Prealgebra Textbook

Second Edition

Chapter 2

Department of Mathematics
College of the Redwoods

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Chapter 2

The Integers

Today, much as we take for granted the fact that there exists a number zero, denoted by 0, such that $a+0=a$ for any whole number a, we similarly take for granted that for any whole number a there exists a unique number $-a$, called the "negative" or "opposite" of a, so that $a+(-a)=0$.

In a natural way, or so it seems to modern-day mathematicians, this easily introduces the concept of a *negative* number. However, history teaches us that the concept of negative numbers was not embraced wholeheartedly by mathematicians until somewhere around the 17th century.

In his work *Arithmetica* (c. 250 AD), the Greek mathematician Diophantus (c. 200-284 AD), who some call the "Father of Algebra," described the equation $4 = 4x + 20$ as "absurd," for how could one talk about an answer less than nothing? Girolamo Cardano (1501-1576), in his seminal work *Ars Magna* (c. 1545 AD) referred to negative numbers as "numeri ficti," while the German mathematician Michael Stifel (1487-1567) referred to them as "numeri absurdi." John Napier (1550-1617) (the creator of logarithms) called negative numbers "defectivi," and Rene Descartes (1596-1650) (the creator of analytic geometry) labeled negative solutions of algebraic equations as "false roots."

On the other hand, there were mathematicians whose treatment of negative numbers resembled somewhat our modern notions of the properties held by negative numbers. The Indian mathematician Brahmagupta described arithmetical rules in terms of fortunes (positive number) and debts (negative numbers). Indeed, in his work *Brahmasphutasiddhanta*, he writes "a fortune subtracted from zero is a debt," which in modern notation would resemble $0-4=-4$.

If you find the study of the integers somewhat difficult, do not be discouraged, as centuries of mathematicians have struggled mightily with the topic. With this thought it mind, let's begin the study of the integers.

2.1 An Introduction to the Integers

As we saw in the introduction to the chapter, negative numbers have a rich and storied history. One of the earliest applications of negative numbers had to do with credits and debits. For example, if \$5 represents a credit or profit, then $-$\$5 represents a debit or loss. Of course, the ancients had a different monetary system than ours, but you get the idea. Note that if a vendor experiences a profit of \$5 on a sale, then a loss of $-$\$5 on a second sale, the vendor *breaks even*. That is, the sum of \$5 and $-$\$5 is zero.

In much the same way, every whole number has an *opposite* or *negative* counterpart.

The Opposite or Negative of a Whole Number. For every whole number a, there is a unique number $-a$, called the **opposite** or **negative** of a, such that $a + (-a) = 0$.

The opposite or negative of any whole number is easily located on the number line.

Number Line Location. To locate the opposite (or negative) of any whole number, first locate the whole number on the number line. The opposite is the reflection of the whole number through the origin (zero).

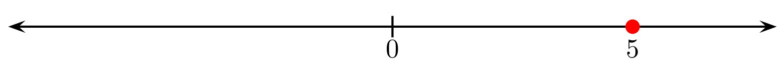
You Try It!

Locate the number -7 and its opposite on the number line.

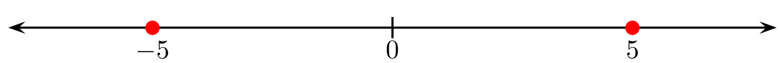
EXAMPLE 1. Locate the whole number 5 and its opposite (negative) on the number line.

Movie

Solution. Draw a number line, then plot the whole number 5 on the line as a shaded dot.



To find its opposite, reflect the number 5 through the origin. This will be the location of the opposite (negative) of the whole number 5, which we indicate by the symbol -5.



Note the symmetry. The whole number 5 is located five units to the right of zero. Its negative is located five units to the left of zero.

□

Important Pronunciation. The symbol -5 is pronounced in one of two ways: (1) "negative five," or (2) "the opposite of five."

In similar fashion, we can locate the opposite or negative of any whole number by reflecting the whole number through the origin (zero), which leads to the image shown in Figure 2.1.

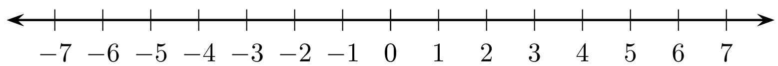


Figure 2.1: The opposite (negative) of any whole number is a reflection of that number through the origin (zero).

The Integers

The collection of numbers arranged on the number line in Figure 2.1 extend indefinitely to the right, and because the numbers on the left are reflections through the origin, the numbers also extend indefinitely to the left. This collection of numbers is called the set of *integers*.

The Integers. The infinite collection of numbers

$$\{\ldots, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, \ldots\}$$

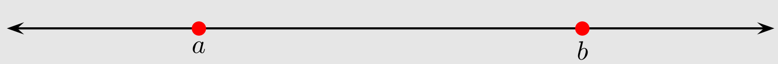
is called the **set of integers**.

The ellipsis ... at each end of this infinite collection means "etcetera," as the integers continue indefinitely to the right and left. Thus, for example, both 23,456 and $-117,191$ are elements of this set and are therefore integers.

Ordering the Integers

As we saw with the whole numbers, as you move to the right on the number line, the numbers get larger; as you move to the left, the numbers get smaller.

Order on the Number Line. Let a and b be integers located on the number line so that the point representing the integer a lies to the left of the point representing the integer b.



Then the integer a is "less than" the integer b and we write

$$a < b$$

Alternatively, we can also say that the integer b is "larger than" the integer a and write

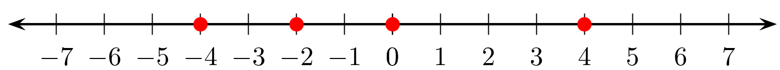
$$b > a.$$

You Try It!

Arrange the numbers -3, -5, 2, and -8 in order, from smallest to largest.

EXAMPLE 2. Arrange the integers 4, 0, -4, and -2 in order, from smallest to largest.

Solution. Place each of the numbers 4, 0, -4, and -2 as shaded dots on the number line.



Thus, -4 is the smallest integer, -2 is the next largest, followed by 0, then 4. Arranging these numbers in order, from smallest to largest, we have

$$-4, \; -2, \; 0, \; 4.$$

Answer: $-8, -5, -3, 2$

□

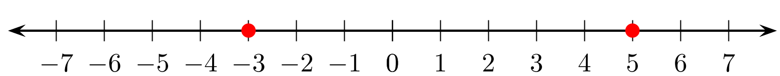
You Try It!

Compare -12 and -11.

EXAMPLE 3. Replace each shaded box with $<$ (less than) or $>$ (greater than) so the resulting inequality is a true statement.

$$-3 \quad \square \quad 5 \qquad\qquad -2 \quad \square \quad -4$$

Solution. For the first case, locate -3 and 5 on the number line as shaded dots.

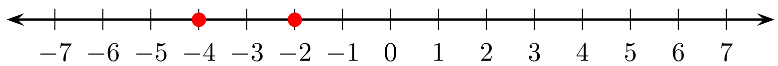


Note that -3 lies to the left of 5, so:

$$-3 \quad \boxed{<} \quad 5$$

That is, -3 is "less than" 5.

In the second case, locate -2 and -4 as shaded dots on the number line.



Note that -2 lies to the right of -4, so:

$$-2 \quad \boxed{>} \quad -4$$

That is, -2 is "greater than" -4.

Answer: $-12 < -11$

Important Observation. In Example 3, note that the "pointy end" of the inequality symbol always points towards the smaller number.

Opposites of Opposites

We stated earlier that every integer has a unique number called its "opposite" or "negative." Thus, the integer -5 is the opposite (negative) of the integer 5. Thus, we can say that the pair -5 and 5 are opposites. Each is the opposite of the other. Logically, this leads us to the conclusion that the opposite of -5 is 5. In symbols, we would write

$$-(-5) = 5.$$

Opposites of Opposites. Let a be an integer. Then the "opposite of the opposite of a is a." In symbols, we write

$$-(-a) = a.$$

We can also state that the "negative of a negative a is a."

You Try It!

EXAMPLE 4. Simplify $-(-13)$ and $-(-119)$.

Simplify: $-(-50)$.

Movie

Solution. The opposite of the opposite of a number returns the original number. That is,

$$-(-13) = 13 \qquad \text{and} \qquad -(-119) = 119.$$

Answer: 50

Positive and Negative

We now define the terms *positive integer* and *negative integer*.

Positive Integer. If a is an integer that lies to the right of zero (the origin) on the number line, then a is a **positive** integer. This means that a is a positive integer if and only if $a > 0$.

Thus, 2, 5, and 117 are positive integers.

Negative Integer. If a is an integer that lies to the left of zero (the origin) on the number line, then a is a **negative** integer. This means that a is a negative integer if and only if $a < 0$.

Thus, -4, -8, and $-1,123$ are negative integers.

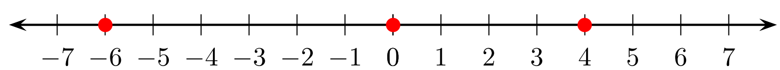
Zero. The integer zero is neither positive nor negative.

You Try It!

Classify -11 as positive, negative, or neither

EXAMPLE 5. Classify each of the following numbers as negative, positive, or neither: 4, -6, and 0.

Solution. Locate 4, -6, and 0 on the number line.



Thus:

- 4 lies to the right of zero. That is, $4 > 0$, making 4 a positive integer.
- -6 lies to the left of zero. That is, $-6 < 0$, making -6 a negative integer.
- The number 0 is neutral. It is neither negative nor positive.

Answer: Negative

□

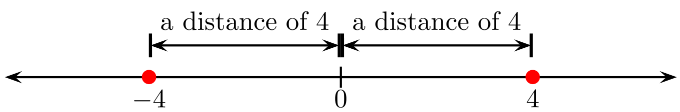
Absolute Value

We define the absolute value of an integer.

Absolute Value. The absolute value of an integer is defined as its distance from the origin (zero).

It is important to note that distance is always a nonnegative quantity (not negative); i.e., distance is either positive or zero.

As an example, we've shaded the integers -4 and 4 on a number line.



The number line above shows two cases:

- The integer -4 is 4 units from zero. Because absolute value measures the distance from zero, $|-4| = 4$.
- The integer 4 is also 4 units from zero. Again, absolute value measures the distance from zero, so $|4| = 4$.

Let's look at another example.

You Try It!

Simplify: $|-33|$.

EXAMPLE 6. Determine the value of each expression: a) $|-7|$, b) $|3|$, and c) $|0|$.

Solution. The absolute value of any integer is equal to the distance that number is from the origin (zero) on the number line. Thus:

a) The integer -7 is 7 units from the origin; hence, $|-7| = 7$.

b) The integer 3 is 3 units from the origin; hence, $|3| = 3$.

c) The integer 0 is 0 units from the origin; hence, $|0| = 0$.

Answer: 33

□

You Try It!

Simplify: $-|-50|$.

EXAMPLE 7. Determine the value of each expression: a) $-(-8)$ and b) $-|-8|$

Solution. These are distinctly different problems.

a) The opposite of -8 is 8. That is, $-(-8) = 8$.

b) However, in this case, we take the absolute value of -8 first, which is 8, then the opposite of that result to get -8. That is,

$$\begin{aligned} -|-8| &= -(8) && \text{First: } |-8| = 8. \\ &= -8 && \text{Second: The opposite of 8 is } -8. \end{aligned}$$

Answer: 50

Applications

There are a number of applications that benefit from the use of integers.

You Try It!

The following table contains record low temperatures (degrees Fahrenheit) for Jackson Hole, Wyoming for the indicated months.

Month	Temp
Sept.	14
Oct.	2
Nov.	-27
Dec.	-49
Jan	-50

Create a bar graph of temperature versus months.

EXAMPLE 8. Profits and losses for the first six months of the fiscal year for a small business are shown in Table 2.1. Profits and losses are measured in thousands of dollars. A positive number represents a profit, while a negative number represents a loss. Create a bar graph representing the profits and losses for this small business for each month of the first half of the fiscal year.

Month	Jan	Feb	Mar	Apr	May	Jun
Profit/Loss	10	12	7	-2	-4	5

Table 2.1: Profit and loss are measured in thousands of dollars.

Solution. Start by labeling the horizontal axis with the months of the fiscal year. Once you've completed that task, scale the vertical axis to accommodate the Profit/Loss values recorded in **Table 2.1**. Finally, starting at the 0 level on the horizontal axis, sketch bars having heights equal to the profit and loss for each month.

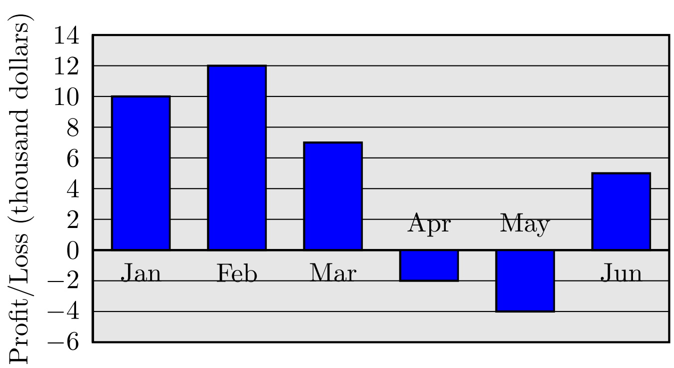


Figure 2.2: Profit and loss bar graph.

Note that the bars in Figure 2.2 for the months January, February, March, and June have heights greater than zero, representing a profit in each of those months. The bars for the months April and May have heights that are less than zero, representing a loss for each of those months.

You Try It!

EXAMPLE 9. Table 2.2 contains the low temperature recordings (degrees Fahrenheit) on five consecutive days in Fairbanks, Alaska, 1995. Create a line graph for the data in Table 2.2.

Date	Jan 21	Jan 22	Jan 23	Jan 24	Jan 25
Low Temp	1	10	5	-20	-28

Table 2.2: Temperature readings are in degrees Fahrenheit.

Solution. Start by labeling the horizontal axis with the days in January that the temperatures occurred. Scale the vertical axis to accommodate the temperatures given in Table 2.2. Finally, plot points on each day at a height that equals the temperature for that given day. Connect consecutive pairs of points with line segments to produce the line graph shown in Figure 2.3.

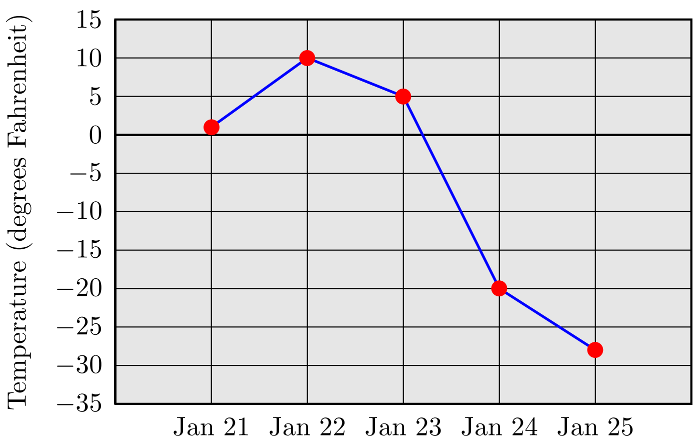


Figure 2.3: Plottting daily low temps versus the date they occurred.

Note that the points in Figure 2.3 have heights greater than zero for January 21-23, representing temperatures greater than zero. The points representing January 24-45 have negative heights corresponding to the negative temperatures of Table 2.2.

A man stands on the roof of a multistory building and throws a baseball vertically upward. The height (in feet) of the ball above the edge of the roof at measured times (in seconds) is given in the following table.

Time	Height
0	0
1	24
2	16
3	-24
4	-96

Create a line graph of the height of the ball versus time in the air.

