therefore, 11 trips are required.

117. The street is $5280 \cdot 2 = 10560$ feet long. 10560 divided by 105 is 100, with a remainder of 60. Therefore, 100 lights are required along the interior of the street. Counting the ends, 102 lights are required.

119. To find the number of articles Eli writes in one week, multiply the numbers 4 and 5. Your answer will be in articles per week.

$$4 \cdot 5 = 20$$

Therefore, Eli writes 20 articles each week.

121. To find the number of yards in 27 laps, multiply 25 by 2 to get 50 yards for one round trip lap. Then multiply that 50 yards by 27.

$$2 \cdot 25 \cdot 27 = 1350$$

Therefore, Wendell swims 1,350 yards when he does 27 laps.

123. To find the minimum amount of hay a horse could eat over a year, multiply the 12 pounds each day by 365 days.

$$12 \cdot 365 = 4,380$$

Therefore, the minimum amout of hay a horse could eat will be 4,380 pounds.

125. To find the cost for non-residents over four years, multiply the 22,000 cost for one year by 4.

$$22,000 \cdot 4 = 88,000$$

Therefore, non-resident undergraduate could expect to pay $88,000 to attend UC for four years.

127. The formula for the area of a rectangle is

$$A = LW,$$

where L is the length and W is the width of the rectangle, respectively. Substitute $L = 48\,\text{mi}$ and $W = 28\,\text{mi}$ and simplify.

$$\begin{aligned} A &= (48\,\text{mi})(28\,\text{mi}) && \text{Substitute } L = 48\,\text{mi and } W = 28\,\text{mi.} \\ &= 1344\,\text{mi}^2 && \text{Multiply: } 48 \cdot 28 = 1344. \end{aligned}$$

Note also that $\text{mi} \cdot \text{mi} = \text{mi}^2$. Hence, the area of the rectangle is $A = 1344$ square miles.

129. To find the cost to lay the sidewalk, first find the total area in square feet. The total area of the rectangular sidewalk will be the length times the width.

$$\begin{aligned} \text{Area} &= LW \\ &= 80 \cdot 4 \\ &= 320. \end{aligned}$$

The total area is 320 square feet. To find the cost of laying the sidewalk, multiply the area by 8.

$$320 \cdot 8 = 2,560$$

Therefore, the total cost for the sidewalk is \$2,560.

131. Pairs of sunspots means two at a time. If the total count of sunspots is 72, dividing by 2 will give us the number of pairs.

$$72 \div 2 = 36$$

There are 36 pairs of sunspots.

1.4 Prime Factorization

1. Because

$$\begin{aligned} 30 &= 1 \cdot 30 \\ 30 &= 2 \cdot 15 \\ 30 &= 3 \cdot 10 \\ 30 &= 5 \cdot 6 \end{aligned}$$

the divisors of 30 are: 1, 2, 3, 5, 6, 10, 15, 30.

3. Because

$$83 = 1 \cdot 83$$

the divisors of 83 are: 1, 83.

5. Because

$$\begin{aligned} 91 &= 1 \cdot 91 \\ 91 &= 7 \cdot 13 \end{aligned}$$

the divisors of 91 are: 1, 7, 13, 91.

7. Because

$$\begin{aligned} 75 &= 1 \cdot 75 \\ 75 &= 3 \cdot 25 \\ 75 &= 5 \cdot 15 \end{aligned}$$

the divisors of 75 are: 1, 3, 5, 15, 25, 75.

9. Because

$$\begin{aligned} 64 &= 1 \cdot 64 \\ 64 &= 2 \cdot 32 \\ 64 &= 4 \cdot 16 \\ 64 &= 8 \cdot 8 \end{aligned}$$

the divisors of 64 are: 1, 2, 4, 8, 16, 32, 64.

11. Because

$$\begin{aligned} 14 &= 1 \cdot 14 \\ 14 &= 2 \cdot 7 \end{aligned}$$

the divisors of 14 are: 1, 2, 7, 14.

13. A number is divisible by 2 if it ends in 0, 2, 4, 6, or 8. The number 117 does not end in 0, 2, 4, 6, 8; hence, 117 is **not** divisible by 2.

15. A number is divisible by 2 if it ends in 0, 2, 4, 6, or 8. The number 13 does not end in 0, 2, 4, 6, 8; hence, 13 is **not** divisible by 2.

17. A number is divisible by 2 if it ends in 0, 2, 4, 6, or 8. The number 105 does not end in 0, 2, 4, 6, 8; hence, 105 is **not** divisible by 2.

19. A number is divisible by 2 if it ends in 0, 2, 4, 6, or 8. The number 31 does not end in 0, 2, 4, 6, 8; hence, 31 is **not** divisible by 2.

21. A number is divisible by 3 if the sum of its digits is divisible by 3.

- Because $5 + 6 + 1 = 12$ is divisible by 3, 561 is divisible by 3.
- Because $5 + 6 + 4 = 15$ is divisible by 3, 564 is divisible by 3.
- Because $8 + 4 + 6 = 18$ is divisible by 3, 846 is divisible by 3.

However,

$$3 + 6 + 4 = 13,$$

which is **not** divisible by 3. Hence, 364 is **not** divisible by 3.

23. A number is divisible by 3 if the sum of its digits is divisible by 3.

- Because $1 + 8 + 6 = 15$ is divisible by 3, 186 is divisible by 3.
- Because $3 + 1 + 5 = 9$ is divisible by 3, 315 is divisible by 3.
- Because $8 + 0 + 4 = 12$ is divisible by 3, 804 is divisible by 3.

However,

$$5 + 5 + 0 = 10,$$

which is **not** divisible by 3. Hence, 550 is **not** divisible by 3.

25. A number is divisible by 3 if the sum of its digits is divisible by 3.

- Because $6 + 6 + 3 = 15$ is divisible by 3, 663 is divisible by 3.
- Because $4 + 1 + 4 = 9$ is divisible by 3, 414 is divisible by 3.
- Because $7 + 8 + 9 = 24$ is divisible by 3, 789 is divisible by 3.

However,

$$8 + 2 + 0 = 10,$$

which is **not** divisible by 3. Hence, 820 is **not** divisible by 3.

27. A number is divisible by 3 if the sum of its digits is divisible by 3.

- Because $3 + 3 + 0 = 6$ is divisible by 3, 330 is divisible by 3.
- Because $6 + 0 + 0 = 6$ is divisible by 3, 600 is divisible by 3.
- Because $1 + 5 + 0 = 6$ is divisible by 3, 150 is divisible by 3.

However,

$$4 + 9 + 3 = 16,$$

which is **not** divisible by 3. Hence, 493 is **not** divisible by 3.

29. A number is divisible by 4 if the last two digits represent a number that is divisible by 4.

- The last two digits of 7648 are 48. Note that 48 is divisible by 4. Hence, 7648 is divisible by 4.
- The last two digits of 4048 are 48. Note that 48 is divisible by 4. Hence, 4048 is divisible by 4.
- The last two digits of 9944 are 44. Note that 44 is divisible by 4. Hence, 9944 is divisible by 4.

However, the last two digits of 3797 are 97. Note that 97 is **not** divisible by 4. Hence, 3797 is **not** divisible by 4.

31. A number is divisible by 4 if the last two digits represent a number that is divisible by 4.

- The last two digits of 4184 are 84. Note that 84 is divisible by 4. Hence, 4184 is divisible by 4.
- The last two digits of 9336 are 36. Note that 36 is divisible by 4. Hence, 9336 is divisible by 4.
- The last two digits of 2460 are 60. Note that 60 is divisible by 4. Hence, 2460 is divisible by 4.

However, the last two digits of 9701 are 1. Note that 1 is **not** divisible by 4. Hence, 9701 is **not** divisible by 4.

33. A number is divisible by 4 if the last two digits represent a number that is divisible by 4.

- The last two digits of 8332 are 32. Note that 32 is divisible by 4. Hence, 8332 is divisible by 4.
- The last two digits of 9816 are 16. Note that 16 is divisible by 4. Hence, 9816 is divisible by 4.
- The last two digits of 7408 are 8. Note that 8 is divisible by 4. Hence, 7408 is divisible by 4.

However, the last two digits of 7517 are 17. Note that 17 is **not** divisible by 4. Hence, 7517 is **not** divisible by 4.

35. A number is divisible by 4 if the last two digits represent a number that is divisible by 4.

- The last two digits of 1244 are 44. Note that 44 is divisible by 4. Hence, 1244 is divisible by 4.
- The last two digits of 7312 are 12. Note that 12 is divisible by 4. Hence, 7312 is divisible by 4.
- The last two digits of 1916 are 16. Note that 16 is divisible by 4. Hence, 1916 is divisible by 4.

However, the last two digits of 7033 are 33. Note that 33 is **not** divisible by 4. Hence, 7033 is **not** divisible by 4.

37. A number is divisible by 5 if and only if it ends with a 0 or 5.

- The number 4120 ends in a 0. Hence, 4120 is divisible by 5.
- The number 8920 ends in a 0. Hence, 8920 is divisible by 5.
- The number 5285 ends in a 5. Hence, 5285 is divisible by 5.

However, the number 9896 does **not** end in a zero or 5. Hence, 9896 is **not** divisible by 5.

39. A number is divisible by 5 if and only if it ends with a 0 or 5.

- The number 8915 ends in a 5. Hence, 8915 is divisible by 5.
- The number 3695 ends in a 5. Hence, 3695 is divisible by 5.
- The number 3005 ends in a 5. Hence, 3005 is divisible by 5.

However, the number 8758 does **not** end in a zero or 5. Hence, 8758 is **not** divisible by 5.

41. A number is divisible by 5 if and only if it ends with a 0 or 5.

- The number 5235 ends in a 5. Hence, 5235 is divisible by 5.
- The number 4240 ends in a 0. Hence, 4240 is divisible by 5.
- The number 4145 ends in a 5. Hence, 4145 is divisible by 5.

However, the number 2363 does **not** end in a zero or 5. Hence, 2363 is **not** divisible by 5.

43. A number is divisible by 5 if and only if it ends with a 0 or 5.

- The number 5550 ends in a 0. Hence, 5550 is divisible by 5.
- The number 4065 ends in a 5. Hence, 4065 is divisible by 5.
- The number 5165 ends in a 5. Hence, 5165 is divisible by 5.

However, the number 1269 does **not** end in a zero or 5. Hence, 1269 is **not** divisible by 5.

45. A number is divisible by 6 if and only if it is divisible by both 2 and 3. A number is divisible by 2 if it ends in 0 2, 4, 6, or 8. A number is divisible by 3 if the sum of its digits is divisible by 3.

- Because $9 + 9 + 0 = 18$ is divisible by 3, 990 is divisible by 3. Because 990 ends in a 0, it is also divisible by 2. Hence, 990 is divisible by 6.
- Because $5 + 2 + 8 = 15$ is divisible by 3, 528 is divisible by 3. Because 528 ends in a 8, it is also divisible by 2. Hence, 528 is divisible by 6.
- Because $3 + 7 + 2 = 12$ is divisible by 3, 372 is divisible by 3. Because 372 ends in a 2, it is also divisible by 2. Hence, 372 is divisible by 6.

Because 328 ends in a 8, it is divisible by 2. However,

$$3 + 2 + 8 = 13,$$

which is **not** divisible by 3. Hence, 328 is **not** divisible by 3. Because 328 is **not** divisible by both 2 anbd 3, it is not divisible by 6.

47. A number is divisible by 6 if and only if it is divisible by both 2 and 3. A number is divisible by 2 if it ends in 0 2, 4, 6, or 8. A number is divisible by 3 if the sum of its digits is divisible by 3.

