Prealgebra Textbook

Second Edition

Chapter 2 Odd Solutions

Department of Mathematics
College of the Redwoods

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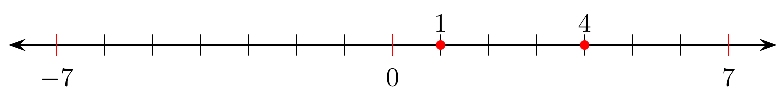
Contents

Chapter 2

The Integers

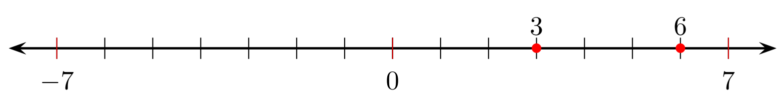
2.1 Introduction to the Integers

1. Locate the number 4 on the number line, then move three units to the left to locate the second number.



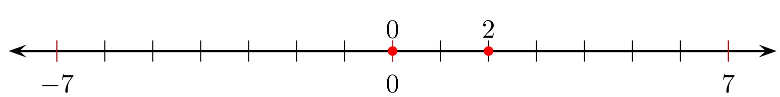
Hence, the number 1 lies three units to the left of the number 4.

3. Locate the number 6 on the number line, then move three units to the left to locate the second number.



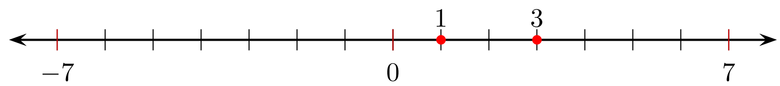
Hence, the number 3 lies three units to the left of the number 6.

5. Locate the number 0 on the number line, then move two units to the right to locate the second number.



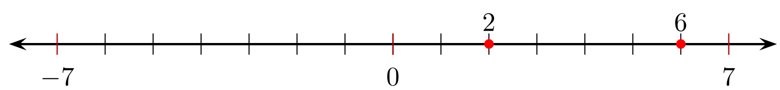
Hence, the number 2 lies two units to the right of the number 0.

7. Locate the number 1 on the number line, then move two units to the right to locate the second number.



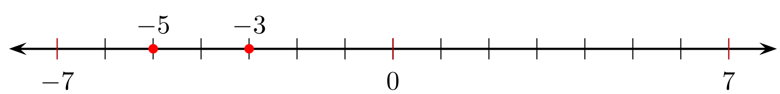
Hence, the number 3 lies two units to the right of the number 1.

9. Locate the number 6 on the number line, then move four units to the left to locate the second number.



Hence, the number 2 lies four units to the left of the number 6.

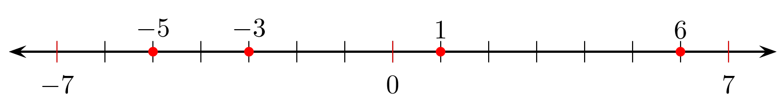
11. Locate the number -5 on the number line, then move two units to the right to locate the second number.



Hence, the number -3 lies two units to the right of the number -5.

13.

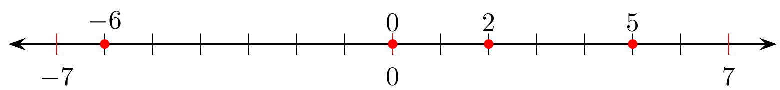
i) Arrange the integers 6, 1, -3, and -5 on a number line.



ii) The order on the number line, left to right, is also from smallest to largest. Therefore, in order, smallest to largest: -5, -3, 1, 6

15.

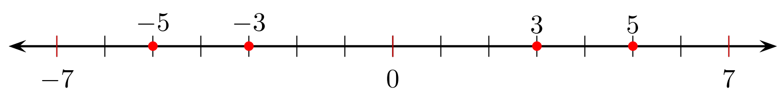
i) Arrange the integers 5, −6, 0, and 2 on a number line.



ii) The order on the number line, left to right, is also from smallest to largest. Therefore, in order, smallest to largest: −6, 0, 2, 5

17.