❧ ❧ ❧ Exercises ☙ ☙ ☙

In Exercises 1-12, for each of the following questions, provide a number line sketch with your answer.

1. What number lies three units to the left of 4 on the number line?

2. What number lies three units to the left of 1 on the number line?

3. What number lies three units to the left of 6 on the number line?

4. What number lies four units to the left of -2 on the number line?

5. What number lies two units to the right of 0 on the number line?

6. What number lies four units to the right of -2 on the number line?

7. What number lies two units to the right of 1 on the number line?

8. What number lies two units to the right of -4 on the number line?

9. What number lies four units to the left of 6 on the number line?

10. What number lies two units to the left of 0 on the number line?

11. What number lies two units to the right of -5 on the number line?

12. What number lies three units to the right of -6 on the number line?

In Exercises 13-24, for each of the following sets of integers, perform the following tasks:

i) Plot each of the integers on a numberline.

ii) List the numbers in order, from smallest to largest.

13. $6, 1, -3$, and -5

14. $5, -3, -5$, and 2

15. $5, -6, 0$, and 2

16. $4, 2, 6$, and -4

17. $-3, -5, 3$, and 5

18. $-4, 5, 2$, and -6

19. $-5, 4, 2$, and -3

20. $6, 1, -3$, and -1

21. $3, 5, -5$, and -1

22. $-4, 6, -2$, and 3

23. $-2, -4, 3$, and -6

24. $2, -6, -4$, and 5

In Exercises 25-36, in each of the following exercises, enter the inequality symbol $<$ or the symbol $>$ in the shaded box in order that the resulting inequality is a true statement.

25. $-4 \ \square \ 0$

26. $-4 \ \square \ 3$

27. $-2 \ \square \ -1$

28. $3 \ \square \ 0$

29. $-3 \ \square \ -1$

30. $6 \ \square \ 5$

31. $3 \ \boxed{\phantom{<}} \ 6$

32. $-4 \ \boxed{\phantom{<}} \ -2$

33. $-3 \ \boxed{\phantom{<}} \ -6$

34. $0 \ \boxed{\phantom{<}} \ -3$

35. $-1 \ \boxed{\phantom{<}} \ 4$

36. $1 \ \boxed{\phantom{<}} \ -4$

In Exercises 37-52, simplify each of the following expressions.

37. $-(-4)$.

38. $-(-6)$.

39. $|7|$.

40. $|1|$.

41. $|5|$.

42. $|3|$.

43. $-|-11|$.

44. $-|-1|$.

45. $|-5|$.

46. $|-1|$.

47. $-|-20|$.

48. $-|-8|$.

49. $|-4|$.

50. $|-3|$.

51. $-(-2)$.

52. $-(-17)$.

In Exercises 53-64, for each of the following exercises, provide a number line sketch with your answer.

53. Find two integers on the number line that are 2 units away from the integer 2.

54. Find two integers on the number line that are 2 units away from the integer -3.

55. Find two integers on the number line that are 4 units away from the integer -3.

56. Find two integers on the number line that are 2 units away from the integer -2.

57. Find two integers on the number line that are 3 units away from the integer -2.

58. Find two integers on the number line that are 4 units away from the integer 1.

59. Find two integers on the number line that are 2 units away from the integer 3.

60. Find two integers on the number line that are 3 units away from the integer 3.

61. Find two integers on the number line that are 3 units away from the integer 0.

62. Find two integers on the number line that are 4 units away from the integer 2.

63. Find two integers on the number line that are 2 units away from the integer 0.

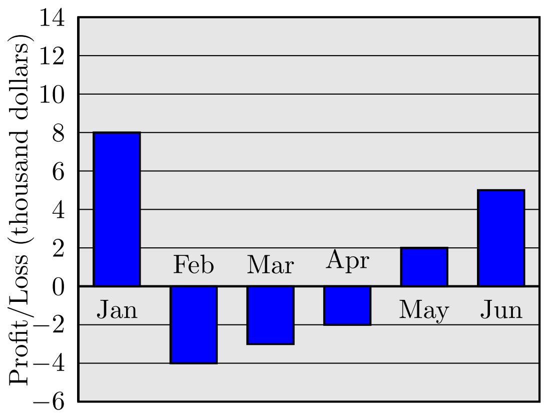
64. Find two integers on the number line that are 3 units away from the integer 1.

---

65. **Dam.** Utah's lowest point of elevation is 2,350 feet above sea level and occurs at Beaver Dam Wash. Express the height as an integer.

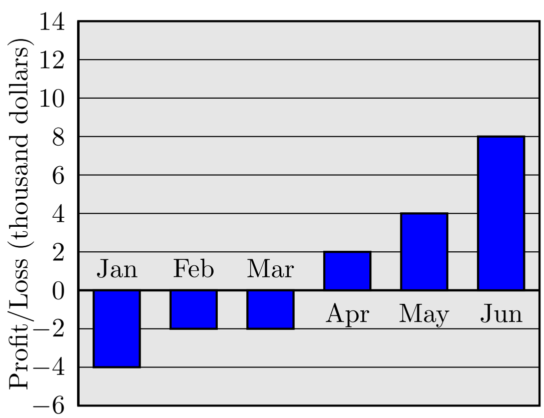
66. **Underwater Glider.** The National Oceanic and Atmospheric Administration's underwater glider samples the bottom of the Atlantic Ocean at 660 feet below sea level. Express that depth as an integer. *Associated Press Times-Standard 4/19/09*

67. Profits and losses for the first six months of the fiscal year for a small business are shown in the following bar chart.



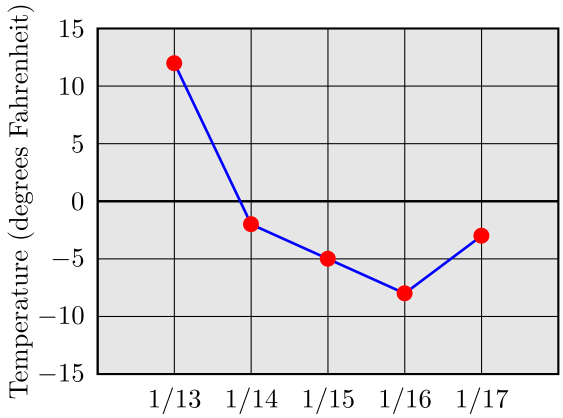
Create a table showing the profit and loss for each month. Use positive integers for the profit and negative integers for the loss. Create a line graph using the data in your table.

68. Profits and losses for the first six months of the fiscal year for a small business are shown in the following bar chart.

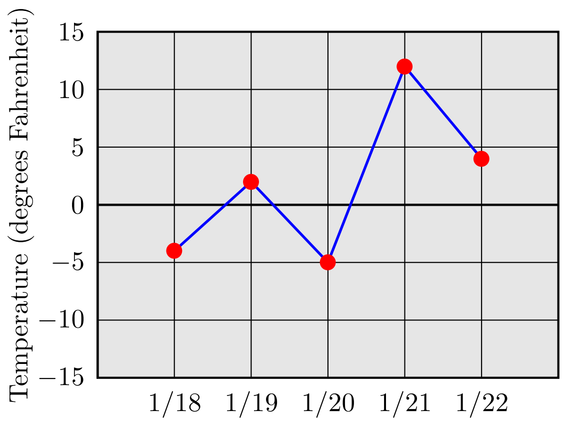


Create a table showing the profit and loss for each month. Use positive integers for the profit and negative integers for the loss. Create a line graph using the data in your table.

69. The following line graph displays the low temperature recordings (degrees Fahrenheit) on five consecutive days in Big Bear, California. What was the lowest temperature reading for the week and on what date did it occur?



70. The following line graph displays the low temperature recordings (degrees Fahrenheit) on five consecutive days in Ogden, Utah. What was the lowest temperature reading for the week and on what date did it occur?



71. The following table contains the low temperature recordings (degrees Fahrenheit) on five consecutive days in Littletown, Ohio. Create a line graph for the data.

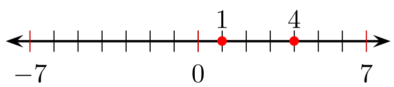
Date	Feb 11	Feb 12	Feb 13	Feb 14	Feb 15
Low Temp	10	−2	−5	−12	8

72. The following table contains the low temperature recordings (degrees Fahrenheit) on five consecutive days in MyTown, Ottawa. Create a line graph for the data.

Date	Apr 20	Apr 21	Apr 22	Apr 23	Apr 24
Low Temp	−10	−2	8	5	−5

❧ ❧ ❧ Answers ☙ ☙ ☙

1.



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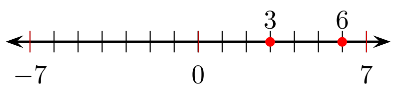

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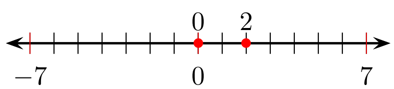

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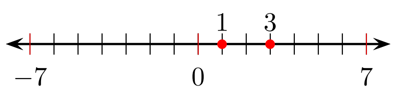

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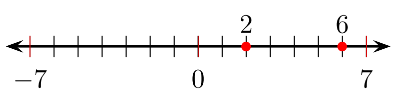

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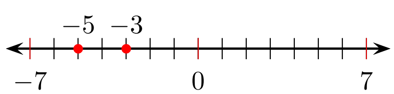
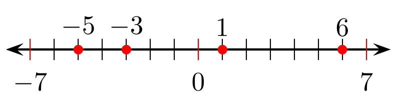


13.

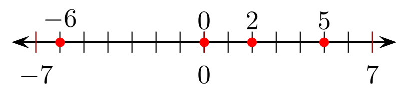
i) Arrange the integers 6, 1, -3, and -5 on a number line.



ii) $-5, -3, 1, 6$

15.

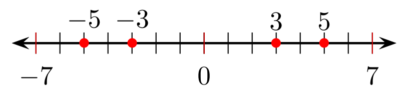
i) Arrange the integers 5, -6, 0, and 2 on a number line.



ii) $-6, 0, 2, 5$

17.

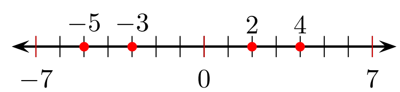
i) Arrange the integers -3, -5, 3, and 5 on a number line.



ii) $-5, -3, 3, 5$

19.

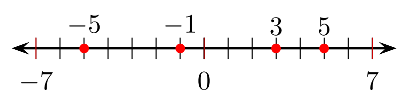
i) Arrange the integers -5, 4, 2, and -3 on a number line.



ii) $-5, -3, 2, 4$

21.

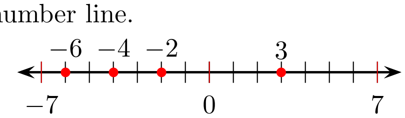
i) Arrange the integers 3, 5, -5, and -1 on a number line.



ii) $-5, -1, 3, 5$

23.

i) Arrange the integers -2, -4, 3, and -6 on a number line.



ii) $-6, -4, -2, 3$

25. $-4 < 0$

27. $-2 < -1$

29. $-3 < -1$

31. $3 < 6$

33. $-3 > -6$

35. $-1 < 4$

37. 4

39. 7

41. 5

43. -11

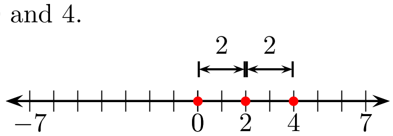
45. 5

47. -20

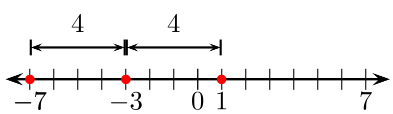
49. 4

51. 2

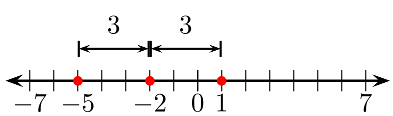
53. 0 and 4.



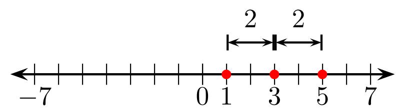
55. -7 and 1.



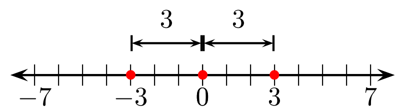
57. -5 and 1.



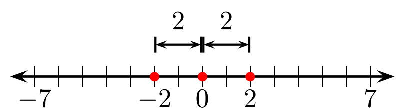
59. 1 and 5.



61. -3 and 3.



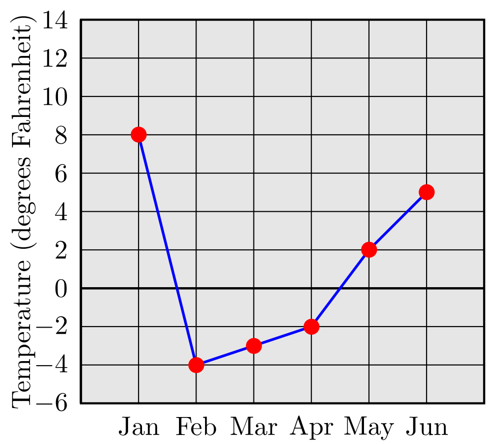
63. -2 and 2.



65. 2,350 feet

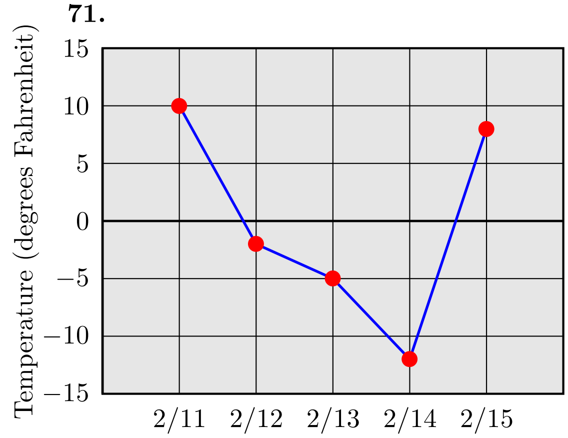
67.

Month	Profit/Loss
Jan	8
Feb	-4
Mar	-3
Apr	-2
May	2
Jun	5



69. Approximately -8° Fahrenheit on January 16.

71.



2.2 Adding Integers

Like our work with the whole numbers, addition of integers is best explained through the use of number line diagrams. However, before we start, let's take a moment to discuss the concept of a vector.

Vectors. A *vector* is a mathematical object that possesses two import qualities: (1) magnitude or length, and (2) directon.

Vectors are a fundamental problem solving tool in mathematics, science, and engineering. In physics, vectors are used to represent forces, position, velocity, and acceleration, while engineers use vectors to represent both internal and external forces on structures, such as bridges and buildings. In this course, and in this particular section, we will concentrate on the use of vectors to help explain addition of integers.

Vectors on the Number Line

Consider the number line in Figure 2.4.

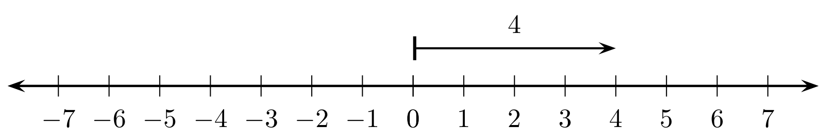


Figure 2.4: A vector representing positive four.

Above the line we've drawn a vector with tail starting at the integer 0 and arrowhead finishing at the integer 4. There are two important things to note about this vector:

1. The magnitude (length) of the vector in Figure 2.4 is four.
2. The vector in Figure 2.4 points to the right.

We will agree that the vector in Figure 2.4 represents positive four.

It is not important that the vector start at the origin. Consider, for example, the vector pictured in Figure 2.5.