- Because $7 + 4 + 4 = 15$ is divisible by 3, 744 is divisible by 3. Because 744 ends in a 4, it is also divisible by 2. Hence, 744 is divisible by 6.
- Because $1 + 7 + 4 = 12$ is divisible by 3, 174 is divisible by 3. Because 174 ends in a 4, it is also divisible by 2. Hence, 174 is divisible by 6.
- Because $9 + 2 + 4 = 15$ is divisible by 3, 924 is divisible by 3. Because 924 ends in a 4, it is also divisible by 2. Hence, 924 is divisible by 6.

Because 538 ends in a 8, it is divisible by 2. However,

$$5 + 3 + 8 = 16,$$

which is **not** divisible by 3. Hence, 538 is **not** divisible by 3. Because 538 is **not** divisible by both 2 anbd 3, it is not divisible by 6.

49. A number is divisible by 6 if and only if it is divisible by both 2 and 3. A number is divisible by 2 if it ends in 0 2, 4, 6, or 8. A number is divisible by 3 if the sum of its digits is divisible by 3.

- Because $4 + 7 + 4 = 15$ is divisible by 3, 474 is divisible by 3. Because 474 ends in a 4, it is also divisible by 2. Hence, 474 is divisible by 6.
- Because $6 + 3 + 6 = 15$ is divisible by 3, 636 is divisible by 3. Because 636 ends in a 6, it is also divisible by 2. Hence, 636 is divisible by 6.
- Because $2+3+4 = 9$ is divisible by 3, 234 is divisible by 3. Because 234 ends in a 4, it is also divisible by 2. Hence, 234 is divisible by 6.

Because 586 ends in a 6, it is divisible by 2. However,

$$5 + 8 + 6 = 19,$$

which is **not** divisible by 3. Hence, 586 is **not** divisible by 3. Because 586 is **not** divisible by both 2 anbd 3, it is not divisible by 6.

51. A number is divisible by 6 if and only if it is divisible by both 2 and 3. A number is divisible by 2 if it ends in 0 2, 4, 6, or 8. A number is divisible by 3 if the sum of its digits is divisible by 3.

- Because $6 + 7 + 8 = 21$ is divisible by 3, 678 is divisible by 3. Because 678 ends in a 8, it is also divisible by 2. Hence, 678 is divisible by 6.
- Because $1 + 6 + 8 = 15$ is divisible by 3, 168 is divisible by 3. Because 168 ends in a 8, it is also divisible by 2. Hence, 168 is divisible by 6.

- Because $7+0+2=9$ is divisible by 3, 702 is divisible by 3. Because 702 ends in a 2, it is also divisible by 2. Hence, 702 is divisible by 6.

Because 658 ends in a 8, it is divisible by 2. However,

$$6+5+8=19,$$

which is **not** divisible by 3. Hence, 658 is **not** divisible by 3. Because 658 is **not** divisible by both 2 anbd 3, it is not divisible by 6.

53. A number is divisible by 8 if the last three digits represent a number that is divisible by 8.

- The last three digits of 2640 are 640. Note that 640 is divisible by 8. Hence, 2640 is divisible by 8.
- The last three digits of 8216 are 216. Note that 216 is divisible by 8. Hence, 8216 is divisible by 8.
- The last three digits of 1792 are 792. Note that 792 is divisible by 8. Hence, 1792 is divisible by 8.

However, the last three digits of 5418 are 418. Note that 418 is **not** divisible by 8. Hence, 5418 is **not** divisible by 8.

55. A number is divisible by 8 if the last three digits represent a number that is divisible by 8.

- The last three digits of 2208 are 208. Note that 208 is divisible by 8. Hence, 2208 is divisible by 8.
- The last three digits of 9016 are 16. Note that 16 is divisible by 8. Hence, 9016 is divisible by 8.
- The last three digits of 3208 are 208. Note that 208 is divisible by 8. Hence, 3208 is divisible by 8.

However, the last three digits of 8506 are 506. Note that 506 is **not** divisible by 8. Hence, 8506 is **not** divisible by 8.

57. A number is divisible by 8 if the last three digits represent a number that is divisible by 8.

- The last three digits of 4712 are 712. Note that 712 is divisible by 8. Hence, 4712 is divisible by 8.
- The last three digits of 7640 are 640. Note that 640 is divisible by 8. Hence, 7640 is divisible by 8.

- The last three digits of 3192 are 192. Note that 192 is divisible by 8. Hence, 3192 is divisible by 8.

However, the last three digits of 2594 are 594. Note that 594 is **not** divisible by 8. Hence, 2594 is **not** divisible by 8.

59. A number is divisible by 8 if the last three digits represent a number that is divisible by 8.

- The last three digits of 1232 are 232. Note that 232 is divisible by 8. Hence, 1232 is divisible by 8.
- The last three digits of 7912 are 912. Note that 912 is divisible by 8. Hence, 7912 is divisible by 8.
- The last three digits of 9808 are 808. Note that 808 is divisible by 8. Hence, 9808 is divisible by 8.

However, the last three digits of 7850 are 850. Note that 850 is **not** divisible by 8. Hence, 7850 is **not** divisible by 8.

61. A number is divisible by 9 if the sum of its digits is divisible by 9.

- Because $2 + 1 + 6 = 9$ is divisible by 9, 216 is divisible by 9.
- Because $2 + 9 + 7 = 18$ is divisible by 9, 297 is divisible by 9.
- Because $4 + 7 + 7 = 18$ is divisible by 9, 477 is divisible by 9.

However,

$$9 + 9 + 1 = 19,$$

which is **not** divisible by 9. Hence, 991 is **not** divisible by 9.

63. A number is divisible by 9 if the sum of its digits is divisible by 9.

- Because $6 + 7 + 5 = 18$ is divisible by 9, 675 is divisible by 9.
- Because $1 + 5 + 3 = 9$ is divisible by 9, 153 is divisible by 9.
- Because $2 + 3 + 4 = 9$ is divisible by 9, 234 is divisible by 9.

However,

$$9 + 3 + 7 = 19,$$

which is **not** divisible by 9. Hence, 937 is **not** divisible by 9.

65. A number is divisible by 9 if the sum of its digits is divisible by 9.

- Because $7 + 8 + 3 = 18$ is divisible by 9, 783 is divisible by 9.
- Because $5 + 9 + 4 = 18$ is divisible by 9, 594 is divisible by 9.
- Because $2 + 1 + 6 = 9$ is divisible by 9, 216 is divisible by 9.

However,

$$9 + 2 + 8 = 19,$$

which is **not** divisible by 9. Hence, 928 is **not** divisible by 9.

67. A number is divisible by 9 if the sum of its digits is divisible by 9.

- Because $4 + 2 + 3 = 9$ is divisible by 9, 423 is divisible by 9.
- Because $8 + 0 + 1 = 9$ is divisible by 9, 801 is divisible by 9.
- Because $9 + 3 + 6 = 18$ is divisible by 9, 936 is divisible by 9.

However,

$$6 + 7 + 6 = 19,$$

which is **not** divisible by 9. Hence, 676 is **not** divisible by 9.

69. The only factors of 19 are 1 and 19. Hence, 19 is a prime number.

71. The only factors of 41 are 1 and 41. Hence, 41 is a prime number.

73. 27 is a composite number. The prime factorization is $27 = 3 \cdot 3 \cdot 3$.

75. 91 is a composite number. The prime factorization is $91 = 7 \cdot 13$.

77. 21 is a composite number. The prime factorization is $21 = 3 \cdot 7$.

79. The only factors of 23 are 1 and 23. Hence, 23 is a prime number.

81. $224 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 7$

83. $108 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

85. $243 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

87. $160 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$

89. $32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

91. $360 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$

93. $144 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

95. $48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$

97. $216 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

99. Evaluate exponents first. Then multiply.

$$\begin{aligned} 5^2 \cdot 4^1 &= 25 \cdot 4 && \text{Exponents first: } 5^2 = 25 \text{ and } 4^1 = 4. \\ &= 100 && \text{Multiply: } 25 \cdot 4 = 100. \end{aligned}$$

101. The exponent 1 tells us to write the base 0 as a factor 1 times. That is,

$$\begin{aligned} 0^1 &= (0) && \text{Write the base 1 time.} \\ &= 0 && \text{Multiply.} \end{aligned}$$

103. Evaluate exponents first. Then multiply.

$$\begin{aligned} 3^3 \cdot 0^2 &= 27 \cdot 0 && \text{Exponents first: } 3^3 = 27 \text{ and } 0^2 = 0. \\ &= 0 && \text{Multiply: } 27 \cdot 0 = 0. \end{aligned}$$

105. The exponent 1 tells us to write the base 4 as a factor 1 times. That is,

$$\begin{aligned} 4^1 &= (4) && \text{Write the base 1 time.} \\ &= 4 && \text{Multiply.} \end{aligned}$$

107. The exponent 3 tells us to write the base 4 as a factor 3 times. That is,

$$\begin{aligned} 4^3 &= (4)(4)(4) && \text{Write the base 3 times.} \\ &= 64 && \text{Multiply.} \end{aligned}$$

109. Evaluate exponents first. Then multiply.

$$\begin{aligned} 3^3 \cdot 1^2 &= 27 \cdot 1 && \text{Exponents first: } 3^3 = 27 \text{ and } 1^2 = 1. \\ &= 27 && \text{Multiply: } 27 \cdot 1 = 27. \end{aligned}$$

111. The formula for the area of a square is

$$A = s^2.$$

Substitute 28 inches for s.

$$\begin{aligned} A &= (28\,\text{in})^2 && \text{Substitute 28 in for } s. \\ &= (28\,\text{in})(28\,\text{in}) && \text{Square.} \\ &= 784\,\text{in}^2 && \text{Multiply.} \end{aligned}$$

Note that $(\text{in})(\text{in}) = \text{in}^2$. Thus, the area of the square is 784 square inches.

113. The formula for the area of a square is

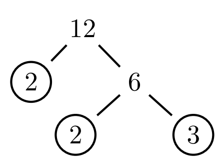
$$A = s^2.$$

Substitute 22 inches for s.

$$\begin{aligned} A &= (22\,\text{in})^2 && \text{Substitute 22 in for } s. \\ &= (22\,\text{in})(22\,\text{in}) && \text{Square.} \\ &= 484\,\text{in}^2 && \text{Multiply.} \end{aligned}$$

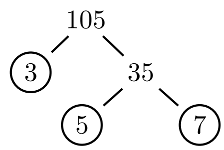
Note that $(\text{in})(\text{in}) = \text{in}^2$. Thus, the area of the square is 484 square inches.

115.



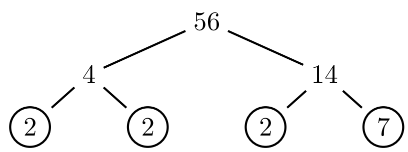
Therefore, $12 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$.

117.



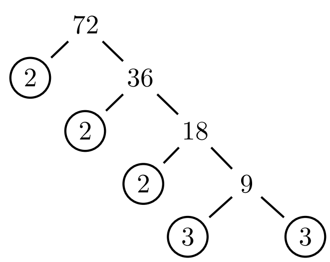
Therefore, $105 = 3 \cdot 5 \cdot 7$.

119.



Therefore, $56 = 2 \cdot 2 \cdot 2 \cdot 7 = 2^3 \cdot 7$

121.



Therefore, $72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 2^3 \cdot 3^2$

123. Follow the algorithm of Eratosthenes.

i) Strike out all multiples of 2 (4, 6, 8, etc.)

ii) The list's next number that has not been struck out is a prime number.

iii) Strike out from the list all multiples of the number you identified in step (ii).

iv) Repeat steps (ii) and (iii) until you can no longer strike any more multiples.

v) All unstruck numbers in the list are primes.