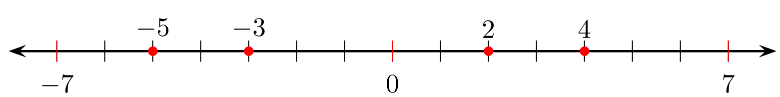
i) Arrange the integers −3, −5, 3, and 5 on a number line.



ii) The order on the number line, left to right, is also from smallest to largest. Therefore, in order, smallest to largest: −5, −3, 3, 5

19.

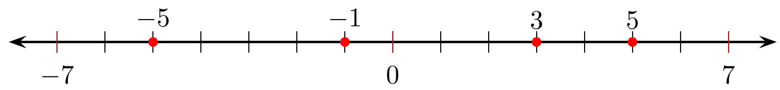
i) Arrange the integers −5, 4, 2, and −3 on a number line.



ii) The order on the number line, left to right, is also from smallest to largest. Therefore, in order, smallest to largest: −5, −3, 2, 4

21.

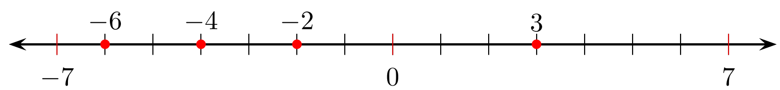
i) Arrange the integers 3, 5, -5, and -1 on a number line.



ii) The order on the number line, left to right, is also from smallest to largest. Therefore, in order, smallest to largest: -5, -1, 3, 5

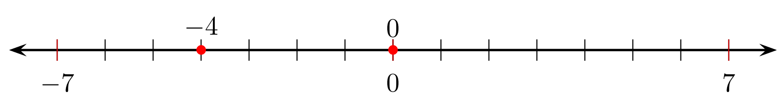
23.

i) Arrange the integers -2, -4, 3, and -6 on a number line.



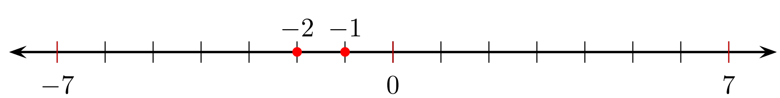
ii) The order on the number line, left to right, is also from smallest to largest. Therefore, in order, smallest to largest: -6, -4, -2, 3

25. If you locate the numbers -4 and 0 on a number line, you can see that -4 lies to the left of 0.



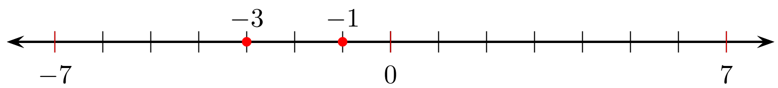
Hence, $-4 < 0$.

27. If you locate the numbers -2 and -1 on a number line, you can see that -2 lies to the left of -1.



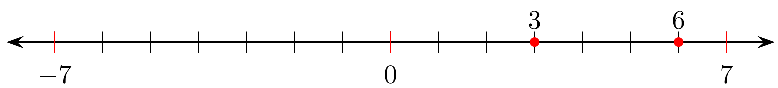
Hence, $-2 < -1$.

29. If you locate the numbers -3 and -1 on a number line, you can see that -3 lies to the left of -1.



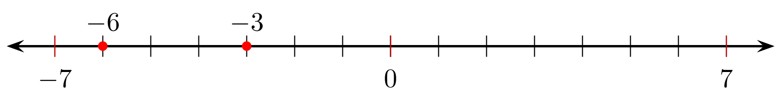
Hence, $-3 < -1$.