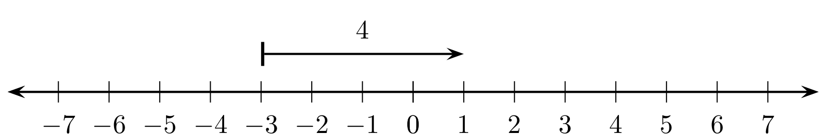


Figure 2.5: A vector representing positive four.

Again, there are two important observations to be made:

1. The magnitude (length) of the vector in Figure 2.5 is four.
2. The vector in Figure 2.5 points to the right.

Hopefully, you have the idea. Any vector that has length 4 and points to the right will represent positive four, regardless of its starting or finishing point.

Conversely, consider the vector in Figure 2.6, which starts at the integer 4 and finishes at the integer -3.

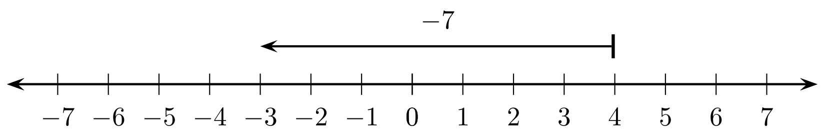


Figure 2.6: A vector representing negative seven.

Two observations:

1. The magnitude (length) of the vector in Figure 2.6 is seven.
2. The vector in Figure 2.6 points to the left.

We will agree that the vector in Figure 2.6 represents negative seven. We could select different starting and finishing points for our vector, but as long as the vector has length seven and points to the left, it represents the integer -7.

Important Observation. A vector pointing to the right represents a **positive** number. A vector pointing to the left represents a **negative** number.

Magnitude and Absolute Value

In Figure 2.4 and Figure 2.5, the vectors pictured represent the integer positive four. Note that the absolute value of four is four; that is, $|4| = 4$. Note also that this absolute value is the magnitude or length of the vectors representing the integer positive four in Figure 2.4 and Figure 2.5.

In Figure 2.6, the vector pictured represents the integer -7. Note that $|-7| = 7$. This shows that the absolute value represents the magnitude or length of the vector representing -7.

Magnitude and Absolute Value. If a is an integer, then $|a|$ gives the magnitude or length of the vector that represents the integer a.

Adding Integers with Like Signs

Because the positive integers are also whole numbers, we've already seen how to add to them in Section 1.2.

You Try It!

Use a number line diagram to show the sum $5 + 7$.

EXAMPLE 1. Find the sum $3 + 4$.

Solution. To add the positive integers 3 and 4, proceed as follows.

1. Start at the integer 0, then draw a vector 3 units in length pointing to the right, as shown in Figure 2.7. This arrow has magnitude (length) three and represents the positive integer 3.

2. Draw a second vector of length four that points to the right, starting at the end of the first vector representing the positive integer 3. This arrow has magnitude (length) four and represents the positive integer 4.

3. The sum of the positive integers 3 and 4 could be represented by a vector that starts at the integer 0 and ends at the positive integer 7. However, we prefer to mark this sum on the number line as a solid dot at the positive integer 7. This integer represents the sum of the positive integers 3 and 4.

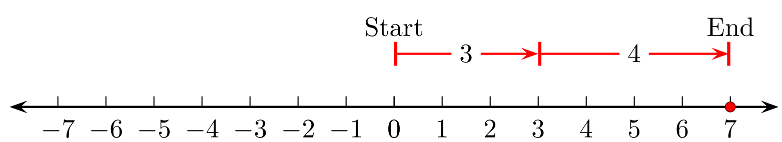


Figure 2.7: Adding two positive integers on the number line.

Thus, $3 + 4 = 7$.

Answer: 12.

Negative integers are added in a similar fashion.

You Try It!

Use a number line diagram to show the sum $-7 + (-3)$.

EXAMPLE 2. Find the sum $-3 + (-4)$.

Solution. To add the negative integers -3 and -4, proceed as follows.

1. Start at the integer 0, then draw a vector 3 units in length pointing to the left, as shown in Figure 2.8. This arrow has magnitude (length) three and represents the negative integer -3.

2. Draw a second vector of length four that points to the left, starting at the end of the first vector representing the negative integer -3. This arrow has magnitude (length) four and represents the negative integer -4.

3. The sum of the negative integers -3 and -4 could be represented by a vector that starts at the integer 0 and ends at the negative integer -7. However, we prefer to mark this sum on the number line as a solid dot at the negative integer -7. This integer represents the sum of the negative integers -3 and -4.

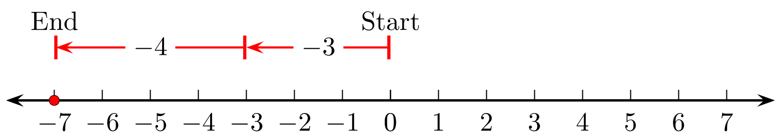


Figure 2.8: Adding two positive integers on the number line.

Thus, $-3 + (-4) = -7$.

Drawing on Physical Intuition. Imagine that you are "walking the number line" in Figure 2.8. You start at the origin (zero) and take 3 paces *to the left*. Next, you walk an additional four paces *to the left*, landing at the number -7.

Answer: -10.

□

It should come as no surprise that the procedure used to add two negative integers comprises two steps.

Adding Two Negative Integers. To add two negative integers, proceed as follows:

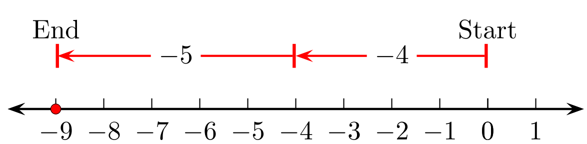
1. Add the magnitudes of the integers.
2. Prefix the common negative sign.

You Try It!

Find the sum: $-5 + (-9)$.

EXAMPLE 3. Find the sums: (a) $-4 + (-5)$, (b) $-12 + (-9)$, and (c) $-2 + (-16)$. **Solution.** We'll examine three separate but equivalent approaches, as discussed in the narrative above.

a) The number line schematic



shows that $(-4) + (-5) = -9$.

b) Drawing on physical intuition, start at zero, walk 12 units to the left, then an additional 9 units to the left. You should find yourself 21 units to the left of the origin (zero). Hence, $-12 + (-9) = -21$.

c) Following the algorithm above in "Adding Two Negative Integers," first add the magnitudes of -2 and -16; that is, $2+16 = 18$. Now prefix the common sign. Hence, $-2 + (-16) = -18$.

Answer: -14.

Adding Integers with Unlike Signs

Adding integers with unlike signs is no harder than adding integers with like signs.

You Try It!

Use a number line diagram to show the sum $-9 + 2$.

EXAMPLE 4. Find the sum $-8 + 4$.

Solution. To find the sum $-8 + 4$, proceed as follows:

1. Start at the integer 0, then draw a vector eight units in length pointing to the left, as shown in Figure 2.9. This arrow has magnitude (length) eight and represents the negative integer -8.

2. Draw a second vector of length four that points to the right, starting at the end of the first vector representing the negative integer -8. This arrow (also shown in Figure 2.9) has magnitude (length) four and represents the positive integer 4.

3. The sum of the negative integers -8 and 4 could be represented by a vector that starts at the integer 0 and ends at the negative integer -4. However, we prefer to mark this sum on the number line as a solid dot at the negative integer -4. This integer represents the sum of the integers -8 and 4.

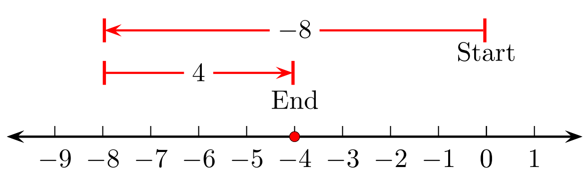


Figure 2.9: Adding -8 and 4 on the number line.

Thus, $-8 + 4 = -4$.

Drawing on Physical Intuition. Imagine you are "walking the number line in Figure 2.9. You start at the origin (zero) and walk eight paces *to the left*. Next, turn around and walk four paces *to the right*, landing on the number -4.

Answer: -7.

Note that adding integers with unlike signs is a *subtractive process*. This is due to the reversal of direction experienced in drawing Figure 2.9 in Example 4.

Adding Two Integers with Unlike Signs. To add two integers with unlike signs, proceed as follows:

1. Subtract the smaller magnitude from the larger magnitude.
2. Prefix the sign of the number with the larger magnitude.

For example, to find the sum $-8+4$ of Example 4, we would note that the integers -8 and 4 have magnitudes 8 and 4, respectively. We would then apply the process outlined in "Adding Two Integers with Unlike Signs."

1. Subtract the smaller magnitude from the larger magnitude; that is, $8 - 4 = 4$.
2. Prefix the sign of the number with the larger magnitude. Because -8 has the larger magnitude and its sign is negative, we prefix a negative sign to the difference of the magnitudes. Thus, $-8 + 4 = -4$.

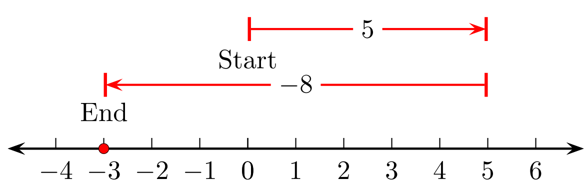
You Try It!

Use a number line diagram to show the sum $5 + (-11)$.

EXAMPLE 5. Find the sums: (a) $5+(-8)$, (b) $-12+16$, and (c) $-117+115$.

Solution. We'll examine three separate but equivalent approaches, as discussed in the narrative above.

a) The number line schematic



shows that $5 + (-8) = -3$.

b) Drawing on physical intuition, start at zero, walk 12 units to the left, then turn around and walk 16 units to the right. You should find yourself 4 units to the right of the origin (zero). Hence, $-12 + 16 = 4$.

c) Following the algorithm in “Adding Two Integers with Unlike Signs,” subtract the smallar magnitude from the larger magnitude, thus $117 - 115 = 2$. Because -117 has the larger magnitude and its sign is negative, we prefix a negative sign to the difference of the magnitudes. Thus, $-117 + 115 = -2$.

Answer: -6.

Properties of Addition of Integers

You will be pleased to learn that the properties of addition for whole numbers also apply to addition of integers.

The Commutative Property of Addition. Let a and b represent two integers. Then,

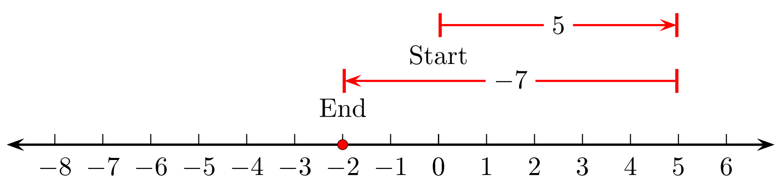
$$a + b = b + a.$$

You Try It!

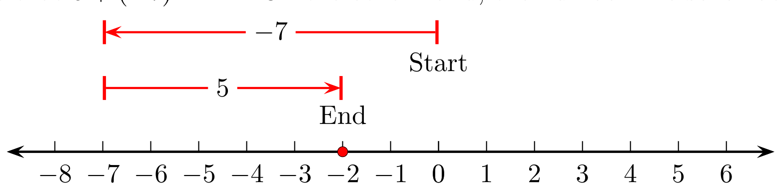
Use a number line diagram to show that $-8 + 6$ is the same as $6 + (-8)$.

EXAMPLE 6. Show that $5 + (-7) = -7 + 5$.

Solution. The number line schematic



shows that $5 + (-7) = -2$. On the other hand, the number line schematic



shows that $-7 + 5 = -2$. Therefore, $5 + (-7) = -7 + 5$.

Addition of integers is also associative.

The Associative Property of Addition. Let a, b, and c represent integers. Then,

$$(a + b) + c = a + (b + c).$$

You Try It!

Show that the expression $(-8+5)+3$ is the same as $-8+(5+3)$ by simplifying each of the two expressions independently.

EXAMPLE 7. Show that $(-9+6)+2 = -9+(6+2)$.

Solution. On the left, the grouping symbols demand that we add -9 and 6 first. Thus,

$$\begin{aligned}(-9+6)+2 &= -3+2\\ &= -1.\end{aligned}$$

On the right, the grouping symbols demand that we add 6 and 2 first. Thus,

$$\begin{aligned}-9+(6+2) &= -9+8\\ &= -1.\end{aligned}$$

Both sides simplify to -1. Therefore, $(-9+6)+2 = -9+(6+2)$.

The Additive Identity Property. The integer zero is called the *additive identity*. If a is any integer, then

$$a+0=a \qquad \text{and} \qquad 0+a=a.$$

Thus, for example, $-8+0=-8$ and $0+(-113)=-113$.

Finally, every integer has a unique opposite, called its *additive inverse*.

The Additive Inverse Property. Let a represent any integer. Then there is a unique integer $-a$, called the **opposite** or **additive inverse** of a, such that

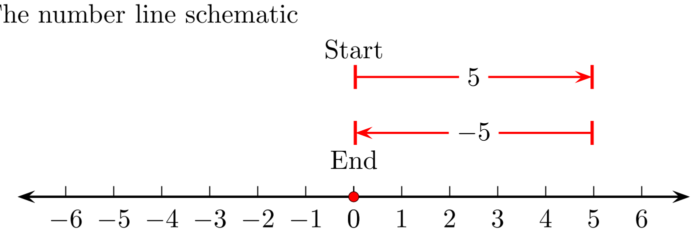
$$a+(-a)=0 \qquad \text{and} \qquad -a+a=0.$$

You Try It!

Use a number line diagram to show that $9+(-9)=0$.

EXAMPLE 8. Show that $5+(-5)=0$.

Solution. The number line schematic



clearly shows that $5+(-5)=0$.

Important Observation. We have used several equivalent phrases to pronounce the integer $-a$. We've used "the opposite of a," "negative a," and "the additive inverse of a." All are equivalent pronunciations.

Grouping for Efficiency

Order of operations require that we perform all additions as they occur, working from left to right.

You Try It!

EXAMPLE 9. Simplify $-7 + 8 + (-9) + 12$.

Simplify: $-8 + 9 + (-4) + 2$.

Solution. We perform the additions as they occur, working left to right.

$$\begin{aligned} -7 + 8 + (-9) + 12 &= 1 + (-9) + 12 && \text{Working left to right, } -7 + 8 = 1. \\ &= -8 + 12 && \text{Working left to right, } 1 + (-9) = -8. \\ &= 4 && -8 + 12 = 4 \end{aligned}$$

Thus, $-7 + 8 + (-9) + 12 = 4$.

Answer: -1

The commutative property of addition tells us that changing the order of addition does not change the answer. The associative property of addition tells us that a sum is not affected by regrouping. Let's work Example 9 again, first grouping positive and negative numbers together.

You Try It!

EXAMPLE 10. Simplify $-7 + 8 + (-9) + 12$.

Simplify: $-11 + 7 + (-12) + 3$.

Solution. The commutative and associative properties allows us to change the order of addition and regroup.

$$\begin{aligned} -7 + 8 + (-9) + 12 &= -7 + (-9) + 8 + 12 && \text{Use the commutative property to change the order.} \\ &= [-7 + (-9)] + [8 + 12] && \text{Use the associative property to regroup.} \\ &= -16 + 20 && \text{Add the negatives. Add the positives.} \\ &= 4 && \text{One final addition.} \end{aligned}$$

Thus, $-7 + 8 + (-9) + 12 = 4$.

Answer: -13

At first glance, there seems to be no advantage in using the technique in Example 10 over the technique used in Example 9. However, the technique in Example 10 is much quicker in practice, particularly if you eliminate some of the explanatory steps.

Efficient Grouping. When asked to find the sum of a number of integers, it is most efficient to first add all the positive integers, then add the negatives, then add the results.

You Try It!

Simplify: $-11 + 3 + (-2) + 7$.