2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~	11
~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~	~~21~~
~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~	31
~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~	41
~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~	~~51~~
~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~	61
~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~	71
~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~	~~81~~
~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~	~~91~~
~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~	

The numbers that remain unstruck are prime. Thus, the primes less than 100 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97

1.5 Order of Operations

1. Order of operations demands that we do multiplications first.

$$\begin{aligned} 5 + 2 \cdot 2 &= 5 + 4 && \text{Multiply: } 2 \cdot 2 = 4. \\ &= 9 && \text{Add: } 5 + 4 = 9. \end{aligned}$$

3. Order of operations demands that we do multiplications first.

$$\begin{aligned} 23 - 7 \cdot 2 &= 23 - 14 && \text{Multiply: } 7 \cdot 2 = 14. \\ &= 9 && \text{Subtract: } 23 - 14 = 9. \end{aligned}$$

5. Multiply first, then add.

$$\begin{aligned} 4 \cdot 3 + 2 \cdot 5 &= 12 + 10 && \text{Multiply: } 4 \cdot 3 = 12 \text{ and } 2 \cdot 5 = 10. \\ &= 22 && \text{Add: } 12 + 10 = 22. \end{aligned}$$

7. Multiply first, then add.

$$\begin{aligned} 6 \cdot 5 + 4 \cdot 3 &= 30 + 12 && \text{Multiply: } 6 \cdot 5 = 30 \text{ and } 4 \cdot 3 = 12. \\ &= 42 && \text{Add: } 30 + 12 = 42. \end{aligned}$$

9. Order of operations demands that we do multiplications first.

$$\begin{aligned} 9 + 2 \cdot 3 &= 9 + 6 && \text{Multiply: } 2 \cdot 3 = 6. \\ &= 15 && \text{Add: } 9 + 6 = 15. \end{aligned}$$

11. Order of operations demands that we do multiplications first.

$$\begin{aligned} 32 - 8 \cdot 2 &= 32 - 16 && \text{Multiply: } 8 \cdot 2 = 16. \\ &= 16 && \text{Subtract: } 32 - 16 = 16. \end{aligned}$$

13. Multiplications and divisions hold no precedence over one another. They must be performed in the order that they appear, moving left to right through the expression.

$$\begin{aligned} 45 \div 3 \cdot 5 &= 15 \cdot 5 && \text{Divide: } 45 \div 3 = 15. \\ &= 75 && \text{Multiply: } 15 \cdot 5 = 75. \end{aligned}$$

15. Move left to right, performing multiplications and divisions in the order that they appear.

$$\begin{aligned} 2 \cdot 9 \div 3 \cdot 18 &= 18 \div 3 \cdot 18 && \text{Multiply: } 2 \cdot 9 = 18. \\ &= 6 \cdot 18 && \text{Divide: } 18 \div 3 = 6. \\ &= 108 && \text{Multiply: } 6 \cdot 18 = 108. \end{aligned}$$

17. Multiplications and divisions hold no precedence over one another. They must be performed in the order that they appear, moving left to right through the expression.

$$\begin{aligned} 30 \div 2 \cdot 3 &= 15 \cdot 3 && \text{Divide: } 30 \div 2 = 15. \\ &= 45 && \text{Multiply: } 15 \cdot 3 = 45. \end{aligned}$$

19. Additions and subtractions hold no precedence over one another. They must be performed as they appear, moving left to right through the expression.

$$\begin{aligned} 8 - 6 + 1 &= 2 + 1 && \text{Subtract: } 8 - 6 = 2. \\ &= 3 && \text{Add: } 2 + 1 = 3. \end{aligned}$$

21. Move left to right, performing multiplications and divisions in the order that they appear.

$$\begin{aligned}
14 \cdot 16 \div 16 \cdot 19 &= 224 \div 16 \cdot 19 && \text{Multiply: } 14 \cdot 16 = 224.\\
&= 14 \cdot 19 && \text{Divide: } 224 \div 16 = 14.\\
&= 266 && \text{Multiply: } 14 \cdot 19 = 266.
\end{aligned}$$

23. Move left to right, performing multiplications and divisions in the order that they appear. Then do the same with additions and subtractions.

$$\begin{aligned}
15 \cdot 17 + 10 \div 10 - 12 \cdot 4 &= 255 + 10 \div 10 - 12 \cdot 4 && \text{Multiply: } 15 \cdot 17 = 255.\\
&= 255 + 1 - 12 \cdot 4 && \text{Divide: } 10 \div 10 = 1.\\
&= 255 + 1 - 48 && \text{Multiply: } 12 \cdot 4 = 48.\\
&= 256 - 48 && \text{Add: } 255 + 1 = 256.\\
&= 208 && \text{Subtract: } 256 - 48 = 208.
\end{aligned}$$

25. Additions and subtractions hold no precedence over one another. They must be performed as they appear, moving left to right through the expression.

$$\begin{aligned}
22 - 10 + 7 &= 12 + 7 && \text{Subtract: } 22 - 10 = 12.\\
&= 19 && \text{Add: } 12 + 7 = 19.
\end{aligned}$$

27. Move left to right, performing multiplications and divisions in the order that they appear. Then do the same with additions and subtractions.

$$\begin{aligned}
20 \cdot 10 + 15 \div 5 - 7 \cdot 6 &= 200 + 15 \div 5 - 7 \cdot 6 && \text{Multiply: } 20 \cdot 10 = 200.\\
&= 200 + 3 - 7 \cdot 6 && \text{Divide: } 15 \div 5 = 3.\\
&= 200 + 3 - 42 && \text{Multiply: } 7 \cdot 6 = 42.\\
&= 203 - 42 && \text{Add: } 200 + 3 = 203.\\
&= 161 && \text{Subtract: } 203 - 42 = 161.
\end{aligned}$$

29. Evaluate the expression in the parentheses first, divide, then add.

$$\begin{aligned}
9 + 8 \div \{4 + 4\} &= 9 + 8 \div 8 && \text{Parens: } 4 + 4 = 8.\\
&= 9 + 1 && \text{Divide: } 8 \div 8 = 1.\\
&= 10 && \text{Add: } 9 + 1 = 10.
\end{aligned}$$

31. Evaluate the expression in the brackets first, then multiply, then subtract.

$$\begin{aligned} 7 \cdot [8 - 5] - 10 &= 7 \cdot 3 - 10 && \text{Subtract: } 8 - 5 = 3. \\ &= 21 - 10 && \text{Multiply: } 7 \cdot 3 = 21. \\ &= 11 && \text{Subtract: } 21 - 10 = 11. \end{aligned}$$

33. Evaluate the expression in the parentheses first, then divide.

$$\begin{aligned} (18 + 10) \div (2 + 2) &= 28 \div 4 && \text{Parens: } 18 + 10 = 28 \text{ and } 2 + 2 = 4. \\ &= 7 && \text{Divide: } 28 \div 4 = 7. \end{aligned}$$

35. Evaluate the expression in the parentheses first, then multiply, then subtract.

$$\begin{aligned} 9 \cdot (10 + 7) - 3 \cdot (4 + 10) &= 9 \cdot 17 - 3 \cdot 14 && \text{Add: } 10 + 7 = 17 \\ & && \text{and } 4 + 10 = 14. \\ &= 153 - 42 && \text{Multiply: } 9 \cdot 17 = 153 \\ & && \text{and } 3 \cdot 14 = 42. \\ &= 111 && \text{Subtract: } 153 - 42 = 111 \end{aligned}$$

37. Evaluate the expression in the braces first, then multiply, then divide.

$$\begin{aligned} 2 \cdot \{8 + 12\} \div 4 &= 2 \cdot 20 \div 4 && \text{Add: } 8 + 12 = 20. \\ &= 40 \div 4 && \text{Multiply: } 2 \cdot 20 = 40. \\ &= 10 && \text{Divide: } 40 \div 4 = 10. \end{aligned}$$

39. Evaluate the expression in the parentheses first, then multiply, then add.

$$\begin{aligned} 9 + 6 \cdot (12 + 3) &= 9 + 6 \cdot 15 && \text{Add: } 12 + 3 = 15. \\ &= 9 + 90 && \text{Multiply: } 6 \cdot 15 = 90. \\ &= 99 && \text{Add: } 9 + 90 = 99. \end{aligned}$$

41. Evaluate the expression in the innermost parentheses first.

$$2 + 9 \cdot [7 + 3 \cdot (9 + 5)] = 2 + 9 \cdot [7 + 3 \cdot 14] \qquad \text{Add: } 9 + 5 = 14.$$

Now evaluate the expression inside the brackets.

$$\begin{aligned} &= 2 + 9 \cdot [7 + 42] && \text{Multiply: } 3 \cdot 14 = 42. \\ &= 2 + 9 \cdot 49 && \text{Add: } 7 + 42 = 49. \\ &= 2 + 441 && \text{Multiply: } 9 \cdot 49 = 441. \\ &= 443 && \text{Add: } 2 + 441 = 443. \end{aligned}$$

43. Evaluate the expression in the innermost parentheses first.

$$7 + 3 \cdot [8 + 8 \cdot (5 + 9)] = 7 + 3 \cdot [8 + 8 \cdot 14] \qquad \text{Add: } 5 + 9 = 14.$$

Now evaluate the expression inside the brackets.

$$\begin{aligned} &= 7 + 3 \cdot [8 + 112] && \text{Multiply: } 8 \cdot 14 = 112. \\ &= 7 + 3 \cdot 120 && \text{Add: } 8 + 112 = 120. \\ &= 7 + 360 && \text{Multiply: } 3 \cdot 120 = 360. \\ &= 367 && \text{Add: } 7 + 360 = 367. \end{aligned}$$

45. When parentheses are nested, evaluate the innermost parentheses first.

$$\begin{aligned} 6 - 5[11 - (2 + 8)] &= 6 - 5[11 - 10] && \text{Inner parens first: } 2 + 8 = 10. \\ &= 6 - 5[1] && \text{Brackets next: } 11 - 10 = 1. \\ &= 6 - 5 && \text{Multiply: } 5[1] = 5. \\ &= 1 && \text{Subtract: } 6 - 5 = 1. \end{aligned}$$

47. When parentheses are nested, evaluate the innermost parentheses first.

$$\begin{aligned} 11 - 1[19 - (2 + 15)] &= 11 - 1[19 - 17] && \text{Inner parens first: } 2 + 15 = 17. \\ &= 11 - 1[2] && \text{Brackets next: } 19 - 17 = 2. \\ &= 11 - 2 && \text{Multiply: } 1[2] = 2. \\ &= 9 && \text{Subtract: } 11 - 2 = 9. \end{aligned}$$

49. Evaluate the expressions in the innermost parentheses first.

$$4\{7[9 + 3] - 2[3 + 2]\} = 4\{7[12] - 2[5]\} \qquad \text{Add: } 9 + 3 = 12 \text{ and } 3 + 2 = 5.$$

Now evaluate the expression inside the braces.

$$\begin{aligned} &= 4\{84 - 10\} && \text{Multiply: } 7[12] = 84 \\ & && \text{and } 2[5] = 10. \\ &= 4\{74\} && \text{Subtract: } 84 - 10 = 74. \\ &= 296 && \text{Multiply: } 4\{74\} = 296. \end{aligned}$$

51. Evaluate the expression in the innermost parentheses first.

$$9 \cdot [3 + 4 \cdot (5 + 2)] = 9 \cdot [3 + 4 \cdot 7] \qquad \text{Add: } 5 + 2 = 7.$$

Now evaluate the expression inside the brackets.

$$\begin{aligned} &= 9 \cdot [3 + 28] && \text{Multiply: } 4 \cdot 7 = 28. \\ &= 9 \cdot 31 && \text{Add: } 3 + 28 = 31. \\ &= 279 && \text{Multiply: } 9 \cdot 31 = 279. \end{aligned}$$

53. Evaluate the expressions in the innermost parentheses first.

$3\{8[6+5]-8[7+3]\} = 3\{8[11]-8[10]\}$ Add: $6+5=11$ and $7+3=10$.