31. If you locate the numbers 3 and 6 on a number line, you can see that 3 lies to the left of 6.



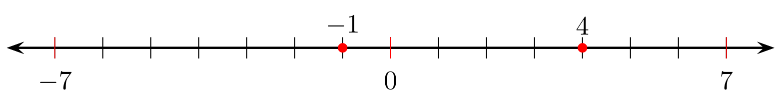
Hence, $3 < 6$.

33. If you locate the numbers -3 and -6 on a number line, you can see that -3 lies to the right of -6.



Hence, $-3 > -6$.

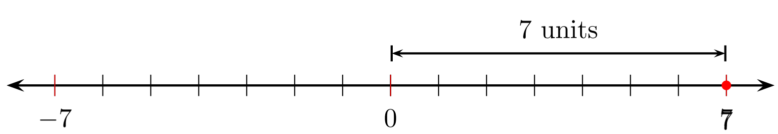
35. If you locate the numbers -1 and 4 on a number line, you can see that -1 lies to the left of 4.



Hence, $-1 < 4$.

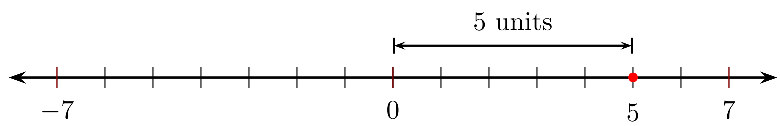
37. The opposite of -4 is 4. That is, $-(-4) = 4$.

39. The number 7 is 7 units from the origin.



Hence, $|7| = 7$.

41. The number 5 is 5 units from the origin.



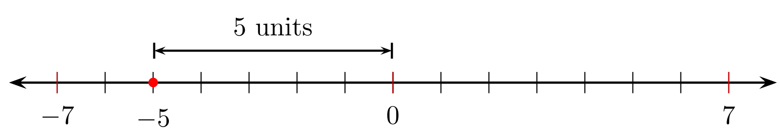
Hence, $|5| = 5$.

43. You must first take the absolute value of -11, which is 11. Then you must take the opposite of this result. That is,

$$\begin{aligned} -|-11| &= -(11) && \text{First: } |-11| = 11. \\ &= -11 && \text{Second: The opposite of 11 is } -11. \end{aligned}$$

Hence, $|-11| = -11$.

45. The number -5 is 5 units from the origin.



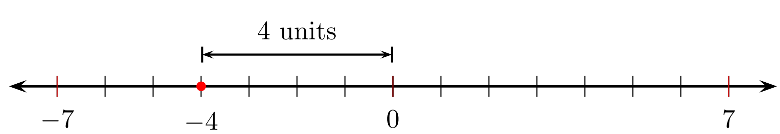
Hence, $|-5| = 5$.

47. You must first take the absolute value of -20, which is 20. Then you must take the opposite of this result. That is,

$$\begin{aligned} -|-20| &= -(20) && \text{First: } |-20| = 20. \\ &= -20 && \text{Second: The opposite of 20 is } -20. \end{aligned}$$

Hence, $|-20| = -20$.

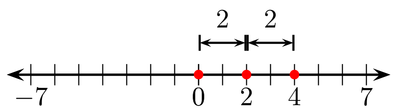
49. The number -4 is 4 units from the origin.



Hence, $|-4| = 4$.

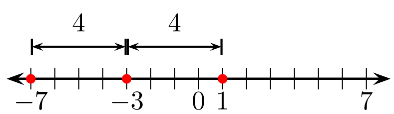
51. The opposite of -2 is 2. That is, $-(-2) = 2$.

53. Plot the integer 2 on the number line, then move 2 units to the left and 2 units to the right to find the required integers.



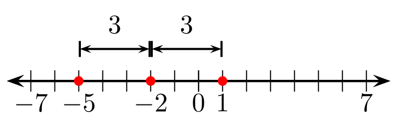
Thus, 0 and 4 are 2 units away from 2.

55. Plot the integer -3 on the number line, then move 4 units to the left and 4 units to the right to find the required integers.