EXAMPLE 11. Simplify $-7 + 8 + (-9) + 12$.

Solution. Add the positive integers first, then the negatives, then add the results.

$$\begin{aligned} -7 + 8 + (-9) + 12 &= 20 + (-16) && \text{Add the positives: } 8 + 12 = 20. \\ & && \text{Add the negatives: } -7 + (-9) = -16. \\ &= 4 && \text{Add the results: } 20 + (-16) = 4. \end{aligned}$$

Thus, $-7 + 8 + (-9) + 12 = 4$.

Answer: -7.

Using Correct Notation. Never write $+-$! That is, the notation

$$9 + -4 \qquad \text{and} \qquad -8 + -6$$

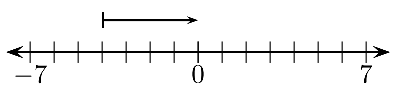
should not be used. Instead, use grouping symbols as follows:

$$9 + (-4) \qquad \text{and} \qquad -8 + (-6)$$

❧ ❧ ❧ Exercises ☙ ☙ ☙

In Exercises 1-12, what integer is represented by the given vector?

1.



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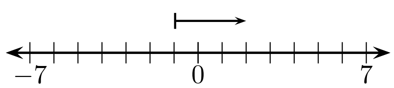
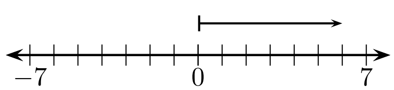
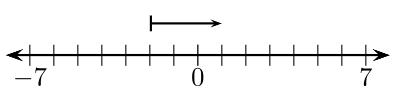


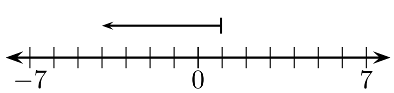
3.



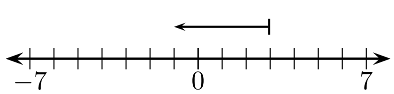
4.



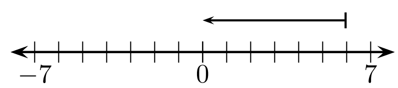
5.



6.



7.



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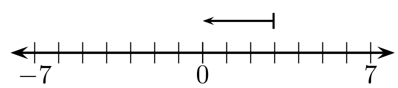

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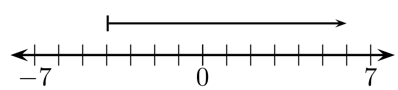
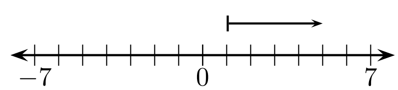
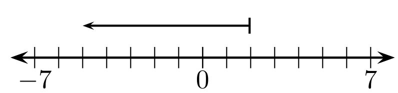


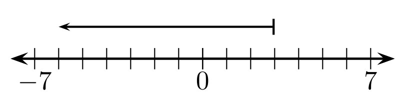
10.



11.



12.



In Exercises 13-36, find the sum of the given integers.

13. $-15 + 1$

14. $-1 + 18$

15. $18 + (-10)$

16. $2 + (-19)$

17. $-10 + (-12)$

18. $-1 + (-7)$

19. $5 + 10$

20. $1 + 12$

21. $2 + 5$

22. $14 + 1$

23. $19 + (-15)$

24. $20 + (-17)$

25. $-2 + (-7)$

26. $-14 + (-6)$

27. $-6 + 16$

28. $-2 + 14$

29. $-11 + (-6)$

30. $-7 + (-8)$

31. $14 + (-9)$

32. $5 + (-15)$

33. $10 + 11$

34. $14 + 11$

35. $-13 + 1$

36. $-8 + 2$

In Exercises 37-52, state the property of addition depicted by the given identity.

37. $-1 + (3 + (-8)) = (-1 + 3) + (-8)$

38. $-4 + (6 + (-5)) = (-4 + 6) + (-5)$

39. $7 + (-7) = 0$

40. $14 + (-14) = 0$

41. $15 + (-18) = -18 + 15$

42. $14 + (-8) = -8 + 14$

43. $-15 + 0 = -15$

44. $-11 + 0 = -11$

45. $-7 + (1 + (-6)) = (-7 + 1) + (-6)$

46. $-4 + (8 + (-1)) = (-4 + 8) + (-1)$

47. $17 + (-2) = -2 + 17$

48. $5 + (-13) = -13 + 5$

49. $-4 + 0 = -4$

50. $-7 + 0 = -7$

51. $19 + (-19) = 0$

52. $5 + (-5) = 0$

In Exercises 53-64, state the additive inverse of the given integer.

53. 18

54. 10

55. 12

56. 15

57. -16

58. -4

59. 11

60. 13

61. -15

62. -19

63. -18

64. -9

In Exercises 65-80, find the sum of the given integers.

65. $6 + (-1) + 3 + (-4)$

66. $6 + (-3) + 2 + (-7)$

67. $15 + (-1) + 2$

68. $11 + (-16) + 16$

69. $-17 + 12 + 3$

70. $-5 + (-3) + 2$

71. $7 + 20 + 19$

72. $14 + (-14) + (-20)$

73. $4 + (-8) + 2 + (-5)$

74. $6 + (-3) + 7 + (-2)$

75. $7 + (-8) + 2 + (-1)$

76. $8 + (-9) + 5 + (-3)$

77. $9 + (-3) + 4 + (-1)$

78. $1 + (-9) + 7 + (-6)$

79. $9 + 10 + 2$

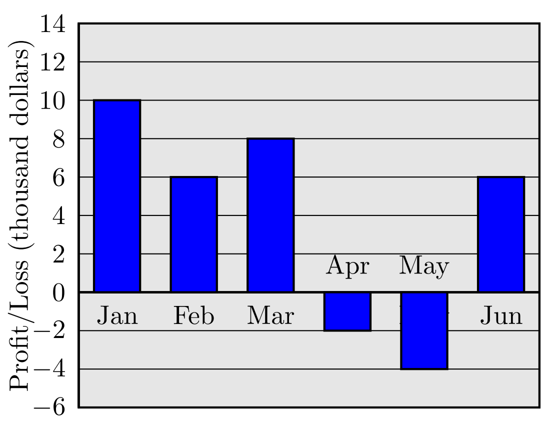
80. $-6 + 15 + (-18)$

---

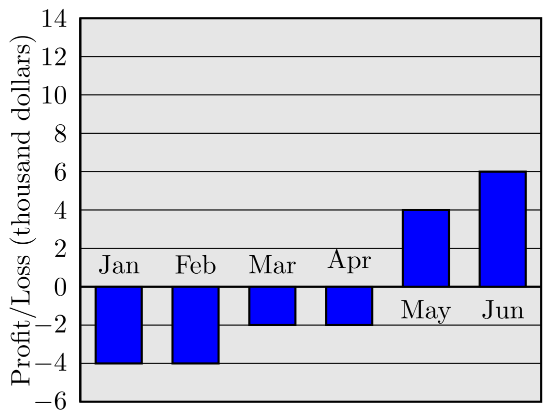
81. **Bank Account.** Gerry opened a new bank account, depositing a check for \$215. He then made several withdraws of \$40, \$75, and \$20 before depositing another check for \$185. How much is in Gerry's account now?

82. **Dead Sea Sinking.** Due to tectonic plate movement, the Dead Sea is sinking about 1 meter each year. If it's currently -418 meters now, what will Dead Sea elevation be in 5 years? Write an expression that models this situation and compute the result.

83. **Profit and Loss.** Profits and losses for the first six months of the fiscal year for a small business are shown in the following bar chart. Sum the profits and losses from each month. Was there a net profit or loss over the six-month period? How much?



84. **Profit and Loss.** Profits and losses for the first six months of the fiscal year for a small business are shown in the following bar chart. Sum the profits and losses from each month. Was there a net profit or loss over the six-month period? How much?



Answers

1. 4

3. 6

5. -5

7. -6

9. 10

11. -7

13. -14

15. 8

17. -22

19. 15

21. 7

23. 4

25. -9

27. 10

29. -17

31. 5

33. 21

35. -12

37. Associative property of addition

39. Additive inverse property

41. Commutative property of addition

43. Additive identity property

45. Associative property of addition

47. Commutative property of addition

49. Additive identity property

51. Additive inverse property

53. -18

55. -12

57. 16

59. -11

61. 15

63. 18

65. 4

67. 16

69. -2

71. 46

73. -7

75. 0

77. 9

79. 21

81. \$265

83. Net Profit: \$24,000

2.3 Subtracting Integers

In Section 1.2, we stated that "Subtraction is the opposite of addition." Thus, to subtract 4 from 7, we walked seven units to the right on the number line, but then walked 4 units in the opposite direction (to the left), as shown in Figure 2.10.

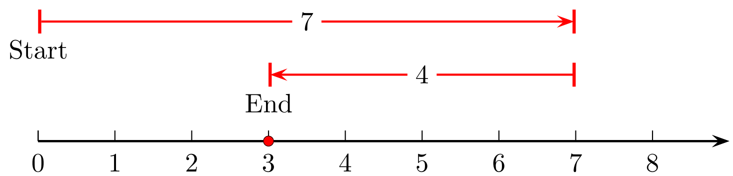


Figure 2.10: Subtraction requires that we reverse direction.

Thus, $7 - 4 = 3$.

The key phrase is "add the opposite." Thus, the subtraction $7-4$ becomes the addition $7+(-4)$, which we would picture on the number line as shown in Figure 2.11.

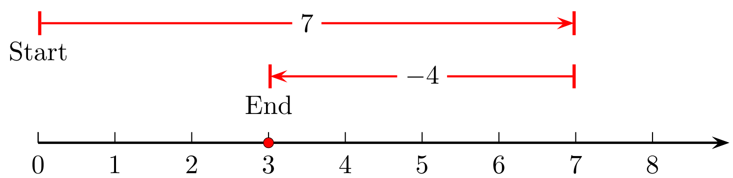


Figure 2.11: Subtraction means *add the opposite*.

Figure 2.10 and Figure 2.11 provide ample evidence that the subtraction $7-4$ is identical to the addition $7+(-4)$. Again, subtraction means "add the opposite." That is, $7 - 4 = 7 + (-4)$.

Defining Subtraction. Subtraction means "add the opposite." That is, if a and b are any integers, then

$$a - b = a + (-b).$$

Thus, for example, $-123-150 = -123+(-150)$ and $-57-(-91) = -57+91$. In each case, subtraction means "add the opposite." In the first case, subtracting 150 is the same as adding -150. In the second case, subtracting -91 is the same as adding 91.

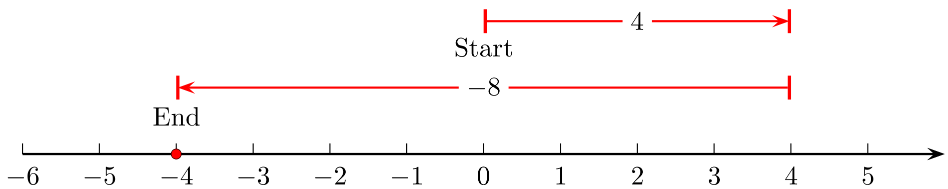
You Try It!

Use each of the techniques in parts (a), (b), and (c) of Example 1 to evaluate the difference $-11-(-9)$.

EXAMPLE 1. Find the differences: (a) $4-8$, (b) $-15-13$, and (c) $-117-(-115)$.

Solution. In each case, subtraction means "add the opposite."

a) Change the subtraction to addition with the phrase "subtraction means add the opposite." That is, $4-8=4+(-8)$. We can now perform this addition on the number line.



Thus, $4-8=4+(-8)=-4$.

b) First change the subtraction into addition by "adding the opposite." That is, $-15-13=-15+(-13)$. We can now use physical intuition to perform the addition. Start at the origin (zero), walk 15 units to the left, then an additional 13 units to the left, arriving at the answer -28. That is,

$$\begin{aligned}-15-13&=-15+(-13)\\&=-28.\end{aligned}$$

c) First change the subtraction into addition by "adding the opposite." That is, $-117-(-115)=-117+115$. Using "Adding Two Integers with Unlike Signs" from Section 2.2, first subtract the smaller magnitude from the larger magnitude; that is, $117-115=2$. Because -117 has the larger magnitude and its sign is negative, prefix a negative sign to the difference in magnitudes. Thus,

$$\begin{aligned}-117-(-115)&=-117+115\\&=-2.\end{aligned}$$

Answer: -2

□

Order of Operations

We will now apply the "Rules Guiding Order of Operations" from Section 1.5 to a number of example exercises.

You Try It!

Simplify: $-3-(-9)-11$.

EXAMPLE 2. Simplify $-5-(-8)-7$.

Solution. We work from left to right, changing each subtraction by "adding the opposite."

$$\begin{aligned} -5-(-8)-7 &= -5+8+(-7) && \text{Add the opposite of } -8 \text{, which is } 8. \text{ Add the opposite of } 7 \text{, which is } -7.\\ &= 3+(-7) && \text{Working left to right, } -5+8=3.\\ &= -4 && 3+(-7)=-4. \end{aligned}$$

Answer: -5

□

Grouping symbols say "do me first."

You Try It!

Simplify: $-3-(-3-3)$.

EXAMPLE 3. Simplify $-2-(-2-4)$.

Solution. Parenthetical expressions must be evaluated first.

$$\begin{aligned} -2-(-2-4) &= -2-(-2+(-4)) && \text{Simplify the parenthetical expression first. Add the opposite of } 4 \text{, which is } -4.\\ &= -2-(-6) && \text{Inside the parentheses, } -2+(-4)=-6.\\ &= -2+6 && \text{Subtracting a } -6 \text{ is the same as adding a } 6.\\ &= 4 && \text{Add: } -2+6=4. \end{aligned}$$

Answer: 3

□

Change as a Difference

Suppose that when I leave my house in the early morning, the temperature outside is 40° Fahrenheit. Later in the day, the temperature measures 60° Fahrenheit. How do I measure the change in the temperature?

The Change in a Quantity. To measure the change in a quantity, always subtract the *former* measurement from the *latter* measurement. That is:

Change in a Quantity	=	Latter Measurement	−	Former Measurement

Thus, to measure the change in temperature, I perform a subtraction as follows:

Change in Temperature	=	Latter Measurement	−	Former Measurement
	=	60° F	−	40° F
	=	20° F		

Note that the positive answer is in accord with the fact that the temperature has increased.

You Try It!

EXAMPLE 4. Suppose that in the afternoon, the temperature measures 65° Fahrenheit, then late evening the temperature drops to 44° Fahrenheit. Find the change in temperature.

Marianne awakes to a morning temperature of 54° Fahrenheit. A storm hits, dropping the temperature to 43° Fahrenheit.Find the change in temperature.

Solution. To measure the change in temperature, we must subtract the former measurement from the latter measurement.

Change in Temperature	=	Latter Measurement	−	Former Measurement
	=	44° F	−	65° F
	=	−11° F		

Note that the negative answer is in accord with the fact that the temperature has decreased. There has been a "change" of −11° Fahrenheit.

Answer: −11° Fahrenheit

□

You Try It!

EXAMPLE 5. Sometimes a bar graph is not the most appropriate visualization for your data. For example, consider the bar graph in Figure 2.12 depicting the Dow Industrial Average for seven consecutive days in March of 2009. Because the bars are of almost equal height, it is difficult to detect fluctuation or change in the Dow Industrial Average.

Let's determine the change in the Dow Industrial average on a day-to-day basis. Remember to subtract the latter measurement minus the former (current day minus former day). This gives us the following changes.