Now evaluate the expression inside the braces.

$$
\begin{aligned}
&= 3\{88-80\} && \text{Multiply: } 8[11]=88 \\
& && \text{and } 8[10]=80. \\
&= 3\{8\} && \text{Subtract: } 88-80=8. \\
&= 24 && \text{Multiply: } 3\{8\}=24.
\end{aligned}
$$

55. Evaluate the expression in the innermost parentheses first.

$3\cdot[2+4\cdot(9+6)] = 3\cdot[2+4\cdot 15]$ Add: $9+6=15$.

Now evaluate the expression inside the brackets.

$$
\begin{aligned}
&= 3\cdot[2+60] && \text{Multiply: } 4\cdot 15=60. \\
&= 3\cdot 62 && \text{Add: } 2+60=62. \\
&= 186 && \text{Multiply: } 3\cdot 62=186.
\end{aligned}
$$

57. Subtract inside the parentheses first, then evaluate the exponent.

$$
\begin{aligned}
(5-2)^2 &= (3)^2 && \text{Parentheses: } (5-2)=(3). \\
&= 9 && \text{Exponent: } (3)^2=9.
\end{aligned}
$$

59. Add inside the parentheses first, then evaluate the exponent.

$$
\begin{aligned}
(4+2)^2 &= (6)^2 && \text{Parentheses: } (4+2)=(6). \\
&= 36 && \text{Exponent: } (6)^2=36.
\end{aligned}
$$

61. Evaluate the exponents first, then add.

$$
\begin{aligned}
2^3+3^3 &= 8+3^3 && \text{Exponent: } 2^3=8. \\
&= 8+27 && \text{Exponent: } 3^3=27. \\
&= 35 && \text{Add: } 8+27=35.
\end{aligned}
$$

63. Evaluate the exponents first, then subtract.

$$
\begin{aligned}
2^3-1^3 &= 8-1^3 && \text{Exponent: } 2^3=8. \\
&= 8-1 && \text{Exponent: } 1^3=1. \\
&= 7 && \text{Subtract: } 8-1=7.
\end{aligned}
$$

65. Evaluate the exponent first, then multiply, then add.

$$\begin{aligned}
12 \cdot 5^2 + 8 \cdot 9 + 4 &= 12 \cdot 25 + 8 \cdot 9 + 4 && \text{Exponent: } 5^2 = 25. \\
&= 300 + 8 \cdot 9 + 4 && \text{Multiply: } 12 \cdot 25 = 300. \\
&= 300 + 72 + 4 && \text{Multiply: } 8 \cdot 9 = 72. \\
&= 376 && \text{Add: } 300 + 72 + 4 = 376.
\end{aligned}$$

67. Evaluate the exponent first, then multiply, then subtract and add working left to right.

$$\begin{aligned}
9 - 3 \cdot 2 + 12 \cdot 10^2 &= 9 - 3 \cdot 2 + 12 \cdot 100 && \text{Exponent: } 10^2 = 100. \\
&= 9 - 6 + 12 \cdot 100 && \text{Multiply: } 3 \cdot 2 = 6. \\
&= 9 - 6 + 1200 && \text{Multiply: } 12 \cdot 100 = 1200. \\
&= 3 + 1200 && \text{Subtract: } 9 - 6 = 3. \\
&= 1203 && \text{Add: } 3 + 1200 = 1203.
\end{aligned}$$

69. The parenthetical expression must be evaluated first, then the exponent, then the subtraction.

$$\begin{aligned}
4^2 - (13 + 2) &= 4^2 - 15 && \text{Parentheses first: } 13 + 2 = 15. \\
&= 16 - 15 && \text{Exponent next: } 4^2 = 16. \\
&= 1 && \text{Subtract: } 16 - 15 = 1.
\end{aligned}$$

71. The parenthetical expression must be evaluated first, then the exponent, then the subtraction.

$$\begin{aligned}
3^3 - (7 + 12) &= 3^3 - 19 && \text{Parentheses first: } 7 + 12 = 19. \\
&= 27 - 19 && \text{Exponent next: } 3^3 = 27. \\
&= 8 && \text{Subtract: } 27 - 19 = 8.
\end{aligned}$$

73. We must evaluate the innermost grouping symbols first.

$$\begin{aligned}
19 + 3[12 - (2^3 + 1)] &= 19 + 3[12 - (8 + 1)] && \text{Exponent first: } 2^3 = 8. \\
&= 19 + 3[12 - 9] && \text{Add: } 8 + 1 = 9. \\
&= 19 + 3[3] && \text{Brackets next: } 12 - 9 = 3. \\
&= 19 + 9 && \text{Multiply: } 3[3] = 9. \\
&= 28 && \text{Add: } 19 + 9 = 28.
\end{aligned}$$

75. We must evaluate the innermost grouping symbols first.

$$\begin{aligned}
17+7[13-(2^2+6)] &= 17+7[13-(4+6)] && \text{Exponent first: } 2^2=4.\\
&= 17+7[13-10] && \text{Add: } 4+6=10.\\
&= 17+7[3] && \text{Brackets next: } 13-10=3.\\
&= 17+21 && \text{Multiply: } 7[3]=21.\\
&= 38 && \text{Add: } 17+21=38.
\end{aligned}$$

77. The parenthetical expression must be evaluated first, then the exponent, then the subtraction.

$$\begin{aligned}
4^3-(12+1) &= 4^3-13 && \text{Parentheses first: } 12+1=13.\\
&= 64-13 && \text{Exponent next: } 4^3=64.\\
&= 51 && \text{Subtract: } 64-13=51.
\end{aligned}$$

79. We must evaluate the innermost grouping symbols first.

$$\begin{aligned}
5+7[11-(2^2+1)] &= 5+7[11-(4+1)] && \text{Exponent first: } 2^2=4.\\
&= 5+7[11-5] && \text{Add: } 4+1=5.\\
&= 5+7[6] && \text{Brackets next: } 11-5=6.\\
&= 5+42 && \text{Multiply: } 7[6]=42.\\
&= 47 && \text{Add: } 5+42=47.
\end{aligned}$$

81. We must simplify numerator and denominator separately, then divide.

$$\begin{aligned}
\frac{13+35}{3(4)} &= \frac{48}{12} && \text{Numerator: } 13+35=48.\\
& && \text{Denominator: } 3(4)=12.\\
&= 4 && \text{Divide: } 48/12=4.
\end{aligned}$$

83. In the numerator, we need to evaluate the parentheses first. To do so, multiply first, then subtract.

$$\begin{aligned}
\frac{64-(8\cdot 6-3)}{4\cdot 7-9} &= \frac{64-(48-3)}{4\cdot 7-9} && \text{Multiply: } 8\cdot 6=48.\\
&= \frac{64-45}{4\cdot 7-9} && \text{Subtract: } 48-3=45.
\end{aligned}$$

In the denominator, we must multiply first.

$$= \frac{64-45}{28-9} \qquad \text{Multiply: } 4\cdot 7=28.$$

Now we can subtract in both numerator and denominator.

$$\begin{aligned} &= \frac{19}{19} && \text{Numerator: } 64 - 45 = 19. \\ & && \text{Numerator: } 28 - 9 = 19. \\ &= 1 && \text{Divide: } 19/19 = 1. \end{aligned}$$

85. Simplify numerator and denominator separately, then divide.

$$\begin{aligned} \frac{2+13}{4-1} &= \frac{15}{3} && \text{Numerator: } 2 + 13 = 15. \\ & && \text{Denominator: } 4 - 1 = 3. \\ &= 5 && \text{Divide: } 15/3 = 5. \end{aligned}$$

87. Simplify numerator and denominator separately, then divide.

$$\begin{aligned} \frac{17+14}{9-8} &= \frac{31}{1} && \text{Numerator: } 17 + 14 = 31. \\ & && \text{Denominator: } 9 - 8 = 1. \\ &= 31 && \text{Divide: } 31/1 = 31. \end{aligned}$$

89. We must simplify numerator and denominator separately, then divide.

$$\begin{aligned} \frac{37+27}{8(2)} &= \frac{64}{16} && \text{Numerator: } 37 + 27 = 64. \\ & && \text{Denominator: } 8(2) = 16. \\ &= 4 && \text{Divide: } 64/16 = 4. \end{aligned}$$

91. In the numerator, we need to evaluate the parentheses first. To do so, multiply first, then subtract.

$$\begin{aligned} \frac{40 - (3 \cdot 7 - 9)}{8 \cdot 2 - 2} &= \frac{40 - (21 - 9)}{8 \cdot 2 - 2} && \text{Multiply: } 3 \cdot 7 = 21. \\ &= \frac{40 - 12}{8 \cdot 2 - 2} && \text{Subtract: } 21 - 9 = 12. \end{aligned}$$

In the denominator, we must multiply first.

$$\begin{aligned} &= \frac{40 - 12}{16 - 2} && \text{Multiply: } 8 \cdot 2 = 16. \end{aligned}$$

Now we can subtract in both numerator and denominator.

$$\begin{aligned} &= \frac{28}{14} && \text{Numerator: } 40 - 12 = 28. \\ & && \text{Numerator: } 16 - 2 = 14. \\ &= 2 && \text{Divide: } 28/14 = 2. \end{aligned}$$

93. Distribute, multiply, then add.

$$\begin{aligned} 5 \cdot (8+4) &= 5 \cdot 8 + 5 \cdot 4 && \text{Distribute 5 times each term in parens.} \\ &= 40 + 20 && \text{Multiply: } 5 \cdot 8 = 40 \text{ and } 5 \cdot 4 = 20. \\ &= 60 && \text{Add: } 40 + 20 = 60. \end{aligned}$$

95. Distribute, multiply, then subtract.

$$\begin{aligned} 7 \cdot (8-3) &= 7 \cdot 8 - 7 \cdot 3 && \text{Distribute 7 times each term in parens.} \\ &= 56 - 21 && \text{Multiply: } 7 \cdot 8 = 56 \text{ and } 7 \cdot 3 = 21. \\ &= 35 && \text{Add: } 56 - 21 = 35. \end{aligned}$$

97. Distribute, multiply, then subtract.

$$\begin{aligned} 6 \cdot (7-2) &= 6 \cdot 7 - 6 \cdot 2 && \text{Distribute 6 times each term in parens.} \\ &= 42 - 12 && \text{Multiply: } 6 \cdot 7 = 42 \text{ and } 6 \cdot 2 = 12. \\ &= 30 && \text{Add: } 42 - 12 = 30. \end{aligned}$$

99. Distribute, multiply, then add.

$$\begin{aligned} 4 \cdot (3+2) &= 4 \cdot 3 + 4 \cdot 2 && \text{Distribute 4 times each term in parens.} \\ &= 12 + 8 && \text{Multiply: } 4 \cdot 3 = 12 \text{ and } 4 \cdot 2 = 8. \\ &= 20 && \text{Add: } 12 + 8 = 20. \end{aligned}$$