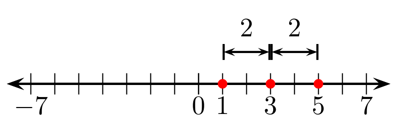
Thus, -7 and 1 are 4 units away from -3.

57. Plot the integer -2 on the number line, then move 3 units to the left and 3 units to the right to find the required integers.



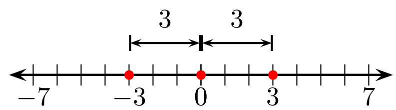
Thus, -5 and 1 are 3 units away from -2.

59. Plot the integer 3 on the number line, then move 2 units to the left and 2 units to the right to find the required integers.



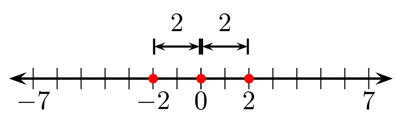
Thus, 1 and 5 are 2 units away from 3.

61. Plot the integer 0 on the number line, then move 3 units to the left and 3 units to the right to find the required integers.



Thus, −3 and 3 are 3 units away from 0.

63. Plot the integer 0 on the number line, then move 2 units to the left and 2 units to the right to find the required integers.



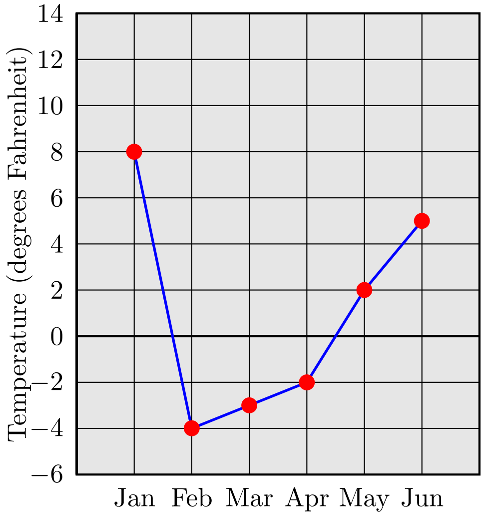
Thus, −2 and 2 are 2 units away from 0.

65. Set a vertical number line with 0 representing sea level, positive numbers representing heights above sea lever, and negative number representing depths below sea level. Thus, 2,350 feet above sea level can be represented by the positive integer 2,350.

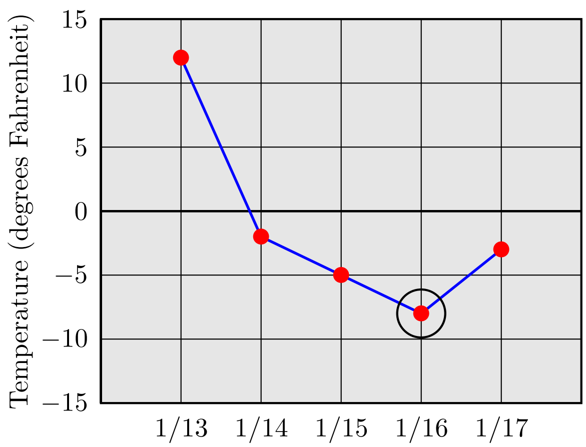
67. Read the heights of the bars to create the following table of profit and loss.

Month	Profit/Loss
Jan	8
Feb	−4
Mar	−3
Apr	−2
May	2
Jun	5

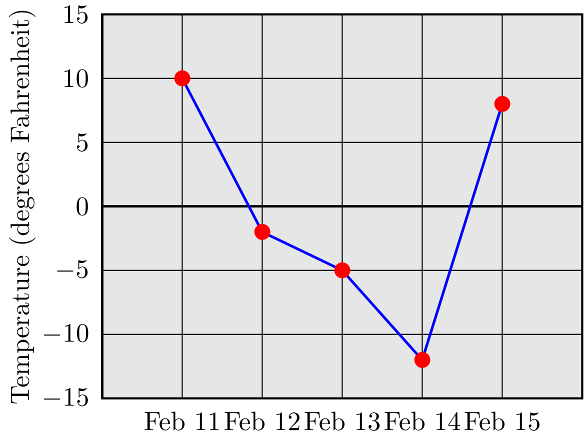
Use the data in the table to create the following line graph.