Consecutive Days	Change in Dow Industrial Average
Sun-Mon	$6900 - 7000 = -100$
Mon-Tues	$6800 - 6900 = -100$
Tues-Wed	$6800 - 6800 = 0$
Wed-Thu	$7000 - 6800 = 200$
Thu-Fri	$7100 - 7000 = 100$
Fri-Sat	$7200 - 7100 = 100$

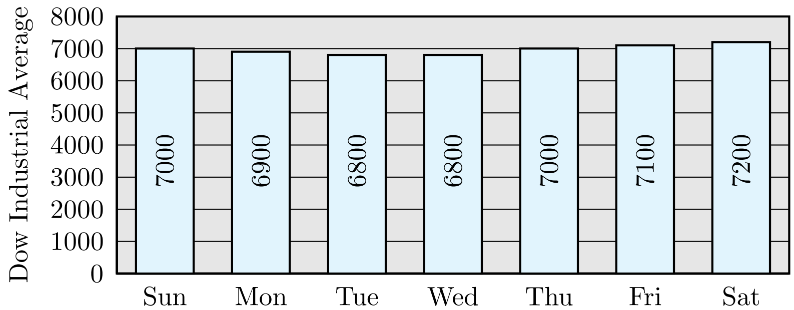


Figure 2.12: Profit and loss bar graph.

We will use the data in the table to construct a line graph. On the horizontal axis, we place the pairs of consecutive days (see Figure 2.13). On the vertical axis we place the Change in the Industrial Dow Average. At each pair of days we plot a point at a height equal to the change in Dow Industrial Average as calculated in our table.

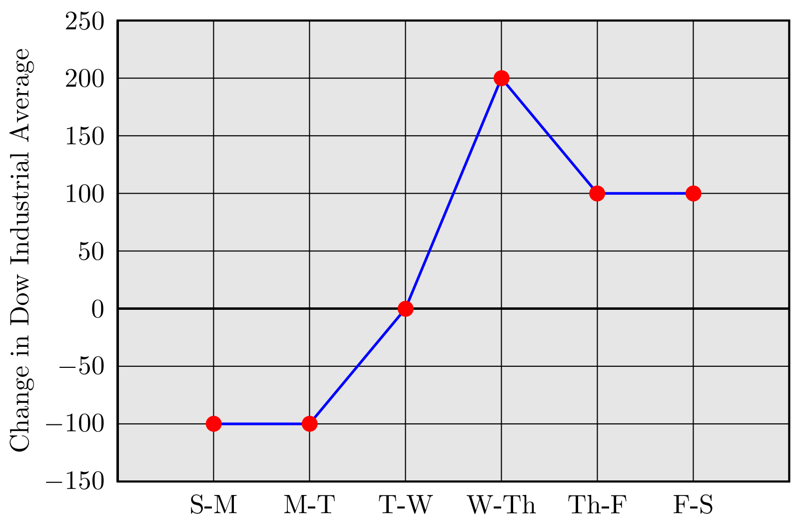


Figure 2.13: Plotting change in Dow versus consecutive days.

Note that the data as displayed by Figure 2.13 more readily shows the changes in the Dow Industrial Average on a day-to-day basis. For example, it is now easy to pick the day that saw the greatest increase in the Dow (from Wednesday to Thursday, the Dow rose 200 points).

□

❧ ❧ ❧ Exercises ☙ ☙ ☙

In Exercises 1-24, find the difference.

1. $16 - 20$

2. $17 - 2$

3. $10 - 12$

4. $16 - 8$

5. $14 - 11$

6. $5 - 8$

7. $7 - (-16)$

8. $20 - (-10)$

9. $-4 - (-9)$

10. $-13 - (-3)$

11. $8 - (-3)$

12. $14 - (-20)$

13. $2 - 11$

14. $16 - 2$

15. $-8 - (-10)$

16. $-14 - (-2)$

17. $13 - (-1)$

18. $12 - (-13)$

19. $-4 - (-2)$

20. $-6 - (-8)$

21. $7 - (-8)$

22. $13 - (-14)$

23. $-3 - (-10)$

24. $-13 - (-9)$

In Exercises 25-34, simplify the given expression.

25. $14 - 12 - 2$

26. $-19 - (-7) - 11$

27. $-20 - 11 - 18$

28. $7 - (-13) - (-1)$

29. $5 - (-10) - 20$

30. $-19 - 12 - (-8)$

31. $-14 - 12 - 19$

32. $-15 - 4 - (-6)$

33. $-11 - (-7) - (-6)$

34. $5 - (-5) - (-14)$

In Exercises 35-50, simplify the given expression.

35. $-2 - (-6 - (-5))$

36. $6 - (-14 - 9)$

37. $(-5 - (-8)) - (-3 - (-2))$

38. $(-6 - (-8)) - (-9 - 3)$

39. $(6 - (-9)) - (3 - (-6))$

40. $(-2 - (-3)) - (3 - (-6))$

41. $-1 - (10 - (-9))$

42. $7 - (14 - (-8))$

43. $3 - (-8 - 17)$

44. $1 - (-1 - 4)$

45. $13-(16-(-1))$

46. $-7-(-3-(-8))$

47. $(7-(-8))-(5-(-2))$

48. $(6-5)-(7-3)$

49. $(6-4)-(-8-2)$

50. $(2-(-6))-(-9-(-3))$

---

51. The first recorded temperature is 42° F. Four hours later, the second temperature is 65° F. What is the change in temperature?

52. The first recorded temperature is 79° F. Four hours later, the second temperature is 46° F. What is the change in temperature?

53. The first recorded temperature is 30° F. Four hours later, the second temperature is 51° F. What is the change in temperature?

54. The first recorded temperature is 109° F. Four hours later, the second temperature is 58° F. What is the change in temperature?

55. Typical temperatures in Fairbanks, Alaska in January are -2 degrees Fahrenheit in the daytime and -19 degrees Fahrenheit at night. What is the change in temperature from day to night?

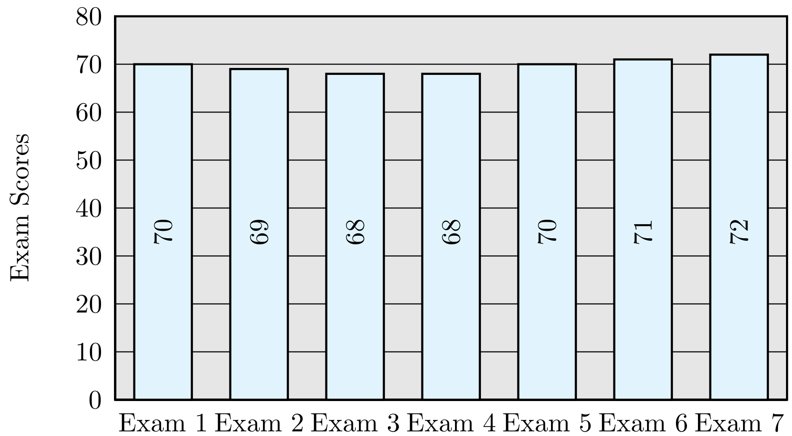
56. Typical summertime temperatures in Fairbanks, Alaska in July are 79 degrees Fahrenheit in the daytime and 53 degrees Fahrenheit at night. What is the change in temperature from day to night?

57. **Communication.** A submarine 1600 feet below sea level communicates with a pilot flying 22,500 feet in the air directly above the submarine. How far is the communique traveling?

58. **Highest to Lowest.** The highest spot on earth is on Mount Everest in Nepal-Tibet at 8,848 meters. The lowest point on the earth's crust is the Mariana's Trench in the North Pacific Ocean at 10,923 meters below sea level. What is the distance between the highest and the lowest points on earth? *Wikipedia http://en.wikipedia.org/wiki/Extremes_on_Earth*

59. **Lowest Elevation.** The lowest point in North America is Death Valley, California at -282 feet. The lowest point on the entire earth's landmass is on the shores of the Dead Sea along the Israel-Jordan border with an elevation of -1,371 feet. How much lower is the Dead Sea shore from Death Valley?

60. Exam Scores. Freida's scores on her first seven mathematics exams are shown in the following bar chart. Calculate the differences between consecutive exams, then create a line graph of the differences on each pair of consecutive exams. Between which two pairs of consecutive exams did Freida show the most improvement?



Answers

1. -4

3. -2

5. 3

7. 23

9. 5

11. 11

13. -9

15. 2

17. 14

19. -2

21. 15

23. 7

25. 0

27. -49

29. -5

31. -45

33. 2

35. -1

37. 4

39. 6

41. -20

43. 28

45. -4

47. 8

49. 12

51. 23° F

53. 21° F

55. -17 degrees Fahrenheit

57. 24,100 feet

59. 1,089 feet lower

2.4 Multiplication and Division of Integers

Before we begin, let it be known that the integers satisfy the same properties of multiplication as do the whole numbers.

Integer Properties of Multiplication.

Commutative Property. If a and b are integers, then their product commutes. That is,

$$a \cdot b = b \cdot a.$$

Associative Property. If a, b, and c are integers, then their product is associative. That is,

$$(a \cdot b) \cdot c = a \cdot (b \cdot c).$$

Multiplicative Identity Property. If a is any integer, then

$$a \cdot 1 = a \qquad \text{and} \qquad 1 \cdot a = a.$$

Because multiplying any integer by 1 returns the identical integer, the integer 1 is called the **multiplicative identity**.

In Section 1.3, we learned that multiplication is equivalent to *repeated addition.* For example,

$$3 \cdot 4 = \underbrace{4 + 4 + 4}_{\text{three fours}}.$$

On the number line, three sets of four is equivalent to walking three sets of four units to the right, starting from zero, as shown in Figure 2.14.

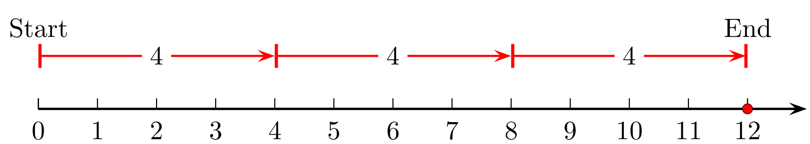


Figure 2.14: Note that $3 \cdot 4 = 4 + 4 + 4$. That is, $3 \cdot 4 = 12$.

This example and a little thought should convince readers that the product of two positive integers will always be a positive integer.

The Product of Two Positive Integers. If a and b are two positive integers, then their product ab is also a positive integer.

For example, $2 \cdot 3 = 6$ and $13 \cdot 117 = 1521$. In each case, the product of two positive integers is a positive integer.

The Product of a Positive Integer and a Negative Integer

If we continue with the idea that multiplication is equivalent to repeated addition, then it must be that

$$3 \cdot (-4) = \underbrace{-4 + (-4) + (-4)}_{\text{three negative fours}}.$$

Pictured on the number line, $3 \cdot (-4)$ would then be equivalent to walking three sets of negative four units (to the left), starting from zero, as shown in Figure 2.15.

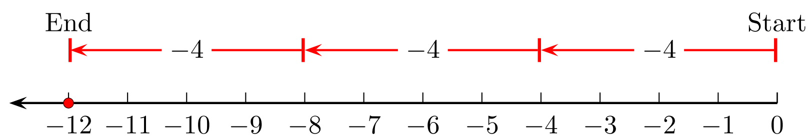


Figure 2.15: Note that $3 \cdot (-4) = -4 + (-4) + (-4)$. That is, $3 \cdot (-4) = -12$.

Note, at least in this particular case, that the product of a positive integer and a negative integer is a negative integer.

We've shown that $3 \cdot (-4) = -12$. However, integer multiplication is commutative, so it must also be true that $-4 \cdot 3 = -12$. That is, the product of a negative integer and a positive integer is also a negative integer. Although not a proof, this argument motivates the following fact about integer multiplication.

The Product of a Positive Integer and a Negative Integer. Two facts are true:

1. If a is a positive integer and b is a negative integer, then the product ab is a negative integer.
2. If a is a negative integer and b is a positive integer, then the product ab is a negative integer.

Thus, for example, $5 \cdot (-12) = -60$ and $-13 \cdot 2 = -26$. In each case the answer is negative because we are taking a product where one of the factors is positive and the other is negative.

The Distributive Property

The integers satisfy the *distributive property.*

The Distributive Property. Let a, b, and c be integers. Then,

$$a \cdot (b + c) = a \cdot b + a \cdot c.$$

We say that "multiplication is distributive with respect to addition."

Note how the a is "distributed." The a is multiplied times each term in the parentheses.

For example, consider the expression $3 \cdot (4 + 5)$. We can evaluate this expression according to the order of operations, simplifying the expression inside the parentheses first.

$$\begin{aligned} 3 \cdot (4 + 5) &= 3 \cdot 9 && (2.1) \\ &= 27 && (2.2) \end{aligned}$$

But we can also use the distributive property, multiplying each term inside the parentheses by three, then simplifying the result.

$$\begin{aligned} 3 \cdot (4 + 5) &= 3 \cdot 4 + 3 \cdot 5 && \text{Distribute the 3.} \\ &= 12 + 15 && \text{Perform multiplications first:} \\ & && 3 \cdot 4 = 12 \text{ and } 3 \cdot 5 = 15. \\ &= 27 && \text{Add: } 12 + 15 = 27. \end{aligned}$$

Note that evaluating $3 \cdot (4 + 5)$ using the distributive property provides the same result as the evaluation (2.1) using the order of operations.

The Multiplicative Property of Zero

The distributive property can be used to provide proofs of a number of important properties of integers. One important property is the fact that if you multiply an integer by zero, the product is zero. Here is a proof of that fact that uses the distributive property.

Let a be any integer. Then,

$$\begin{aligned} a \cdot 0 &= a \cdot (0 + 0) && \text{Additive Identity Property: } 0 + 0 = 0. \\ a \cdot 0 &= a \cdot 0 + a \cdot 0 && \text{Distribute } a \text{ times each zero in the parentheses.} \end{aligned}$$

Next, to "undo" the effect of adding $a \cdot 0$, subtract $a \cdot 0$ from both sides of the equation.

$$\begin{aligned} a \cdot 0 - a \cdot 0 &= a \cdot 0 + a \cdot 0 - a \cdot 0 && \text{Subtract } a \cdot 0 \text{ from both sides.} \\ 0 &= a \cdot 0 && a \cdot 0 - a \cdot 0 = 0 \text{ on each side.} \end{aligned}$$

Multiplicative Property of Zero. Let a represent any integer. Then

$$a \cdot 0 = 0 \qquad \text{and} \qquad 0 \cdot a = 0.$$

Thus, for example, $-18 \cdot 0 = 0$ and $0 \cdot 122 = 0$.

Multiplying by Minus One

Here is another useful application of the distributive property.

$$\begin{aligned} (-1)a + a &= (-1)a + 1a && \text{Replace } a \text{ with } 1a. \\ &= (-1+1)a && \text{Use the distributive property to factor out } a. \\ &= 0a && \text{Replace } -1+1 \text{ with } 0. \\ &= 0 && \text{Replace } 0a \text{ with } 0. \end{aligned}$$

Thus, $(-1)a + a = 0$. That is, if you add $(-1)a$ to a you get zero. However, the Additive Inverse Property says that $-a$ is the *unique* number that you add to a to get zero. The conclusion must be that $(-1)a = -a$.

Multiplying by Minus One. If a is any integer, then

$$(-1)a = -a.$$

Thus, for example, $-1(4) = -4$ and $-1(-4) = -(-4) = 4$.

This property is rather important, as we will see in future work. Not only does it tell us that $(-1)a = -a$, but it also tells us that if we see $-a$, then it can be interpreted to mean $(-1)a$.