101. First, expand 62 as $60 + 2$, then apply the distributive property.

$$\begin{aligned} 9 \cdot 62 &= 9 \cdot (60 + 2) && \text{Expand: } 62 = 60 + 2. \\ &= 9 \cdot 60 + 9 \cdot 2 && \text{Distribute the 9.} \\ &= 540 + 18 && \text{Multiply: } 9 \cdot 60 = 540 \text{ and } 9 \cdot 2 = 18. \\ &= 558 && \text{Add: } 540 + 18 = 558. \end{aligned}$$

103. First, expand 58 as $50 + 8$, then apply the distributive property.

$$\begin{aligned} 3 \cdot 58 &= 3 \cdot (50 + 8) && \text{Expand: } 58 = 50 + 8. \\ &= 3 \cdot 50 + 3 \cdot 8 && \text{Distribute the 3.} \\ &= 150 + 24 && \text{Multiply: } 3 \cdot 50 = 150 \text{ and } 3 \cdot 8 = 24. \\ &= 174 && \text{Add: } 150 + 24 = 174. \end{aligned}$$

1.6 Solving Equations by Addition and Subtraction

1. Substitute 10 for x in the equation, then simplify both sides of the resulting equation.

$$\begin{aligned} x - 4 &= 6 && \text{Original equation.} \\ 10 - 4 &= 6 && \text{Substitute 10 for } x. \\ 6 &= 6 && \text{Simplify both sides.} \end{aligned}$$

This last equation is a true statement. Hence, 10 is a solution of the equation $x - 4 = 6$. For contrast, substitute 11 for x in the equation and simplify.

$$\begin{aligned} x - 4 &= 6 && \text{Original equation.} \\ 11 - 4 &= 6 && \text{Substitute 11 for } x. \\ 7 &= 6 && \text{Simplify both sides.} \end{aligned}$$

This last equation is **not** a true statement. Hence, 11 is **not** a solution of $x - 4 = 6$. Readers should check that the remaining two given numbers are not solutions by substituting them into the equation and showing that a false statement results.

3. Substitute 4 for x in the equation, then simplify both sides of the resulting equation.

$$\begin{aligned} x + 2 &= 6 && \text{Original equation.} \\ 4 + 2 &= 6 && \text{Substitute 4 for } x. \\ 6 &= 6 && \text{Simplify both sides.} \end{aligned}$$

This last equation is a true statement. Hence, 4 is a solution of the equation $x + 2 = 6$. For contrast, substitute 5 for x in the equation and simplify.

$$\begin{aligned} x + 2 &= 6 && \text{Original equation.} \\ 5 + 2 &= 6 && \text{Substitute 5 for } x. \\ 7 &= 6 && \text{Simplify both sides.} \end{aligned}$$

This last equation is **not** a true statement. Hence, 5 is **not** a solution of $x + 2 = 6$. Readers should check that the remaining two given numbers are not solutions by substituting them into the equation and showing that a false statement results.

5. Substitute 1 for x in the equation, then simplify both sides of the resulting equation.

$$\begin{aligned} x + 2 &= 3 && \text{Original equation.} \\ 1 + 2 &= 3 && \text{Substitute 1 for } x. \\ 3 &= 3 && \text{Simplify both sides.} \end{aligned}$$

This last equation is a true statement. Hence, 1 is a solution of the equation $x + 2 = 3$. For contrast, substitute 2 for x in the equation and simplify.

$$\begin{aligned} x + 2 &= 3 && \text{Original equation.} \\ 2 + 2 &= 3 && \text{Substitute 2 for } x. \\ 4 &= 3 && \text{Simplify both sides.} \end{aligned}$$

This last equation is **not** a true statement. Hence, 2 is **not** a solution of $x + 2 = 3$. Readers should check that the remaining two given numbers are not solutions by substituting them into the equation and showing that a false statement results.

7. Substitute 11 for x in the equation, then simplify both sides of the resulting equation.

$$\begin{aligned} x - 4 &= 7 && \text{Original equation.} \\ 11 - 4 &= 7 && \text{Substitute 11 for } x. \\ 7 &= 7 && \text{Simplify both sides.} \end{aligned}$$

This last equation is a true statement. Hence, 11 is a solution of the equation $x - 4 = 7$. For contrast, substitute 12 for x in the equation and simplify.

$$\begin{aligned} x - 4 &= 7 && \text{Original equation.} \\ 12 - 4 &= 7 && \text{Substitute 12 for } x. \\ 8 &= 7 && \text{Simplify both sides.} \end{aligned}$$

This last equation is **not** a true statement. Hence, 12 is **not** a solution of $x - 4 = 7$. Readers should check that the remaining two given numbers are not solutions by substituting them into the equation and showing that a false statement results.

9. Substitute 1 for x in the equation, then simplify both sides of the resulting equation.

$$\begin{aligned} x + 3 &= 4 && \text{Original equation.} \\ 1 + 3 &= 4 && \text{Substitute 1 for } x. \\ 4 &= 4 && \text{Simplify both sides.} \end{aligned}$$

This last equation is a true statement. Hence, 1 is a solution of the equation $x + 3 = 4$. For contrast, substitute 2 for x in the equation and simplify.

$$\begin{aligned} x + 3 &= 4 && \text{Original equation.} \\ 2 + 3 &= 4 && \text{Substitute 2 for } x. \\ 5 &= 4 && \text{Simplify both sides.} \end{aligned}$$

This last equation is **not** a true statement. Hence, 2 is **not** a solution of $x + 3 = 4$. Readers should check that the remaining two given numbers are not solutions by substituting them into the equation and showing that a false statement results.

11. Substitute 14 for x in the equation, then simplify both sides of the resulting equation.

$$\begin{aligned} x - 6 &= 8 && \text{Original equation.} \\ 14 - 6 &= 8 && \text{Substitute 14 for } x. \\ 8 &= 8 && \text{Simplify both sides.} \end{aligned}$$

This last equation is a true statement. Hence, 14 is a solution of the equation $x - 6 = 8$. For contrast, substitute 15 for x in the equation and simplify.

$$\begin{aligned} x - 6 &= 8 && \text{Original equation.} \\ 15 - 6 &= 8 && \text{Substitute 15 for } x. \\ 9 &= 8 && \text{Simplify both sides.} \end{aligned}$$

This last equation is **not** a true statement. Hence, 15 is **not** a solution of $x - 6 = 8$. Readers should check that the remaining two given numbers are not solutions by substituting them into the equation and showing that a false statement results.

13. To undo the effect of adding 5, subtract 5 from both sides of the equation.

$$\begin{aligned} x + 5 &= 6 && \text{Original equation.} \\ x + 5 - 5 &= 6 - 5 && \text{Subtract 5 from both sides.} \\ x &= 1 && \text{On the left, subtracting 5 ``undoes''} \\ & && \text{the effect of adding 5 and returns } x. \\ & && \text{On the right, } 6 - 5 = 1. \end{aligned}$$

Hence, 1 is a solution of $x + 5 = 6$.

15. To undo the effect of adding 4, subtract 4 from both sides of the equation.

$$\begin{aligned} 5 &= 4 + x && \text{Original equation.} \\ 5 - 4 &= 4 + x - 4 && \text{Subtract 4 from both sides.} \\ x &= 1 && \text{On the right, subtracting 4 ``undoes''} \\ & && \text{the effect of adding 4 and returns } x. \\ & && \text{On the left, } 5 - 4 = 1. \end{aligned}$$

Hence, 1 is a solution of $5 = 4 + x$.

17. To undo the effect of adding 13, subtract 13 from both sides of the equation.

$$
\begin{aligned}
13 + x &= 17 && \text{Original equation.} \\
13 + x - 13 &= 17 - 13 && \text{Subtract 13 from both sides.} \\
x &= 4 && \text{On the left, subtracting 13 ``undoes''} \\
&&& \text{the effect of adding 13 and returns } x. \\
&&& \text{On the right, } 17 - 13 = 4.
\end{aligned}
$$

Hence, 4 is a solution of $13 + x = 17$.

19. To undo the effect of adding 9, subtract 9 from both sides of the equation.

$$
\begin{aligned}
9 + x &= 10 && \text{Original equation.} \\
9 + x - 9 &= 10 - 9 && \text{Subtract 9 from both sides.} \\
x &= 1 && \text{On the left, subtracting 9 ``undoes''} \\
&&& \text{the effect of adding 9 and returns } x. \\
&&& \text{On the right, } 10 - 9 = 1.
\end{aligned}
$$

Hence, 1 is a solution of $9 + x = 10$.

21. To undo the effect of subtracting 3, add 3 to both sides of the equation.

$$
\begin{aligned}
19 &= x - 3 && \text{Original equation.} \\
19 + 3 &= x - 3 + 3 && \text{Add 3 to both sides.} \\
x &= 22 && \text{On the right, adding 3 ``undoes'' the} \\
&&& \text{effect of subtracting 3 and returns } x. \\
&&& \text{On the left, } 19 + 3 = 22.
\end{aligned}
$$

Hence, 22 is a solution of $19 = x - 3$.

23. To undo the effect of subtracting 18, add 18 to both sides of the equation.

$$
\begin{aligned}
x - 18 &= 1 && \text{Original equation.} \\
x - 18 + 18 &= 1 + 18 && \text{Add 18 to both sides.} \\
x &= 19 && \text{On the left, adding 18 ``undoes'' the} \\
&&& \text{effect of subtracting 18 and returns } x. \\
&&& \text{On the right, } 1 + 18 = 19.
\end{aligned}
$$

Hence, 19 is a solution of $x - 18 = 1$.

25. To undo the effect of subtracting 3, add 3 to both sides of the equation.

$x - 3 = 11$	Original equation.
$x - 3 + 3 = 11 + 3$	Add 3 to both sides.
$x = 14$	On the left, adding 3 "undoes" the effect of subtracting 3 and returns x. On the right, $11 + 3 = 14$.

Hence, 14 is a solution of $x - 3 = 11$.

27. To undo the effect of adding 2, subtract 2 from both sides of the equation.

$2 + x = 4$	Original equation.
$2 + x - 2 = 4 - 2$	Subtract 2 from both sides.
$x = 2$	On the left, subtracting 2 "undoes" the effect of adding 2 and returns x. On the right, $4 - 2 = 2$.

Hence, 2 is a solution of $2 + x = 4$.

29. To undo the effect of subtracting 14, add 14 to both sides of the equation.

$x - 14 = 12$	Original equation.
$x - 14 + 14 = 12 + 14$	Add 14 to both sides.
$x = 26$	On the left, adding 14 "undoes" the effect of subtracting 14 and returns x. On the right, $12 + 14 = 26$.

Hence, 26 is a solution of $x - 14 = 12$.

31. To undo the effect of adding 2, subtract 2 from both sides of the equation.

$x + 2 = 8$	Original equation.
$x + 2 - 2 = 8 - 2$	Subtract 2 from both sides.
$x = 6$	On the left, subtracting 2 "undoes" the effect of adding 2 and returns x. On the right, $8 - 2 = 6$.

Hence, 6 is a solution of $x + 2 = 8$.

33. To undo the effect of adding 11, subtract 11 from both sides of the equation.

$$\begin{aligned} 11 + x &= 17 && \text{Original equation.} \\ 11 + x - 11 &= 17 - 11 && \text{Subtract 11 from both sides.} \\ x &= 6 && \text{On the left, subtracting 11 ``undoes''} \\ & && \text{the effect of adding 11 and returns } x. \\ & && \text{On the right, } 17 - 11 = 6. \end{aligned}$$

Hence, 6 is a solution of $11 + x = 17$.