69. We've circled the low temperature in the plot. Note that the low temperature is approximately $-8°$ Fahrenheit and it occurred on January 16.



71.



2.2 Adding Integers

1. The vector starts at -4 and finishes at 0. The vector points to the right and has length 4. Hence, this vector represents the integer 4.

3. The vector starts at 0 and finishes at 6. The vector points to the right and has length 6. Hence, this vector represents the integer 6.

5. The vector starts at 1 and finishes at -4. The vector points to the left and has length 5. Hence, this vector represents the integer -5.

7. The vector starts at 6 and finishes at 0. The vector points to the left and has length 6. Hence, this vector represents the integer -6.

9. The vector starts at -4 and finishes at 6. The vector points to the right and has length 10. Hence, this vector represents the integer 10.

11. The vector starts at 2 and finishes at -5. The vector points to the left and has length 7. Hence, this vector represents the integer -7.

13. To add a negative and a positive integer, subtract the smaller magnitude from the larger magnitude ($15 - 1 = 14$), then prefix the sign of the integer with the larger magnitude. Thus,

$$-15 + 1 = -14.$$

15. To add a positive and a negative integer, subtract the smaller magnitude from the larger magnitude ($18 - 10 = 8$), then prefix the sign of the integer with the larger magnitude. Thus,

$$18 + (-10) = 8.$$

17. To add two negative integers, (1) add their magnitudes ($10 + 12 = 22$), and (2) prefix their common negative sign. Thus,

$$-10 + (-12) = -22$$

19. To add two positive integers, (1) add their magnitudes ($5 + 10 = 15$), and (2) prefix their common negative sign. Thus,

$$5 + 10 = 15$$

21. To add two positive integers, (1) add their magnitudes ($2 + 5 = 7$), and (2) prefix their common negative sign. Thus,

$$2 + 5 = 7$$

23. To add a positive and a negative integer, subtract the smaller magnitude from the larger magnitude ($19 - 15 = 4$), then prefix the sign of the integer with the larger magnitude. Thus,

$$19 + (-15) = 4.$$

25. To add two negative integers, (1) add their magnitudes ($2 + 7 = 9$), and (2) prefix their common negative sign. Thus,

$$-2 + (-7) = -9$$

27. To add a negative and a positive integer, subtract the smaller magnitude from the larger magnitude ($16 - 6 = 10$), then prefix the sign of the integer with the larger magnitude. Thus,

$$-6 + 16 = 10.$$

29. To add two negative integers, (1) add their magnitudes ($11 + 6 = 17$), and (2) prefix their common negative sign. Thus,

$$-11 + (-6) = -17$$

31. To add a positive and a negative integer, subtract the smaller magnitude from the larger magnitude ($14-9=5$), then prefix the sign of the integer with the larger magnitude. Thus,

$$14+(-9)=5.$$

33. To add two positive integers, (1) add their magnitudes ($10+11=21$), and (2) prefix their common negative sign. Thus,

$$10+11=21$$

35. To add a negative and a positive integer, subtract the smaller magnitude from the larger magnitude ($13-1=12$), then prefix the sign of the integer with the larger magnitude. Thus,

$$-13+1=-12.$$

37. The identity

$$-1+(3+(-8))=(-1+3)+(-8)$$

is an example of the associative property of addition.

39. The identity

$$7+(-7)=0$$

is an example of the additive inverse property.

41. The identity

$$15+(-18)=-18+15$$

is an example of the commutative property of addition.

43. The identity

$$-15+0=-15$$

is an example of the additive identity property.