The Product of Two Negative Integers

We can employ the multiplicative property of -1, that is, $(-1)a = -a$ to find the product of two negative numbers.

$$\begin{aligned} (-4)(-3) &= [(-1)(4)](-3) && \text{Replace } -4 \text{ with } (-1)(4). \\ &= (-1)[(4)(-3)] && \text{Use the associative property to regroup.} \\ &= (-1)(-12) && \text{We know: } (4)(-3) = -12. \\ &= -(-12) && (-1)a = -a. \text{ Here } (-1)(-12) = -(-12). \\ &= 12 && -(-a) = a. \text{ Here } -(-12) = 12. \end{aligned}$$

Thus, at least in the case of $(-4)(-3)$, the product of two negative integers is a positive integer. This is true in general.

The Product of Two Negative Integers. If both a and b are negative integers, then their product ab is a positive integer.

Thus, for example, $(-5)(-7) = 35$ and $(-112)(-6) = 672$. In each case the answer is positive, because the product of two negative integers is a positive integer.

Memory Device

Here's a simple memory device to help remember the rules for finding the product of two integers.

Like and Unlike Signs. There are two cases:

Unlike Signs. The product of two integers with *unlike* signs is negative. That is:

$$\begin{aligned}(+)(-) &= -\\(-)(+) &= -\end{aligned}$$

Like Signs. The product of two integers with *like* signs is positive. That is:

$$\begin{aligned}(+)(+) &= +\\(-)(-) &= +\end{aligned}$$

You Try It!

EXAMPLE 1. Simplify: (a) $(-3)(-2)$, (b) $(4)(-10)$, and (c) $(12)(-3)$.

Simplify: (a) $(-12)(4)$ and (b) $(-3)(-11)$.

Solution. In each example, we use the "like" and "unlike" signs approach.

a) Like signs gives a positive result. Hence, $(-3)(-2) = 6$.

b) Unlike signs gives a negative result. Hence, $(4)(-10) = -40$.

c) Unlike signs gives a negative result. Hence, $(12)(-3) = -36$.

Answer: (a) -48, (b) 33

□

You Try It!

Simplify: $(-2)(-3)(4)(-1)$.

EXAMPLE 2. Simplify $(-3)(2)(-4)(-2)$.

Solution. Order of operations demands that we work from left to right.

$$\begin{aligned}(-3)(2)(-4)(-2) &= (-6)(-4)(-2) && \text{Work left to right: } (-3)(2) = -6.\\ &= (24)(-2) && \text{Work left to right: } (-6)(-4) = 24.\\ &= -48 && \text{Multiply: } (24)(-2) = -48.\end{aligned}$$

Answer: -24.

Hence, $(-3)(2)(-4)(-2) = -48$.

□

You Try It!

Simplify: (a) $(-2)^2$ and (b) -2^2.

EXAMPLE 3. Simplify: (a) $(-2)^3$ and (c) $(-3)^4$.

Solution. In each example, use

$$a^m = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{m \text{ times}},$$

then work left to right with the multiplication.

a) Use the definition of an exponent, then order of operations.

$$\begin{aligned}(-2)^3 &= (-2)(-2)(-2) && \text{Write } -2 \text{ as a factor three times.}\\ &= 4(-2) && \text{Work left to right: } (-2)(-2) = 4.\\ &= -8\end{aligned}$$

b) Use the definition of an exponent, then order of operations.

$$\begin{aligned}(-3)^4 &= (-3)(-3)(-3)(-3) && \text{Write } -3 \text{ as a factor four times.}\\ &= 9(-3)(-3) && \text{Work left to right: } (-3)(-3) = 9.\\ &= -27(-3) && \text{Work left to right: } 9(-3) = -27.\\ &= 81\end{aligned}$$

Answer: (a) 4 and (b) -4.

□

Example 3 motivates the following fact.

Even and Odd. Two facts are apparent.

1. If a product contains an odd number of negative factors, then the product is negative.
2. If a product contains an even number of negative factors, then the product is positive.

Thus, for example,

$$(-2)^5 = (-2)(-2)(-2)(-2)(-2) = -32$$

quickly evaluates as -32 as it has an odd number of negative factors. On the other hand,

$$(-2)^6 = (-2)(-2)(-2)(-2)(-2)(-2) = 64$$

quickly evaluates as 64 as it has an even number of negative factors.

Division of Integers

Consider that

$$\frac{12}{3} = 4 \text{ because } 3(4) = 12 \qquad \text{and} \qquad \frac{-12}{-3} = 4 \text{ because } -3(4) = -12.$$

In like manner,

$$\frac{12}{-3} = -4 \text{ because } -3(-4) = 12 \qquad \text{and} \qquad \frac{-12}{3} = -4 \text{ because } 3(-4) = -12.$$

Thus, the rules for dividing integers are the same as the rules for multiplying integers.

Like and Unlike Signs. There are two cases:

Unlike Signs. The quotient of two integers with *unlike* signs is negative. That is,

$$\frac{(+)}{(-)} = -$$
$$\frac{(-)}{(+)} = -$$

Like Signs. The quotient of two integers with *like* signs is positive. That is,

$$\frac{(+)}{(+)} = +$$
$$\frac{(-)}{(-)} = +$$

Thus, for example, $12/(-6) = -2$ and $-44/(-4) = 11$. In the first case, unlike signs gives a negative quotient. In the second case, like signs gives a positive quotient.

One final reminder.

Division by Zero is Undefined. If a is any integer, the quotient

$$\frac{a}{0}$$

is undefined. Division by zero is meaningless.

See the discussion in Section 1.3 for a discussion on division by zero.

You Try It!

Simplify: (a) $-24/4$ and (b) $-28/(-7)$.

EXAMPLE 4. Simplify: (a) $-12/(-4)$, (b) $6/(-3)$, and (c) $-15/0$.

Solution. In each example, we use the "like" and "unlike" signs approach.

a) Like signs gives a positive result. Hence,

$$\frac{-12}{-4} = 3.$$

b) Unlike signs gives a negative result. Hence,

$$\frac{6}{-3} = -2.$$

c) Division by zero is undefined. Hence,

$$\frac{-15}{0}$$

is undefined.

Answer: (a) -6, (b) 4

□

❧ ❧ ❧ Exercises ☙ ☙ ☙

In Exercises 1-16, state the property of multiplication depicted by the given identity.

1. $(-2)\big[(-16)(13)\big] = \big[(-2)(-16)\big](13)$

2. $(10)\big[(-15)(-6)\big] = \big[(10)(-15)\big](-6)$

3. $(-17)(-10) = (-10)(-17)$

4. $(-5)(3) = (3)(-5)$

5. $(4)(11) = (11)(4)$

6. $(-5)(-11) = (-11)(-5)$

7. $16 \cdot \big(8 + (-15)\big) = 16 \cdot 8 + 16 \cdot (-15)$

8. $1 \cdot \big(-16 + (-6)\big) = 1 \cdot (-16) + 1 \cdot (-6)$

9. $(17)\big[(20)(11)\big] = \big[(17)(20)\big](11)$

10. $(14)\big[(-20)(-18)\big] = \big[(14)(-20)\big](-18)$

11. $-19 \cdot 1 = -19$

12. $-17 \cdot 1 = -17$

13. $8 \cdot 1 = 8$

14. $-20 \cdot 1 = -20$

15. $14 \cdot \big(-12 + 7\big) = 14 \cdot (-12) + 14 \cdot 7$

16. $-14 \cdot \big(-3 + 6\big) = -14 \cdot (-3) + (-14) \cdot 6$

In Exercises 17-36, simplify each given expression.

17. $4 \cdot 7$

18. $4 \cdot 2$

19. $3 \cdot (-3)$

20. $7 \cdot (-9)$

21. $-1 \cdot 10$

22. $-1 \cdot 11$

23. $-1 \cdot 0$

24. $-8 \cdot 0$

25. $-1 \cdot (-14)$

26. $-1 \cdot (-13)$

27. $-1 \cdot (-19)$

28. $-1 \cdot (-17)$

29. $2 \cdot 0$

30. $-6 \cdot 0$

31. $-3 \cdot 8$

32. $7 \cdot (-3)$

33. $7 \cdot 9$

34. $6 \cdot 3$

35. $-1 \cdot 5$

36. $-1 \cdot 2$

In Exercises 37-48, simplify each given expression.

37. $(-7)(-1)(3)$

38. $(10)(6)(3)$

39. $(-7)(9)(10)(-10)$

40. $(-8)(-5)(7)(-9)$

41. $(6)(5)(8)$

42. $(7)(-1)(-9)$

43. $(-10)(4)(-3)(8)$

44. $(8)(-2)(-5)(2)$

45. $(6)(-3)(-8)$

46. $(-5)(-4)(1)$

47. $(2)(1)(3)(4)$

48. $(7)(5)(1)(4)$

In Exercises 49-60, compute the exact value.

49. $(-4)^4$

50. $(-3)^4$

51. $(-5)^4$

52. $(-2)^2$

53. $(-5)^2$

54. $(-3)^3$

55. $(-6)^2$

56. $(-6)^4$

57. $(-4)^5$

58. $(-4)^2$

59. $(-5)^3$

60. $(-3)^2$

In Exercises 61-84, simplify each given expression.

61. $-16 \div (-8)$

62. $-33 \div (-3)$

63. $\dfrac{-8}{1}$

64. $\dfrac{40}{-20}$

65. $\dfrac{-1}{0}$

66. $\dfrac{2}{0}$

67. $-3 \div 3$

68. $-58 \div 29$

69. $\dfrac{56}{-28}$

70. $\dfrac{60}{-12}$

71. $0 \div 15$

72. $0 \div (-4)$

73. $\dfrac{63}{21}$

74. $\dfrac{-6}{-1}$

75. $\dfrac{78}{13}$

76. $\dfrac{-84}{-14}$

77. $0 \div 5$

78. $0 \div (-16)$

79. $\dfrac{17}{0}$

80. $\dfrac{-20}{0}$

81. $-45 \div 15$

82. $-28 \div 28$

83. $12 \div 3$

84. $-22 \div (-22)$

---

85. **Scuba.** A diver goes down 25 feet. A second diver then dives down 5 times further than the first diver. Write the final depth of the second diver as an integer.

86. **Investing Loss.** An investing club of five friends has lost $4400 on a trade. If they share the loss equally, write each members' loss as an integer.

❧ ❧ ❧ Answers ☙ ☙ ☙

1. Associative property of multiplication

3. Commutative property of multiplication

5. Commutative property of multiplication

7. Distributive property

9. Associative property of multiplication

11. Multiplicative identity property

13. Multiplicative identity property

15. Distributive property

17. 28

19. -9

21. -10

23. 0

25. 14

27. 19

29. 0

31. -24

33. 63

35. -5

37. 21

39. 6300

41. 240

43. 960

45. 144

47. 24

49. 256

51. 625

53. 25

55. 36

57. -1024

59. -125

61. 2

63. -8

65. Division by zero is undefined.

67. -1

69. -2

71. 0

73. 3

75. 6

77. 0

79. Division by zero is undefined.

81. -3

83. 4

85. -125 feet

2.5 Order of Operations with Integers

For convenience, we repeat the “Rules Guiding Order of Operations” first introduced in Section 1.5.

Rules Guiding Order of Operations. When evaluating expressions, proceed in the following order.

1. Evaluate expressions contained in grouping symbols first. If grouping symbols are nested, evaluate the expression in the innermost pair of grouping symbols first.
2. Evaluate all exponents that appear in the expression.
3. Perform all multiplications and divisions in the order that they appear in the expression, moving left to right.
4. Perform all additions and subtractions in the order that they appear in the expression, moving left to right.

Let’s look at a number of examples that require the use of these rules.

You Try It!

Simplify: -2^2.

EXAMPLE 1. Simplify: (a) $(-3)^2$ and (b) -3^2.

Solution. Recall that for any integer a, we have $(-1)a = -a$. Because negating is equivalent to multiplying by -1, the “Rules Guiding Order of Operations” require that we address grouping symbols and exponents before negation.

a) Because of the grouping symbols, we negate first, then square. That is,

$$\begin{aligned}(-3)^2 &= (-3)(-3)\\ &= 9.\end{aligned}$$

b) There are no grouping symbols in this example. Thus, we must square first, then negate. That is,

$$\begin{aligned}-3^2 &= -(3 \cdot 3)\\ &= -9.\end{aligned}$$

Answer: -4

$\square$

You Try It!

EXAMPLE 2. Simplify: $-2 - 3(5 - 7)$.

Simplify: $-3 - 2(6 - 8)$.

Solution. Grouping symbols first, then multiplication, then subtraction.

$$\begin{aligned} -2 - 3(5 - 7) &= -2 - 3(-2) && \text{Perform subtraction within parentheses.} \\ &= -2 - (-6) && \text{Multiply: } 3(-2) = -6. \\ &= -2 + 6 && \text{Add the opposite.} \\ &= 4 \end{aligned}$$

Answer: 1

You Try It!

EXAMPLE 3. Simplify: $-2(2 - 4)^2 - 3(3 - 5)^3$.

Simplify:

$$-2(5 - 6)^3 - 3(5 - 7)^2$$

.

Solution. Grouping symbols first, then multiplication, and subtraction, in that order.

$$\begin{aligned} -2(2 - 4)^2 - 3(3 - 5)^3 &= -2(-2)^2 - 3(-2)^3 && \text{Perform subtraction within parentheses first.} \\ &= -2(4) - 3(-8) && \text{Exponents are next.} \\ &= -8 - (-24) && \text{Multiplications are next.} \\ &= -8 + 24 && \text{Add the opposite.} \\ &= 16 \end{aligned}$$

Answer: -10

You Try It!

EXAMPLE 4. Simplify: $-24 \div 8(-3)$.

Simplify: $-48 \div 6(-2)$.

Solution. Division has no preference over multiplication, or vice versa. Divisions and multiplications must be performed in the order that they occur, moving left to right.

$$\begin{aligned} -24 \div 8(-3) &= -3(-3) && \text{Division first: } -24 \div 8 = -3. \\ &= 9 \end{aligned}$$

Note that if you multiply first, which would be incorrect, you would get a completely different answer.

Answer: 16

You Try It!

Simplify: $(-4)(-2)^2(-1)^3$.

EXAMPLE 5. Simplify: $(-2)(-3)(-2)^3$.

Solution. Exponents first, then multiplication in the order that it occurs, moving left to right.

$$\begin{aligned}(-2)(-3)(-2)^3 &= (-2)(-3)(-8) && \text{Exponent first: } (-2)^3 = -8.\\ &= 6(-8) && \text{Multiply from left to right: } (-2)(-3) = 6.\\ &= -48\end{aligned}$$

Answer: 16

□

Evaluating Fractions

If a fraction bar is present, evaluate the numerator and denominator separately according to the "Rules Guiding Order of Operations," then perform the division in the final step.

You Try It!

Simplify:

$$\frac{6-2(-6)}{-2-(-2)^2}$$

EXAMPLE 6. Simplify:

$$\frac{-5-5(2-4)^3}{-22-3(-5)}$$

Solution. Evaluate numerator and denominator separately, then divide.

$$\begin{aligned}\frac{-5-5(2-4)^3}{-22-3(-5)} &= \frac{-5-5(-2)^3}{-22-(-15)} && \begin{array}{l}\text{Numerator: parentheses first.}\\ \text{Denominator: multiply } 3(-5) = -15.\end{array}\\ &= \frac{-5-5(-8)}{-22+15} && \begin{array}{l}\text{Numerator: exponent } (-2)^3 = -8.\\ \text{Denominator: add the opposite.}\end{array}\\ &= \frac{-5-(-40)}{-7} && \begin{array}{l}\text{Numerator: multiply } 5(-8) = -40.\\ \text{Denominator: add } -22+15 = -7.\end{array}\\ &= \frac{-5+40}{-7} && \text{Numerator: add the opposite.}\\ &= \frac{35}{-7} && \text{Numerator: } -5+40 = 35.\\ &= -5 && \text{Divide: } 35/-7 = -5\end{aligned}$$

Answer: -3

□

Absolute Value

Absolute value calculates the magnitude of the vector associated with an integer, which is equal to the distance between the number and the origin (zero) on the number line. Thus, for example, $|4| = 4$ and $|-5| = 5$.