35. To undo the effect of adding 13, subtract 13 from both sides of the equation.

$$\begin{aligned} x + 13 &= 17 && \text{Original equation.} \\ x + 13 - 13 &= 17 - 13 && \text{Subtract 13 from both sides.} \\ x &= 4 && \text{On the left, subtracting 13 ``undoes''} \\ & && \text{the effect of adding 13 and returns } x. \\ & && \text{On the right, } 17 - 13 = 4. \end{aligned}$$

Hence, 4 is a solution of $x + 13 = 17$.

37. To undo the effect of adding 3, subtract 3 from both sides of the equation.

$$\begin{aligned} 20 &= 3 + x && \text{Original equation.} \\ 20 - 3 &= 3 + x - 3 && \text{Subtract 3 from both sides.} \\ x &= 17 && \text{On the right, subtracting 3 ``undoes''} \\ & && \text{the effect of adding 3 and returns } x. \\ & && \text{On the left, } 20 - 3 = 17. \end{aligned}$$

Hence, 17 is a solution of $20 = 3 + x$.

39. To undo the effect of adding 8, subtract 8 from both sides of the equation.

$$\begin{aligned} 20 &= 8 + x && \text{Original equation.} \\ 20 - 8 &= 8 + x - 8 && \text{Subtract 8 from both sides.} \\ x &= 12 && \text{On the right, subtracting 8 ``undoes''} \\ & && \text{the effect of adding 8 and returns } x. \\ & && \text{On the left, } 20 - 8 = 12. \end{aligned}$$

Hence, 12 is a solution of $20 = 8 + x$.

41. To undo the effect of subtracting 20, add 20 to both sides of the equation.

$3 = x - 20$	Original equation.
$3 + 20 = x - 20 + 20$	Add 20 to both sides.
$x = 23$	On the right, adding 20 "undoes" the effect of subtracting 20 and returns x. On the left, $3 + 20 = 23$.

Hence, 23 is a solution of $3 = x - 20$.

43. To undo the effect of adding 16, subtract 16 from both sides of the equation.

$x + 16 = 17$	Original equation.
$x + 16 - 16 = 17 - 16$	Subtract 16 from both sides.
$x = 1$	On the left, subtracting 16 "undoes" the effect of adding 16 and returns x. On the right, $17 - 16 = 1$.

Hence, 1 is a solution of $x + 16 = 17$.

45. To undo the effect of subtracting 6, add 6 to both sides of the equation.

$5 = x - 6$	Original equation.
$5 + 6 = x - 6 + 6$	Add 6 to both sides.
$x = 11$	On the right, adding 6 "undoes" the effect of subtracting 6 and returns x. On the left, $5 + 6 = 11$.

Hence, 11 is a solution of $5 = x - 6$.

47. To undo the effect of subtracting 6, add 6 to both sides of the equation.

$18 = x - 6$	Original equation.
$18 + 6 = x - 6 + 6$	Add 6 to both sides.
$x = 24$	On the right, adding 6 "undoes" the effect of subtracting 6 and returns x. On the left, $18 + 6 = 24$.

Hence, 24 is a solution of $18 = x - 6$.

49. To undo the effect of adding 13, subtract 13 from both sides of the equation.

$$\begin{aligned} 18 &= 13 + x && \text{Original equation.} \\ 18 - 13 &= 13 + x - 13 && \text{Subtract 13 from both sides.} \\ x &= 5 && \text{On the right, subtracting 13 "undoes"} \\ &&& \text{the effect of adding 13 and returns } x. \\ &&& \text{On the left, } 18 - 13 = 5. \end{aligned}$$

Hence, 5 is a solution of $18 = 13 + x$.

51. To undo the effect of subtracting 9, add 9 to both sides of the equation.

$$\begin{aligned} x - 9 &= 15 && \text{Original equation.} \\ x - 9 + 9 &= 15 + 9 && \text{Add 9 to both sides.} \\ x &= 24 && \text{On the left, adding 9 "undoes" the} \\ &&& \text{effect of subtracting 9 and returns } x. \\ &&& \text{On the right, } 15 + 9 = 24. \end{aligned}$$

Hence, 24 is a solution of $x - 9 = 15$.

53. We follow the "Requirements for Word Problem Solutions."

1. *Set up a Variable Dictionary.* Let x represent the unknown number in this problem.

2. *Set up an Equation.*

A certain numer	less	12	is	19
x	$-$	12	$=$	19

3. *Solve the Equation.* To undo the effect of subtracting 12, add 12 to both sides of the equation.

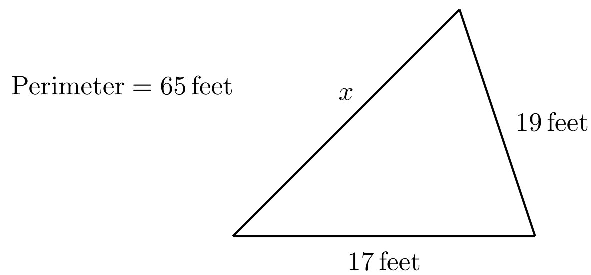
$$\begin{aligned} x - 12 &= 19 && \text{Original equation.} \\ x - 12 + 12 &= 19 + 12 && \text{Add 12 to both sides.} \\ x &= 31 && \text{On the left, adding 12 "undoes"} \\ &&& \text{the effect of subtracting 12 and returns } x. \\ &&& \text{On the right, } 19 + 12 = 31. \end{aligned}$$

4. *Answer the Question.* The unknown number is 31.

5. *Look Back.* Going back to the problem statement, we note that 12 less than our answer 31 is 19. Hence, our solution is correct.

55. We follow the "Requirements for Word Problem Solutions."

1. *Set up a Variable Dictionary.* Let's sketch a triangle (not to scale), label the two known sides and let x represent the length of the unknown side.



2. *Set up an Equation.*

Perimeter	is	First Side	plus	Second Side	plus	Third Side
65	$=$	19	$+$	17	$+$	x

Simplify the right-hand side by adding 19 and 17; i.e., $19 + 17 = 36$.

$$65 = 36 + x$$

3. *Solve the Equation.* To undo the effect of adding 36, subtract 36 from both sides of the equation.

$$\begin{aligned} 65 &= 36 + x && \text{Original equation.} \\ 65 - 36 &= 36 + x - 36 && \text{Subtract 36 from both sides.} \\ 29 &= x && \text{On the right, subtracting 36 “undoes”} \\ & && \text{the effect of adding 36 and returns } x. \\ & && \text{On the left, } 65 - 36 = 29. \end{aligned}$$

4. *Answer the Question.* The third side has length 29 feet.

5. *Look Back.* Going back to the problem statement, we add the three sides of the triangle: $19 + 17 + 29 = 65$. Because this equals the stated perimeter, the solution is correct.

57. We follow the "Requirements for Word Problem Solutions."

1. *Set up a Variable Dictionary.* Let d represent Burt 's deposit.
2. *Set up an Equation.*

Original Balance	plus	Burt 's deposit	is	New Balance
1900	$+$	d	$=$	8050

3. *Solve the Equation.* To undo the effect of adding 1900, subtract 1900 to both sides of the equation.

$$
\begin{aligned}
1900 + d &= 8050 && \text{Original equation.}\\
1900 + d - 1900 &= 8050 - 1900 && \text{Subtract 1900 from both sides.}\\
d &= 6150 && \text{On the left, subtracting 1900 "undoes"}\\
& && \text{the effect of adding 1900 and returns } d.\\
& && \text{On the right, } 8050 - 1900 = 6150.
\end{aligned}
$$

4. *Answer the Question.* Burt 's deposit is \$6150.
5. *Look Back.* Going back to the problem statement, we note that Burt deposits \$6150 to an account containing \$1900, the new balance will be \$8050. Hence our solution is correct.

59. We follow the "Requirements for Word Problem Solutions."

1. *Set up a Variable Dictionary.* Let x represent the unknown number in this problem.
2. *Set up an Equation.*

8	more than	a certain number	is	18
8	$+$	x	$=$	18

3. *Solve the Equation.* To undo the effect of adding 8, subtract 8 from both sides of the equation.

$$
\begin{aligned}
8 + x &= 18 && \text{Original equation.}\\
8 + x - 8 &= 18 - 8 && \text{Subtract 8 from both sides.}\\
x &= 10 && \text{On the left, subtracting 8 "undoes"}\\
& && \text{the effect of adding 8 and returns } x.\\
& && \text{On the right, } 18 - 8 = 10.
\end{aligned}
$$

4. *Answer the Question.* The unknown number is 10.
5. *Look Back.* Going back to the problem statement, we note that 8 more than our answer 10 is 18. Hence, our solution is correct.

61. We follow the "Requirements for Word Problem Solutions."

1. *Set up a Variable Dictionary.* Let B represent the original balance before the withdrawal
2. *Set up an Equation.*

Original Balance	minus	Michelle 's Withdrawal	is	Current Balance
B	$-$	120	$=$	1000

3. *Solve the Equation.* To undo the effect of subtracting 120, add 120 to both sides of the equation.

$$\begin{aligned} B - 120 &= 1000 && \text{Original equation.} \\ B - 120 + 120 &= 1000 + 120 && \text{Add 120 to both sides.} \\ B &= 1120 && \text{On the left, adding 120 "undoes"} \\ & && \text{the effect of subtracting 120 and returns } B. \\ & && \text{On the right, } 1000 + 120 = 1120. \end{aligned}$$

4. *Answer the Question.* The original balance is 1120.
5. *Look Back.* Going back to the problem statement, we note that if the original balance was \$1120 and Michelle withdraws \$120, the result is \$1000. Hence our solution is correct.

63. In our solution, we will carefully address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We can satisfy this requirement by simply stating "Let x represent the increase in foreclosure notices."
2. *Set up an Equation.* "The number of foreclosure notices last year and the increase in notices is the number of foreclosure notices this year." becomes

Last year's notices	plus	the increase in notices	is	this year's notices
650,000	$+$	x	$=$	804,000

3. *Solve the Equation.* To "undo" the addition, subtract 650,000 from both sides of the equation.

$$\begin{aligned}
650,000 + x &= 804,000 && \text{Original equation.} \\
650,000 + x - 650,000 &= 804,000 - 650,000 && \text{Subtract 650,000 from both sides of the equation.} \\
x &= 154,000 && \text{On the left, subtracting 650,000 "undoes" the effect} \\
& && \text{of adding 650,000 and returns } x\text{. On the right,} \\
& && 804,000 - 650,000 = 154,000.
\end{aligned}$$

4. *Answer the Question.* The number of notices is 154,000.

5. *Look Back.* Does increase of 154,000 notices satisfy the words in the original problem? We were told that "The number of foreclosure notices last year and the increase in notices is the number of foreclosure notices this year." Well, 650,000 increased by 154,000 is 804,000.

65. In our solution, we will carefully address each step of the *Requirements for Word Problem Solutions*.

1. *Set up a Variable Dictionary.* We can satisfy this requirement by simply stating "Let x represent the height Ikhana can fly."

2. *Set up an Equation.* "Flying height of Ikhana plus 40,000 more feet is flying height of Global Hawk." becomes

Height of Ikhana	plus	40,000 ft.	is	Height of Global Hawk
x	$+$	40,000	$=$	65,000

3. *Solve the Equation.* To "undo" the addition, subtract 40,000 from both sides of the equation.

$$\begin{aligned}
x + 40,000 &= 65,000 && \text{Original equation.} \\
x + 40,000 - 40,000 &= 65,000 - 40,000 && \text{Subtract 40,000 from both sides of the equation.} \\
x &= 25,000 && \text{On the left, subtracting 40,000 "undoes" the effect} \\
& && \text{of adding 40,000 and returns } x\text{. On the right,} \\
& && 65,000 - 40,000 = 25,000.
\end{aligned}$$

4. *Answer the Question.* The height Ikhana can fly is 25,000 feet.