But absolute value bars also act as grouping symbols, and according to the "Rules Guiding Order of Operations," you should evaluate the expression inside a pair of grouping symbols first.

You Try It!

EXAMPLE 7. Simplify: (a) $-(-3)$ and (b) $-|-3|$.

Simplify: $-|-8|$.

Solution. There is a huge difference between simple grouping symbols and absolute value.

a) This is a case of $-(-a) = a$. Thus, $-(-3) = 3$.

b) This case is much different. The absolute value of -3 is 3, and then the negative of that is -3. In symbols,

$$-|-3| = -3.$$

Answer: -8

□

You Try It!

EXAMPLE 8. Simplify: $-3 - 2|5 - 7|$.

Simplify: $-2 - 4|6 - 8|$.

Solution. Evaluate the expression inside the absolute value bars first. Then multiply, then subtract.

$$\begin{aligned}
-3 - 2|5 - 7| &= -3 - 2|-2| && \text{Subtract inside absolute value bars.} \\
&= -3 - 2(2) && \text{Take the absolute value: } |-2| = 2. \\
&= -3 - 4 && \text{Multiply: } 2(2) = 4. \\
&= -7 && \text{Subtract.}
\end{aligned}$$

Answer: -10

□

❧ ❧ ❧ Exercises ☙ ☙ ☙

In Exercises 1-40, compute the exact value of the given expression.

1. $7 - \dfrac{-14}{2}$

2. $-2 - \dfrac{-16}{4}$

3. $-7 - \dfrac{-18}{9}$

4. $-6 - \dfrac{-7}{7}$

5. -5^4

6. -3^3

7. $9 - 1(-7)$

8. $85 - 8(9)$

9. -6^3

10. -3^5

11. $3 + 9(4)$

12. $6 + 7(-1)$

13. $10 - 72 \div 6 \cdot 3 + 8$

14. $8 - 120 \div 5 \cdot 6 + 7$

15. $6 + \dfrac{14}{2}$

16. $16 + \dfrac{8}{2}$

17. -3^4

18. -2^2

19. $3 - 24 \div 4 \cdot 3 + 4$

20. $4 - 40 \div 5 \cdot 4 + 9$

21. $64 \div 4 \cdot 4$

22. $18 \div 6 \cdot 1$

23. $-2 - 3(-5)$

24. $64 - 7(7)$

25. $15 \div 1 \cdot 3$

26. $30 \div 3 \cdot 5$

27. $8 + 12 \div 6 \cdot 1 - 5$

28. $9 + 16 \div 2 \cdot 4 - 9$

29. $32 \div 4 \cdot 4$

30. $72 \div 4 \cdot 6$

31. $-11 + \dfrac{16}{16}$

32. $4 + \dfrac{-20}{4}$

33. -5^2

34. -4^3

35. $10 + 12(-5)$

36. $4 + 12(4)$

37. $2 + 6 \div 1 \cdot 6 - 1$

38. $1 + 12 \div 2 \cdot 2 - 6$

39. $40 \div 5 \cdot 4$

40. $30 \div 6 \cdot 5$

In Exercises 41-80, simplify the given expression.

41. $-11 + |-1 - (-6)^2|$

42. $13 + |-21 - (-4)^2|$

43. $|0(-4)| - 4(-4)$

44. $|10(-3)| - 3(-1)$

45. $(2 + 3 \cdot 4) - 8$

46. $(11 + 5 \cdot 2) - 10$

47. $(8 - 1 \cdot 12) + 4$

48. $(9 - 6 \cdot 1) + 3$

49. $(6 + 10 \cdot 4) - 6$

50. $(8 + 7 \cdot 6) - 12$

51. $10 + (6 - 4)^3 - 3$

52. $5 + (12 - 7)^2 - 6$

53. $(6 - 8)^2 - (4 - 7)^3$

54. $(3 - 8)^2 - (4 - 9)^3$

55. $|0(-10)| + 4(-4)$

56. $|12(-5)| + 7(-5)$

57. $|8(-1)| - 8(-7)$

58. $|6(-11)| - 7(-1)$

59. $3 + (3 - 8)^2 - 7$

60. $9 + (8 - 3)^3 - 6$

61. $(4 - 2)^2 - (7 - 2)^3$

62. $(1 - 4)^2 - (3 - 6)^3$

63. $8 - |-25 - (-4)^2|$

64. $20 - |-22 - 4^2|$

65. $-4 - |30 - (-5)^2|$

66. $-8 - |-11 - (-6)^2|$

67. $(8 - 7)^2 - (2 - 6)^3$

68. $(2 - 7)^2 - (4 - 7)^3$

69. $4 - (3 - 6)^3 + 4$

70. $6 - (7 - 8)^3 + 2$

71. $-3 + |-22 - 5^2|$

72. $12 + |23 - (-6)^2|$

73. $(3 - 4 \cdot 1) + 6$

74. $(12 - 1 \cdot 6) + 4$

75. $1 - (1 - 5)^2 + 11$

76. $9 - (3 - 1)^3 + 10$

77. $(2 - 6)^2 - (8 - 6)^3$

78. $(2 - 7)^2 - (2 - 4)^3$

79. $|9(-3)| + 12(-2)$

80. $|0(-3)| + 9(-7)$

In Exercises 81-104, simplify the given expression.

81. $\dfrac{4(-10) - 5}{-9}$

82. $\dfrac{-4 \cdot 6 - (-8)}{-4}$

83. $\dfrac{10^2 - 4^2}{2 \cdot 6 - 10}$

84. $\dfrac{3^2 - 9^2}{2 \cdot 7 - 5}$

85. $\dfrac{3^2 + 6^2}{5 - 1 \cdot 8}$

86. $\dfrac{10^2 + 4^2}{1 - 6 \cdot 5}$

87. $\dfrac{-8 - 4}{7 - 13}$

88. $\dfrac{13 - 1}{8 - 4}$

89. $\dfrac{2^2+6^2}{11-4\cdot 4}$

90. $\dfrac{7^2+3^2}{10-8\cdot 1}$

91. $\dfrac{1^2-5^2}{9\cdot 1-5}$

92. $\dfrac{5^2-7^2}{2\cdot 2-12}$

93. $\dfrac{4^2-8^2}{6\cdot 3-2}$

94. $\dfrac{7^2-6^2}{6\cdot 3-5}$

95. $\dfrac{10^2+2^2}{7-2\cdot 10}$

96. $\dfrac{2^2+10^2}{10-2\cdot 7}$

97. $\dfrac{16-(-2)}{19-1}$

98. $\dfrac{-8-20}{-15-(-17)}$

99. $\dfrac{15-(-15)}{13-(-17)}$

100. $\dfrac{7-(-9)}{-1-1}$

101. $\dfrac{4\cdot 5-(-19)}{3}$

102. $\dfrac{10\cdot 7-(-11)}{-3}$

103. $\dfrac{-6\cdot 9-(-4)}{2}$

104. $\dfrac{-6\cdot 2-10}{-11}$

Answers

1. 14

3. -5

5. -625

7. 16

9. -216

11. 39

13. -18

15. 13

17. -81

19. -11

21. 64

23. 13

25. 45

27. 5

29. 32

31. -10

33. -25

35. -50

37. 37

39. 32

41. 26

43. 16

45. 6

47. 0

49. 40

51. 15

53. 31

55. -16

57. 64

59. 21

61. -121

63. -33

65. -9

67. 65

69. 35

71. 44

73. 5

75. -4

77. 8

79. 3

81. 5

83. 42

85. -15

87. 2

89. -8

91. -6

93. -3

95. -8

97. 1

99. 1

101. 13

103. -25

2.6 Solving Equations Involving Integers

Recall (see Section 1.6) that a *variable* is a symbol (usually a letter) that stands for a value that varies. If a variable in an equation is replaced by a number and a true statement results, then that number is called a *solution* of the equation.

You Try It!

Is -4 a solution of $8 - 2x = 5$?

EXAMPLE 1. Is -6 a solution of the equation $2x + 5 = -7$?

Solution. Substitute -6 for x in the equation.

$$\begin{aligned} 2x + 5 &= -7 && \text{Original equation.} \\ 2(-6) + 5 &= -7 && \text{Substitute } -6 \text{ for } x. \\ -12 + 5 &= -7 && \text{On the left, multiply first.} \\ -7 &= -7 && \text{On the left, add.} \end{aligned}$$

Because the last statement is a true statement, -6 is a solution of the equation.

Answer: No.

□

Adding or Subtracting the Same Amount

Two equations having the same set of solutions are *equivalent*. For example, $2x+5 = -7$ and $x = -6$ have the same solutions. Therefore, they are equivalent equations. Certain algebraic operations produce equivalent equations.

Producing Equivalent Equations.

Adding the Same Quantity to Both Sides of an Equation. If we start with the equation

$$a = b,$$

then adding c to both sides of the equation produces the equivalent equation

$$a + c = b + c.$$

Subtracting the Same Quantity from Both Sides of an Equation. If we start with the equation

$$a = b,$$

then subtracting c from both sides of the equation produces the equivalent equation

$$a - c = b - c.$$

That is, adding or subtracting the same amount from both sides of an equation will not change the solutions of the equation.

You Try It!

EXAMPLE 2. Solve for x: $x + 3 = -7$.

Solve for x:

$$x + 9 = -11$$

Solution. To undo the effect of adding 3, subtract 3 from both sides of the equation.

$$\begin{aligned} x + 3 &= -7 && \text{Original equation.} \\ x + 3 - 3 &= -7 - 3 && \text{Subtract 3 from both sides.} \\ x &= -7 + (-3) && \text{Simplify the left hand side. On the right, express subtraction as adding the opposite.} \\ x &= -10 \end{aligned}$$

To check the solution, substitute -10 for x in the original equation and simplify.

$$\begin{aligned} x + 3 &= -7 && \text{Original equation.} \\ -10 + 3 &= -7 && \text{Substitute } -10 \text{ for } x. \\ -7 &= -7 && \text{Simplify both sides.} \end{aligned}$$

Since the last line of the check is a true statement, this confirms that -10 is a solution.

Answer: $x = -20$

You Try It!

EXAMPLE 3. Solve for x: $x - 8 = -11$.

Solve for x:

$$x - 2 = -7$$

Solution. To undo the effect of subtracting 8, add 8 to both sides of the equation.

$$\begin{aligned} x - 8 &= -11 && \text{Original equation.} \\ x - 8 + 8 &= -11 + 8 && \text{Add 8 to both sides.} \\ x &= -3 && \text{Simplify both sides.} \end{aligned}$$

To check the solution, substitute -3 for x in the original equation and simplify.

$$\begin{aligned} x - 8 &= -11 && \text{Original equation.} \\ -3 - 8 &= -11 && \text{Substitute } -3 \text{ for } x. \\ -11 &= -11 && \text{Simplify both sides.} \end{aligned}$$

Since the last line of the check is a true statement, this confirms that -3 is a solution.

Answer: $x = -5$

Sometimes a bit of simplification is in order before you start the solution process.

You Try It!

Solve for y:

$$y + 2(-4) = -8 + 6$$

EXAMPLE 4. Solve for y: $-8 + 2 = y - 11(-4)$.

Solution. First, simplify both sides of the equation.

$$\begin{aligned}
-8 + 2 &= y - 11(-4) && \text{Original equation.}\\
-6 &= y - (-44) && \text{Simplify. On the left, } -8+2=-6. \\
& && \text{On the right, } 11(-4) = -44.\\
-6 &= y + 44 && \text{Express subtraction as adding the opposite.}\\
-6 - 44 &= y + 44 - 44 && \text{Subtract 44 from both sides of the equation.}\\
-6 + (-44) &= y && \text{Express subtraction as addition. Simplify on the right.}\\
-50 &= y
\end{aligned}$$

To check the solution, substitute -50 for y in the original equation and simplify.

$$\begin{aligned}
-8 + 2 &= y - 11(-4) && \text{Original equation.}\\
-8 + 2 &= -50 - 11(-4) && \text{Substitute } -50 \text{ for } y.\\
-6 &= -50 - (-44) && \text{On the left, add. On the right, multiply}\\
& && \text{first: } 11(-4) = -44.\\
-6 &= -50 + 44 && \text{Express subtraction on the right as addition.}\\
-6 &= -6 && \text{On the right, add: } -50 + 44 = -6.
\end{aligned}$$

Since the last line of the check is a true statement, this confirms that -50 is a solution.

Answer: $y = 6$

□

Multiplying or Dividing by the Same Amount

Adding and subtracting is not the only way to produce an equivalent equation.

Producing Equivalent Equations.

Multiplying Both Sides of an Equation by the Same Quantity. If we start with the equation

$$a = b,$$

then multiplying both sides of the equation by c produces the equivalent equation

$$a \cdot c = b \cdot c, \qquad \text{or equivalently,} \qquad ac = bc,$$

provided $c \neq 0$.

Dividing Both Sides of an Equation by the Same Quantity. If we start with the equation

$$a = b,$$

then dividing both sides of the equation by c produces the equivalent equation

$$\frac{a}{c} = \frac{b}{c},$$

provided $c \neq 0$.

That is, multiplying or dividing both sides of an equation by the same amount will not change the solutions of the equation.

You Try It!

EXAMPLE 5. Solve for x: $-3x = 30$.

Solve for z:

$$-4z = -28$$

Solution. To undo the effect of multiplying by -3, divide both sides of the equation by -3.

$$\begin{aligned} -3x &= 30 && \text{Original equation.} \\ \frac{-3x}{-3} &= \frac{30}{-3} && \text{Divide both sides by } -3. \\ x &= -10 && \text{On the left, } -3 \text{ times } x\text{, divided by } -3 \text{ is } x. \\ & && \text{On the right, } 30/(-3) = -10. \end{aligned}$$

To check the solution, substitute -10 for x in the original equation and simplify.

$$\begin{aligned} -3x &= 30 && \text{Original equation.} \\ -3(-10) &= 30 && \text{Substitute } -10 \text{ for } x. \\ 30 &= 30 && \text{Simplify.} \end{aligned}$$

Because the last line of the check is a true statement, this confirms that -10 is a solution.

Answer: $z = 7$

□

You Try It!

EXAMPLE 6. Solve for x: $\dfrac{x}{-2} = -20$.

Solve for t:

$$\frac{t}{3} = -11$$

Solution. To undo the effect of dividing by -2, multiply both sides of the equation by -2.

$$\begin{aligned} \frac{x}{-2} &= -20 && \text{Original equation.} \\ -2\left(\frac{x}{-2}\right) &= -2(-20) && \text{Multiply both sides by } -2. \\ x &= 40 && \text{On the left, } x \text{ divided by } -2 \text{, multiplied by } -2, \\ & && \text{the result is } x. \text{ On the right, } -2(-20) = 40. \end{aligned}$$

To check the solution, substitute 40 foro x in the original equation and simplify.

$$\begin{aligned} \frac{x}{-2} &= -20 && \text{Original equation.} \\ \frac{40}{-2} &= -20 && \text{Substitute 40 for } x. \\ -20 &= -20 && \text{Simplify both sides.} \end{aligned}$$

Because the last line of the check is a true statement, this confirms that 40 is a solution.

Answer: $t = -33$

Combining Operations

Recall the "Wrap" and "Unwrap" discussion from Section 1.6. To wrap a present we: (1) put the gift paper on, (2) put the tape on, and (3) put the decorative bow on. To unwrap the gift, we must "undo" each of these steps in inverse order. Hence, to unwrap the gift we: (1) take off the decorative bow, (2) take off the tape, and (3) take off the gift paper.