5. *Look Back.* Does a flying height of 25,000 feet satisfy the words in the original problem? We were told that "Flying height of Ikhana plus 40,000 more feet is height of Global Hawk." Well, 25,000 feet increased by 40,000 feet is 65,000 feet.

1.7 Solving Equations by Multiplication and Division

1. Substitute 24 for x in the equation, then simplify both sides of the resulting equation.

$$\begin{aligned} \frac{x}{6} &= 4 && \text{Original equation.} \\ \frac{24}{6} &= 4 && \text{Substitute 24 for } x. \\ 4 &= 4 && \text{Simplify both sides.} \end{aligned}$$

This last equation is a true statement. Hence, 24 is a solution of the equation $\frac{x}{6} = 4$. For contrast, substitute 25 for x in the equation and simplify.

$$\begin{aligned} \frac{x}{6} &= 4 && \text{Original equation.} \\ \frac{25}{6} &= 4 && \text{Substitute 25 for } x. \\ \frac{25}{6} &= 4 && \text{Simplify both sides.} \end{aligned}$$

This last equation is **not** a true statement. Hence, 25 is **not** a solution of $\frac{x}{6} = 4$. Readers should check that the remaining two given numbers are not solutions by substituting them into the equation and showing that a false statement results.

3. Substitute 6 for x in the equation, then simplify both sides of the resulting equation.

$$\begin{aligned} \frac{x}{2} &= 3 && \text{Original equation.} \\ \frac{6}{2} &= 3 && \text{Substitute 6 for } x. \\ 3 &= 3 && \text{Simplify both sides.} \end{aligned}$$

This last equation is a true statement. Hence, 6 is a solution of the equation $\frac{x}{2} = 3$. For contrast, substitute 7 for x in the equation and simplify.

$$\begin{aligned} \frac{x}{2} &= 3 && \text{Original equation.} \\ \frac{7}{2} &= 3 && \text{Substitute 7 for } x. \\ \frac{7}{2} &= 3 && \text{Simplify both sides.} \end{aligned}$$

This last equation is **not** a true statement. Hence, 7 is **not** a solution of $\frac{x}{2} = 3$. Readers should check that the remaining two given numbers are not solutions by substituting them into the equation and showing that a false statement results.

5. Substitute 2 for x in the equation, then simplify both sides of the resulting equation.

$$\begin{aligned} 5x &= 10 && \text{Original equation.} \\ 5(2) &= 10 && \text{Substitute 2 for } x. \\ 10 &= 10 && \text{Simplify both sides.} \end{aligned}$$

This last equation is a true statement. Hence, 2 is a solution of the equation $5x = 10$. For contrast, substitute 3 for x in the equation and simplify.

$$\begin{aligned} 5x &= 10 && \text{Original equation.} \\ 5(3) &= 10 && \text{Substitute 3 for } x. \\ 15 &= 10 && \text{Simplify both sides.} \end{aligned}$$

This last equation is **not** a true statement. Hence, 3 is **not** a solution of $5x = 10$. Readers should check that the remaining two given numbers are not solutions by substituting them into the equation and showing that a false statement results.

7. Substitute 5 for x in the equation, then simplify both sides of the resulting equation.

$$\begin{aligned} 5x &= 25 && \text{Original equation.} \\ 5(5) &= 25 && \text{Substitute 5 for } x. \\ 25 &= 25 && \text{Simplify both sides.} \end{aligned}$$

This last equation is a true statement. Hence, 5 is a solution of the equation $5x = 25$. For contrast, substitute 6 for x in the equation and simplify.

$$\begin{aligned} 5x &= 25 && \text{Original equation.} \\ 5(6) &= 25 && \text{Substitute 6 for } x. \\ 30 &= 25 && \text{Simplify both sides.} \end{aligned}$$

This last equation is **not** a true statement. Hence, 6 is **not** a solution of $5x = 25$. Readers should check that the remaining two given numbers are not solutions by substituting them into the equation and showing that a false statement results.

9. Substitute 1 for x in the equation, then simplify both sides of the resulting equation.

$$\begin{aligned} 2x &= 2 && \text{Original equation.} \\ 2(1) &= 2 && \text{Substitute 1 for } x. \\ 2 &= 2 && \text{Simplify both sides.} \end{aligned}$$

This last equation is a true statement. Hence, 1 is a solution of the equation $2x = 2$. For contrast, substitute 2 for x in the equation and simplify.

$$\begin{aligned} 2x &= 2 && \text{Original equation.} \\ 2(2) &= 2 && \text{Substitute 2 for } x. \\ 4 &= 2 && \text{Simplify both sides.} \end{aligned}$$

This last equation is **not** a true statement. Hence, 2 is **not** a solution of $2x = 2$. Readers should check that the remaining two given numbers are not solutions by substituting them into the equation and showing that a false statement results.

11. Substitute 56 for x in the equation, then simplify both sides of the resulting equation.

$$\begin{aligned} \frac{x}{8} &= 7 && \text{Original equation.} \\ \frac{56}{8} &= 7 && \text{Substitute 56 for } x. \\ 7 &= 7 && \text{Simplify both sides.} \end{aligned}$$

This last equation is a true statement. Hence, 56 is a solution of the equation $\frac{x}{8} = 7$. For contrast, substitute 57 for x in the equation and simplify.

$$\begin{aligned} \frac{x}{8} &= 7 && \text{Original equation.} \\ \frac{57}{8} &= 7 && \text{Substitute 57 for } x. \\ \frac{57}{8} &= 7 && \text{Simplify both sides.} \end{aligned}$$

This last equation is **not** a true statement. Hence, 57 is **not** a solution of $\frac{x}{8} = 7$. Readers should check that the remaining two given numbers are not solutions by substituting them into the equation and showing that a false statement results.

13. To undo the effect of dividing by 6, multiply both sides of the equation by 6.

$$
\begin{aligned}
\frac{x}{6} &= 7 && \text{Original equation.} \\
6\left(\frac{x}{6}\right) &= 6(7) && \text{Multiply both sides by 6.} \\
x &= 42 && \text{On the left, multiplying by 6 ``undoes'' the} \\
&&& \text{effect of dividing by 6 and returns } x. \\
&&& \text{On the right, } 6(7) = 42.
\end{aligned}
$$

Hence, 42 is a solution of $\dfrac{x}{6} = 7$.

15. To undo the effect of multiplying by 2, divide both sides of the equation by 2.

$$
\begin{aligned}
2x &= 16 && \text{Original equation.} \\
\frac{2x}{2} &= \frac{16}{2} && \text{Divide both sides by 2.} \\
x &= 8 && \text{On the left, dividing by 2 ``undoes''} \\
&&& \text{the effect of multiplying by 2 and returns } x. \\
&&& \text{On the right, } 16/2 = 8.
\end{aligned}
$$

Hence, 8 is a solution of $2x = 16$.

17. To undo the effect of multiplying by 2, divide both sides of the equation by 2.

$$
\begin{aligned}
2x &= 18 && \text{Original equation.} \\
\frac{2x}{2} &= \frac{18}{2} && \text{Divide both sides by 2.} \\
x &= 9 && \text{On the left, dividing by 2 ``undoes''} \\
&&& \text{the effect of multiplying by 2 and returns } x. \\
&&& \text{On the right, } 18/2 = 9.
\end{aligned}
$$

Hence, 9 is a solution of $2x = 18$.

19. To undo the effect of multiplying by 4, divide both sides of the equation by 4.

$$
\begin{aligned}
4x &= 24 && \text{Original equation.} \\
\frac{4x}{4} &= \frac{24}{4} && \text{Divide both sides by 4.} \\
x &= 6 && \text{On the left, dividing by 4 “undoes”} \\
&&& \text{the effect of multiplying by 4 and returns } x. \\
&&& \text{On the right, } 24/4 = 6.
\end{aligned}
$$

Hence, 6 is a solution of $4x = 24$.

21. To undo the effect of dividing by 4, multiply both sides of the equation by 4.

$$
\begin{aligned}
\frac{x}{4} &= 9 && \text{Original equation.} \\
4\left(\frac{x}{4}\right) &= 4(9) && \text{Multiply both sides by 4.} \\
x &= 36 && \text{On the left, multiplying by 4 “undoes” the} \\
&&& \text{effect of dividing by 4 and returns } x. \\
&&& \text{On the right, } 4(9) = 36.
\end{aligned}
$$

Hence, 36 is a solution of $\dfrac{x}{4} = 9$.

23. To undo the effect of multiplying by 5, divide both sides of the equation by 5.

$$
\begin{aligned}
5x &= 5 && \text{Original equation.} \\
\frac{5x}{5} &= \frac{5}{5} && \text{Divide both sides by 5.} \\
x &= 1 && \text{On the left, dividing by 5 “undoes”} \\
&&& \text{the effect of multiplying by 5 and returns } x. \\
&&& \text{On the right, } 5/5 = 1.
\end{aligned}
$$

Hence, 1 is a solution of $5x = 5$.

25. To undo the effect of multiplying by 5, divide both sides of the equation by 5.

$$\begin{aligned}
5x &= 30 && \text{Original equation.} \\
\frac{5x}{5} &= \frac{30}{5} && \text{Divide both sides by 5.} \\
x &= 6 && \text{On the left, dividing by 5 ``undoes''} \\
&&& \text{the effect of multiplying by 5 and returns } x. \\
&&& \text{On the right, } 30/5 = 6.
\end{aligned}$$

Hence, 6 is a solution of $5x = 30$.

27. To undo the effect of dividing by 3, multiply both sides of the equation by 3.

$$\begin{aligned}
\frac{x}{3} &= 4 && \text{Original equation.} \\
3\left(\frac{x}{3}\right) &= 3(4) && \text{Multiply both sides by 3.} \\
x &= 12 && \text{On the left, multiplying by 3 ``undoes'' the} \\
&&& \text{effect of dividing by 3 and returns } x. \\
&&& \text{On the right, } 3(4) = 12.
\end{aligned}$$

Hence, 12 is a solution of $\dfrac{x}{3} = 4$.

29. To undo the effect of dividing by 8, multiply both sides of the equation by 8.

$$\begin{aligned}
\frac{x}{8} &= 9 && \text{Original equation.} \\
8\left(\frac{x}{8}\right) &= 8(9) && \text{Multiply both sides by 8.} \\
x &= 72 && \text{On the left, multiplying by 8 ``undoes'' the} \\
&&& \text{effect of dividing by 8 and returns } x. \\
&&& \text{On the right, } 8(9) = 72.
\end{aligned}$$

Hence, 72 is a solution of $\dfrac{x}{8} = 9$.

31. To undo the effect of dividing by 7, multiply both sides of the equation by 7.

$$\begin{aligned} \frac{x}{7} &= 8 && \text{Original equation.} \\ 7\left(\frac{x}{7}\right) &= 7(8) && \text{Multiply both sides by 7.} \\ x &= 56 && \text{On the left, multiplying by 7 "undoes" the} \\ &&& \text{effect of dividing by 7 and returns } x. \\ &&& \text{On the right, } 7(8) = 56. \end{aligned}$$

Hence, 56 is a solution of $\dfrac{x}{7} = 8$.

33. To undo the effect of multiplying by 2, divide both sides of the equation by 2.

$$\begin{aligned} 2x &= 8 && \text{Original equation.} \\ \frac{2x}{2} &= \frac{8}{2} && \text{Divide both sides by 2.} \\ x &= 4 && \text{On the left, dividing by 2 "undoes"} \\ &&& \text{the effect of multiplying by 2 and returns } x. \\ &&& \text{On the right, } 8/2 = 4. \end{aligned}$$

Hence, 4 is a solution of $2x = 8$.