Now, imagine a machine that takes its input, then: (1) multiplies the input by 2, and (2) adds 3 to the result. This machine is pictured on the left in Figure 2.16.

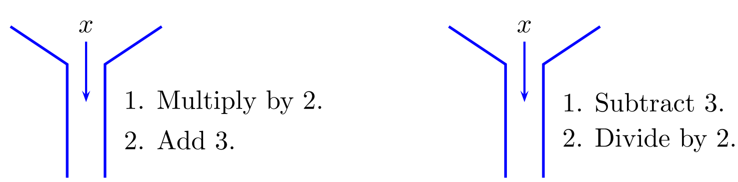


Figure 2.16: The second machine "unwraps" the first machine.

To "unwrap" the effect of the machine on the left, we need a machine that will "undo" each of the steps of the first machine, but in inverse order. The "unwrapping" machine is pictured on the right in Figure 2.16. It will first subtract three from its input, then divide the result by 2. Note that each of these operations "undoes" the corresponding operation of the first machine, but in inverse order.

For example, drop the integer 7 into the first machine on the left in Figure 2.16. First, we double 7, then add 3 to the result. The result is $2(7) + 3 = 17$.

Now, to "unwrap" this result, we drop 17 into the second machine. We first subtract 3, then divide by 2. The result is $(17 - 3)/2 = 7$, the original integer input into the first machine.

Now, consider the equation

$$2x + 3 = 7.$$

On the left, order of operations demands that we first multiply x by 2, then add 3. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will (1) subtract three from both sides of the equation, then (2) divide both sides of the resulting equation by 2.

$$\begin{aligned} 2x + 3 - 3 &= 7 - 3 && \text{Subtract 3 from both sides.} \\ 2x &= 4 && \text{Simplify both sides.} \\ \frac{2x}{2} &= \frac{4}{2} && \text{Divide both sides by 2.} \\ x &= 2 && \text{Simplify both sides.} \end{aligned}$$

Readers should check this solution in the original equation.

You Try It!

Solve for x:

$$\frac{x}{2} + 6 = 4$$

EXAMPLE 7. Solve for x: $\dfrac{x}{4} - 3 = -7$.

Solution. On the left, order of operations demands that we first divide x by 4, then subtract 3. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will (1) add 3 to both sides of the equation, then (2) multiply both sides of the resulting equation by 4.

$$\begin{aligned} \frac{x}{4} - 3 &= -7 && \text{Original equation.} \\ \frac{x}{4} - 3 + 3 &= -7 + 3 && \text{Add 3 to both sides.} \\ \frac{x}{4} &= -4 && \text{Simplify both sides.} \\ 4\left(\frac{x}{4}\right) &= 4(-4) && \text{Multiply both sides by 4.} \\ x &= -16 && \text{Simplify both sides.} \end{aligned}$$

Check. Substitute -16 for x in the original equation.

$$\begin{aligned} \frac{x}{4} - 3 &= -7 && \text{Original equation.} \\ \frac{-16}{4} - 3 &= -7 && \text{Substitute } -16 \text{ for } x. \\ -4 - 3 &= -7 && \text{Divide first: } -16/4 = -4. \\ -7 &= -7 && \text{Subtract: } -4 - 3 = -7. \end{aligned}$$

Because the last line of the check is a true statement, -16 is a solution of the original equation.

Answer: $x = -4$

You Try It!

Solve for r:

$$0 = 9 + 3r$$

EXAMPLE 8. Solve for t: $0 = 8 - 2t$.

Solution. On the right, order of operations demands that we first multiply t by -2, then add 8. To solve this equation for t, we must "undo" each of these operations in inverse order. Thus, we will (1) subtract 8 from both sides of the equation, then (2) divide both sides of the resulting equation by -2.

$0 = 8 - 2t$	Original equation.
$0 - 8 = 8 - 2t - 8$	Subtract 8 from both sides.
$-8 = -2t$	Simplify both sides.
$\dfrac{-8}{-2} = \dfrac{-2t}{-2}$	Divide both sides by -2.
$4 = t$	Simplify both sides.

Check. Substitute 4 for t in the original equation.

$0 = 8 - 2t$	Original equation.
$0 = 8 - 2(4)$	Substitute 4 for t.
$0 = 8 - 8$	Multiply first: $2(4) = 8$.
$0 = 0$	Subtract: $8 - 8 = 0$.

Because the last line in the check is a true statement, 4 is a solution of the original equation.

Answer: $r = -3$

You Try It!

Solve for q:

$$\frac{q}{-2} - 9 = -8 + 3$$

EXAMPLE 9. Solve for p: $-12 + 3 = -8 + 4 + \dfrac{p}{-3}$.

Solution. Always simplify when possible.

$-12 + 3 = -8 + 4 + \dfrac{p}{-3}$	Original equation.
$-9 = -4 + \dfrac{p}{-3}$	Simplify both sides.

On the right, order of operations demands that we first divide p by -3, then add -4. To solve this equation for p, we must "undo" each of these operations in inverse order. Thus, we will (1) add a positive 4 to both sides of the equation, then (2) multiply both sides of the resulting equation by -3.

$$\begin{aligned}
-9 + 4 &= -4 + \frac{p}{-3} + 4 && \text{Add 4 to both sides.} \\
-5 &= \frac{p}{-3} && \text{Simplify both sides.} \\
-3(-5) &= -3\left(\frac{p}{-3}\right) && \text{Multiply both sides by } -3. \\
15 &= p && \text{Simplify both sides.}
\end{aligned}$$

Check. Substitute 15 for p in the original equation.

$$\begin{aligned}
-12 + 3 &= -8 + 4 + \frac{p}{-3} && \text{Original equation.} \\
-12 + 3 &= -8 + 4 + \frac{15}{-3} && \text{Substitute 15 for } p. \\
-9 &= -8 + 4 + (-5) && \text{On the left, add: } -12 + 3 = -9. \text{ On the right, divide: } 15/(-3) = -5. \\
-9 &= -4 + (-5) && \text{On the right, add: } -8 + 4 = -4. \\
-9 &= -9 && \text{On the right, add: } -4 + (-5) = -9.
\end{aligned}$$

Because the last line in the check is a true statement, 15 is a solution of the original equation.

Answer: $q = -8$

Applications

Let's look at some applications of equations involving integers. First, we remind readers that a solution of a word problem must incorporate each of the following steps.

Requirements for Word Problem Solutions.

1. Set up a Variable Dictionary. You must let your readers know what each variable in your problem represents. This can be accomplished in a number of ways:

- Statements such as "Let P represent the perimeter of the rectangle."
- Labeling unknown values with variables in a table.
- Labeling unknown quantities in a sketch or diagram.

2. **Set up an Equation.** Every solution to a word problem must include a carefully crafted equation that accurately describes the constraints in the problem statement.

3. **Solve the Equation.** You must always solve the equation set up in the previous step.

4. **Answer the Question.** This step is easily overlooked. For example, the problem might ask for Jane's age, but your equation's solution gives the age of Jane's sister Liz. Make sure you answer the original question asked in the problem. Your solution should be written in a sentence with appropriate units.

5. **Look Back.** It is important to note that this step does not imply that you should simply check your solution in your equation. After all, it's possible that your equation incorrectly models the problem's situation, so you could have a valid solution to an incorrect equation. The important question is: "Does your answer make sense based on the words in the original problem statement."

You Try It!

After withdrawing \$125 from his account, Allen finds that he is overdrawn by \$15. What was his account balance before his withdrawal?

EXAMPLE 10. A student's bank account is overdrawn. After making a deposit of \$120, he finds that his account is still overdrawn by an amount of \$75. What was his balance before he made his deposit?

Solution. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, the unknown is the original balance in the student's account. Let B represent this original balance.

2. *Set up an Equation.* A positive integer represents a healthy balance, while a negative number represents an account that is overdrawn. After the student's deposit, the account is still overdrawn by \$75. We will say that this balance is $-\$75$. Thus,

Original Balance	plus	Student Deposit	equals	Current Balance
B	$+$	\$120	$=$	$-\$75$

3. *Solve the Equation.* To "undo" the addition, subtract 120 from both sides of the equation.

$$\begin{aligned} B + 120 &= -75 && \text{Original equation.} \\ B + 120 - 120 &= -75 - 120 && \text{Subtract 120 from both sides.} \\ B &= -195 && \text{Simplify both sides.} \end{aligned}$$

4. *Answer the Question.* The original balance was overdrawn to the tune of \$195.

5. *Look Back.* If the original balance was overdrawn by \$195, then we let −\$195 represent this balance. The student makes a deposit of \$120. Add this to the original balance to get −\$195 + \$120 = −\$75, the correct current balance.

Answer: \$110

□

You Try It!

EXAMPLE 11. Three more than twice a certain number is −11. Find the unknown number.

Five less than twice a certain number is −7. Find the unknown number.

Solution. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* "Three more than twice a certain number" becomes:

Three	more than	Twice a Certain Number	is	−11
3	$+$	$2x$	$=$	-11

3. *Solve the Equation.* On the left, order of operations requires that we first multiply x by 2, then add 3. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will (1) subtract 3 from both sides of the equation, then (2) divide both sides of the resulting equation by 2.

$$\begin{aligned} 3 + 2x &= -11 && \text{Original equation.} \\ 3 + 2x - 3 &= -11 - 3 && \text{Subtract 3 from both sides.} \\ 2x &= -14 && \text{Simplify both sides.} \\ \frac{2x}{2} &= \frac{-14}{2} && \text{Divide both sides by 2.} \\ x &= -7 && \text{Simplify both sides.} \end{aligned}$$

4. *Answer the Question.* The unknown number is -7.

5. *Look Back.* Does the answer satisfy the problem constraints? Three more than twice -7 is three more than -14, or -11. So the solution is correct.

Answer: -1

❧ ❧ ❧ Exercises ☙ ☙ ☙

1. Is -11 a solution of $2x+3=-19$?

2. Is -8 a solution of $2x+7=-9$?

3. Is 6 a solution of $3x+1=19$?

4. Is -6 a solution of $2x+7=-5$?

5. Is 12 a solution of $4x+5=-8$?

6. Is -8 a solution of $-3x+8=18$?

7. Is 15 a solution of $2x+6=-9$?

8. Is 3 a solution of $-4x+1=-20$?

9. Is -15 a solution of $-3x+6=-17$?

10. Is -18 a solution of $-3x+9=-9$?

11. Is -6 a solution of $-2x+3=15$?

12. Is 7 a solution of $-3x+5=-16$?

In Exercises 13-28, solve the given equation for x.

13. $x-13=11$

14. $x-6=12$

15. $x-3=6$

16. $x-3=-19$

17. $x+10=17$

18. $x+3=9$

19. $x-6=1$

20. $x-10=12$

21. $x-15=-12$

22. $x-2=13$

23. $x+11=-19$

24. $x+3=17$

25. $x+2=1$

26. $x+2=-20$

27. $x+5=-5$

28. $x+14=-15$

In Exercises 29-44, solve the given equation for x.

29. $-x=-20$

30. $5x=-35$

31. $\dfrac{x}{-7}=10$

32. $\dfrac{x}{-6}=-20$

33. $\dfrac{x}{-10}=12$

34. $\dfrac{x}{2}=11$

35. $\dfrac{x}{9}=-16$

36. $\dfrac{x}{-3}=-7$

37. $-10x=20$

38. $-17x=-85$

39. $14x=84$

40. $-10x=-40$

41. $-2x=28$

42. $-14x=42$

43. $\dfrac{x}{-10}=15$

44. $\dfrac{x}{-8}=-1$

In Exercises 45-68, solve the given equation for x.

45. $-4x - 4 = 16$

46. $-6x - 14 = 4$

47. $4x - 4 = 76$

48. $-5x - 15 = 45$

49. $5x - 14 = -79$

50. $15x - 2 = 43$

51. $-10x - 16 = 24$

52. $2x - 7 = -11$

53. $9x + 5 = -85$

54. $8x + 8 = -16$

55. $7x + 15 = -55$

56. $2x + 2 = -38$

57. $-x + 8 = 13$

58. $-5x + 20 = -50$

59. $12x - 15 = -3$

60. $-19x - 17 = -36$

61. $4x - 12 = -56$

62. $7x - 16 = 40$

63. $19x + 18 = 113$

64. $-6x + 20 = -64$

65. $-14x + 12 = -2$

66. $-9x + 5 = 104$

67. $14x + 16 = 44$

68. $-14x + 10 = -60$

---

69. Two less than eight times an unknown number is -74. Find the unknown number.

70. Six less than three times an unknown number is 21. Find the unknown number.

71. Eight more than two times an unknown number is 0. Find the unknown number.

72. Five more than eight times an unknown number is -35. Find the unknown number.

73. The number -6 is 2 more than an unknown number. Find the unknown number.

74. The number -4 is 7 more than an unknown number. Find the unknown number.

75. Three more than eight times an unknown number is -29. Find the unknown number.

76. Four more than nine times an unknown number is 85. Find the unknown number.

77. Alan's scores on his first three exams are 79, 61, and 54. What must Alan score on his next exam to average 71 for all four exams?

78. Benny's scores on his first three exams are 54, 68, and 54. What must Benny score on his next exam to average 61 for all four exams?

79. The quotient of two integers is 5. One of the integers is -2. Find the other integer.

80. The quotient of two integers is 3. One of the integers is -7. Find the other integer.

81. The quotient of two integers is 9. One of the integers is -8. Find the other integer.

82. The quotient of two integers is 9. One of the integers is -2. Find the other integer.

83. The number -5 is 8 more than an unknown number. Find the unknown number.

84. The number -6 is 8 more than an unknown number. Find the unknown number.

85. A student's bank account is overdrawn. After making a deposit of \$260, he finds that his account is still overdrawn by an amount of \$70. What was his balance before he made his deposit?

86. A student's bank account is overdrawn. After making a deposit of \$300, he finds that his account is still overdrawn by an amount of \$70. What was his balance before he made his deposit?

87. A student's bank account is overdrawn. After making a deposit of \$360, he finds that his account is still overdrawn by an amount of \$90. What was his balance before he made his deposit?

88. A student's bank account is overdrawn. After making a deposit of \$260, he finds that his account is still overdrawn by an amount of \$50. What was his balance before he made his deposit?

89. The number -10 is -5 times larger than an unknown number. Find the unknown number.

90. The number -3 is -3 times larger than an unknown number. Find the unknown number.

91. The number -15 is -5 times larger than an unknown number. Find the unknown number.

92. The number -16 is 4 times larger than an unknown number. Find the unknown number.

93. Two less than nine times an unknown number is 7. Find the unknown number.

94. Four less than two times an unknown number is 8. Find the unknown number.

95. Mark's scores on his first three exams are 79, 84, and 71. What must Mark score on his next exam to average 74 for all four exams?

96. Alan's scores on his first three exams are 85, 90, and 61. What must Alan score on his next exam to average 77 for all four exams?

❧ ❧ ❧ Answers ☙ ☙ ☙

1. Yes

3. Yes

5. No

7. No

9. No

11. Yes

13. 24

15. 9

17. 7

19. 7

21. 3

23. -30

25. -1

27. -10

29. 20

31. -70

33. -120

35. -144

37. -2

39. 6

41. -14

43. -150

45. -5

47. 20

49. -13

51. -4

53. -10

55. -10

57. -5

59. 1

61. -11

63. 5

65. 1

67. 2

69. -9

71. -4

73. -8

75. -4

77. 90

79. -10

81. -72

83. -13

85. $-\$330$

87. $-\$450$

89. 2

91. 3

93. 1

95. 62

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