35. To undo the effect of dividing by 8, multiply both sides of the equation by 8.

$$\begin{aligned} \frac{x}{8} &= 5 && \text{Original equation.} \\ 8\left(\frac{x}{8}\right) &= 8(5) && \text{Multiply both sides by 8.} \\ x &= 40 && \text{On the left, multiplying by 8 "undoes" the} \\ &&& \text{effect of dividing by 8 and returns } x. \\ &&& \text{On the right, } 8(5) = 40. \end{aligned}$$

Hence, 40 is a solution of $\dfrac{x}{8} = 5$.

37. In our solution, we will carefully address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We can set up our variable dictionary by simply stating “Let N represent the number of bookcases purchased.

2. *Set up an equation.* If you multiply the price of a bookcase by the number of bookcases purchased, you get the total price of the purchase. In words and symbols,

Unit price	times	Number of bookcases	equals	Total price
370	$\cdot$	N	$=$	4810

3. *Solve the Equation.* To "undo" the multiplication by 370, divide both sides of the equation by 370.

$$\begin{aligned} 370N &= 4810 && \text{Our equation.} \\ \frac{370N}{370} &= \frac{4810}{370} && \text{Divide both sides by 370.} \\ N &= 13 && \text{On the left, dividing by 370 "undoes" the effect} \\ & && \text{of multiplying by 370 and returns } N\text{. On the right,} \\ & && 4810/370 = 13. \end{aligned}$$

4. *Answer the Question.* The total number of bookcases purchased is 13.

5. *Look Back.* Does the solution 13 satisfy the words of the original problem? To find the total purchase price, multiply the individual price \$370 times the number of bookcases purchased, 13. That is, $370 \cdot 13 = 4810$, which matches the given total price of the purchase, so our answer is correct.

39. We follow the "Requirements for Word Problem Solutions."

1. *Set up a Variable Dictionary.* Let x represent the unknown number in this problem.

2. *Set up an Equation.*

An unknown number	divided by	3	is	2
x	$\div$	3	$=$	2

This can also be written in the following form.

$$\frac{x}{3} = 2$$

3. *Solve the Equation.* To undo the effect of dividing by 3, multiply both sides of the equation by 3.

$$\begin{aligned} \frac{x}{3} &= 2 && \text{Original equation.} \\ 3\left(\frac{x}{3}\right) &= 3(2) && \text{Multiply both sides by 3.} \\ x &= 6 && \text{On the left, multiply by 3 ``undoes''} \\ & && \text{the effect of dividing by 3 and returns } x. \\ & && \text{On the right, } 3(2) = 6. \end{aligned}$$

4. *Answer the Question.* The unknown number is 6.

5. *Look Back.* Going back to the problem statement, we note that 6 divided by 3 is 2. Hence, our solution is correct.

41. In our solution, we will carefully address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We can set up our variable dictionary by simply stating "Let T represent the total points accumulated by the class."

2. *Set up an equation.* To find the average score on the exam, take the total points accumulated by the class, then divide by the number of students in the class. In words and symbols,

Total Points	divided by	Number of Students	equals	Average Score
T	$\div$	29	$=$	80

An equivalent representation is

$$\frac{T}{29} = 80.$$

3. *Solve the Equation.* To "undo" the division by 29, multiply both sides of the equation by 29.

$$\begin{aligned} \frac{T}{29} &= 80 && \text{Our equation.} \\ 29 \cdot \frac{T}{29} &= 80 \cdot 29 && \text{Multiply both sides of the equation by 29.} \\ T &= 2320 && \text{On the left, multiplying by 29 ``undoes'' the effect} \\ & && \text{of dividing by 29 and returns } T. \text{ On the right,} \\ & && 80 \cdot 29 = 2320. \end{aligned}$$

4. *Answer the Question.* The total points accumulated by the class on the exam is 2,320.

5. *Look Back.* Does the solution 2,320 satisfy the words of the original problem? To find the average on the exam, divide the total points 2,320 by 29, the number of students in the class. Note that this gives an average score of $2320 \div 29 = 80$. The answer works!

43. We follow the "Requirements for Word Problem Solutions."

1. *Set up a Variable Dictionary.* Let x represent the unknown number in this problem.

2. *Set up an Equation.*

An unknown number	divided by	9	is	5
x	$\div$	9	$=$	5

This can also be written in the following form.

$$\frac{x}{9} = 5$$

3. *Solve the Equation.* To undo the effect of dividing by 9, multiply both sides of the equation by 9.

$$\frac{x}{9} = 5 \qquad \text{Original equation.}$$

$$9\left(\frac{x}{9}\right) = 9(5) \qquad \text{Multiply both sides by 9.}$$

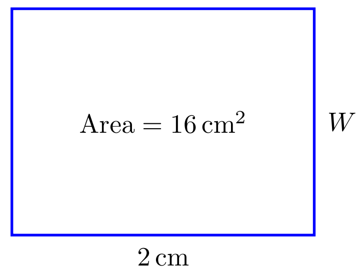
$$x = 45 \qquad \text{On the left, multiply by 9 "undoes" the effect of dividing by 9 and returns } x\text{. On the right, } 9(5) = 45.$$

4. *Answer the Question.* The unknown number is 45.

5. *Look Back.* Going back to the problem statement, we note that 45 divided by 9 is 5. Hence, our solution is correct.

45. We follow the "Requirements for Word Problem Solutions."

1. *Set up a Variable Dictionary.* Let W represent the width of the rectangle.



2. *Set up an Equation.* We know that the area of a rectangle is found by multiplying the length and width. That is,

$$A = LW.$$

Substitute 16 for A and 2 for L.

$$16 = 2W$$

3. *Solve the Equation.* To undo the effect of multiplying by 2, divide both sides of the equation by 2.

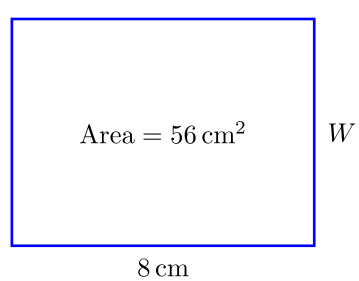
$$16 = 2W. \quad \text{Original equation.}$$
$$\frac{16}{2} = \frac{2W}{2} \quad \text{Divide both sides by 2.}$$
$$8 = W \quad \text{On the right, dividing by 2 ``undoes'' the effect of multiplying by 2 and returns } W. \text{ On the left, } 16/2 = 8.$$

4. *Answer the Question.* The width of the rectangle is 8 cm.

5. *Look Back.* Going back to the problem statement, if we multiply 2 cm by 8 cm, we get 16 square cm. Because this equals the stated area, the solution is correct.

47. We follow the "Requirements for Word Problem Solutions."

1. *Set up a Variable Dictionary.* Let W represent the width of the rectangle.



2. *Set up an Equation.* We know that the area of a rectangle is found by multiplying the length and width. That is,

$$A = LW.$$

Substitute 56 for A and 8 for L.

$$56 = 8W$$

3. *Solve the Equation.* To undo the effect of multiplying by 8, divide both sides of the equation by 8.

$56 = 8W.$	Original equation.
$\dfrac{56}{8} = \dfrac{8W}{8}$	Divide both sides by 8.
$7 = W$	On the right, dividing by 8 "undoes" the effect of multiplying by 8 and returns W. On the left, $56/8 = 7$.

4. *Answer the Question.* The width of the rectangle is 7 cm.

5. *Look Back.* Going back to the problem statement, if we multiply 8 cm by 7 cm, we get 56 square cm. Because this equals the stated area, the solution is correct.

49. In our solution, we will carefully address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We can set up our variable dictionary by simply stating "Let N represent the number of stereos purchased.

2. *Set up an equation.* If you multiply the price of a stereo by the number of stereos purchased, you get the total price of the purchase. In words and symbols,

Unit price	times	Number of stereos	equals	Total price
430	$\cdot$	N	$=$	6020

3. *Solve the Equation.* To "undo" the multiplication by 430, divide both sides of the equation by 430.

$430N = 6020$	Our equation.
$\dfrac{430N}{430} = \dfrac{6020}{430}$	Divide both sides by 430.
$N = 14$	On the left, dividing by 430 "undoes" the effect of multiplying by 430 and returns N. On the right, $6020/430 = 14$.

4. *Answer the Question.* The total number of stereos purchased is 14.

5. *Look Back.* Does the solution 14 satisfy the words of the original problem? To find the total purchase price, multiply the individual price \$430 times the number of stereos purchased, 14. That is, $430 \cdot 14 = 6020$, which matches the given total price of the purchase, so our answer is correct.

51. In our solution, we will carefully address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We can set up our variable dictionary by simply stating "Let T represent the total points accumulated by the class."

2. *Set up an equation.* To find the average score on the exam, take the total points accumulated by the class, then divide by the number of students in the class. In words and symbols,

Total Points	divided by	Number of Students	equals	Average Score
T	$\div$	35	$=$	74

An equivalent representation is

$$\frac{T}{35} = 74.$$

3. *Solve the Equation.* To "undo" the division by 35, multiply both sides of the equation by 35.

$$\frac{T}{35} = 74 \quad \text{Our equation.}$$

$$35 \cdot \frac{T}{35} = 74 \cdot 35 \quad \text{Multiply both sides of the equation by 35.}$$

$$T = 2590$$

On the left, multiplying by 35 "undoes" the effect of dividing by 35 and returns T. On the right, $74 \cdot 35 = 2590$.

4. *Answer the Question.* The total points accumulated by the class on the exam is 2,590.

5. *Look Back.* Does the solution 2,590 satisfy the words of the original problem? To find the average on the exam, divide the total points 2,590 by 35, the number of students in the class. Note that this gives an average score of $2590 \div 35 = 74$. The answer works!

53. We follow the "Requirements for Word Problem Solutions."

1. *Set up a Variable Dictionary.* Let x represent the unknown number in this problem.
2. *Set up an Equation.*

5	times	a certain number	is	20
5	$\cdot$	x	$=$	20

3. *Solve the Equation.* To undo the effect of multiplying by 5, divide both sides of the equation by 5.

$$
\begin{aligned}
5x &= 20 && \text{Original equation.}\\
\frac{5x}{5} &= \frac{20}{5} && \text{Divide both sides by 5.}\\
x &= 4 && \text{On the left, dividing by 5 ``undoes''}\\
&&& \text{the effect of multiplying by 5 and returns } x.\\
&&& \text{On the right, } 20/5 = 4.
\end{aligned}
$$

4. *Answer the Question.* The unknown number is 4.
5. *Look Back.* Going back to the problem statement, we note that 5 times our answer 4 is 20. Hence, our solution is correct.

55. We follow the "Requirements for Word Problem Solutions."

1. *Set up a Variable Dictionary.* Let x represent the unknown number in this problem.
2. *Set up an Equation.*

3	times	a certain number	is	21
3	$\cdot$	x	$=$	21

3. *Solve the Equation.* To undo the effect of multiplying by 3, divide both sides of the equation by 3.

$$
\begin{aligned}
3x &= 21 && \text{Original equation.}\\
\frac{3x}{3} &= \frac{21}{3} && \text{Divide both sides by 3.}\\
x &= 7 && \text{On the left, dividing by 3 ``undoes''}\\
&&& \text{the effect of multiplying by 3 and returns } x.\\
&&& \text{On the right, } 21/3 = 7.
\end{aligned}
$$

4. *Answer the Question.* The unknown number is 7.

5. *Look Back.* Going back to the problem statement, we note that 3 times our answer 7 is 21. Hence, our solution is correct.