45. The identity

$$-7+(1+(-6))=(-7+1)+(-6)$$

is an example of the associative property of addition.

47. The identity

$$17 + (-2) = -2 + 17$$

is an example of the commutative property of addition.

49. The identity

$$-4 + 0 = -4$$

is an example of the additive identity property.

51. The identity

$$19 + (-19) = 0$$

is an example of the additive inverse property.

53. Because

$$18 + (-18) = 0,$$

the additive inverse of 18 is -18.

55. Because

$$12 + (-12) = 0,$$

the additive inverse of 12 is -12.

57. Because

$$-16 + 16 = 0,$$

the additive inverse of -16 is 16. Alternatively, the additive inverse of -16 is $-(-16)$, which equals 16.

59. Because

$$11 + (-11) = 0,$$

the additive inverse of 11 is -11.

61. Because

$$-15 + 15 = 0,$$

the additive inverse of -15 is 15. Alternatively, the additive inverse of -15 is $-(-15)$, which equals 15.

63. Because

$$-18 + 18 = 0,$$

the additive inverse of -18 is 18. Alternatively, the additive inverse of -18 is $-(-18)$, which equals 18.

65. Because the commutative and associative properties allow us to add in any order, we add the positive integers first, then the negatives. Then we take a final sum.

$$\begin{aligned} 6 + (-1) + 3 + (-4) &= 9 + (-5) && \text{Add positives: } 6 + 3 = 9; \\ & && \text{Add negatives: } -1 + (-4) = -5. \\ &= 4 && \text{Add: } 9 + (-5) = 4. \end{aligned}$$

67. Perform the additions in the order that they occur, moving from left to right.

$$\begin{aligned} 15 + (-1) + 2 &= 14 + 2 && \text{Add first two terms: } 15 + (-1) = 14. \\ &= 16 && \text{Add: } 14 + 2 = 16. \end{aligned}$$

69. Perform the additions in the order that they occur, moving from left to right.

$$\begin{aligned} -17 + 12 + 3 &= -5 + 3 && \text{Add first two terms: } -17 + 12 = -5. \\ &= -2 && \text{Add: } -5 + 3 = -2. \end{aligned}$$

71. Perform the additions in the order that they occur, moving from left to right.

$$\begin{aligned} 7 + 20 + 19 &= 27 + 19 && \text{Add first two terms: } 7 + 20 = 27. \\ &= 46 && \text{Add: } 27 + 19 = 46. \end{aligned}$$

73. Because the commutative and associative properties allow us to add in any order, we add the positive integers first, then the negatives. Then we take a final sum.

$$\begin{aligned} 4 + (-8) + 2 + (-5) &= 6 + (-13) && \text{Add positives: } 4 + 2 = 6; \\ & && \text{Add negatives: } -8 + (-5) = -13. \\ &= -7 && \text{Add: } 6 + (-13) = -7. \end{aligned}$$

75. Because the commutative and associative properties allow us to add in any order, we add the positive integers first, then the negatives. Then we take a final sum.

$$\begin{aligned} 7 + (-8) + 2 + (-1) &= 9 + (-9) && \text{Add positives: } 7 + 2 = 9; \\ & && \text{Add negatives: } -8 + (-1) = -9. \\ &= 0 && \text{Add: } 9 + (-9) = 0. \end{aligned}$$

77. Because the commutative and associative properties allow us to add in any order, we add the positive integers first, then the negatives. Then we take a final sum.

$$\begin{aligned} 9 + (-3) + 4 + (-1) &= 13 + (-4) && \text{Add positives: } 9 + 4 = 13; \\ & && \text{Add negatives: } -3 + (-1) = -4. \\ &= 9 && \text{Add: } 13 + (-4) = 9. \end{aligned}$$

79. Perform the additions in the order that they occur, moving from left to right.

$$\begin{aligned} 9 + 10 + 2 &= 19 + 2 && \text{Add first two terms: } 9 + 10 = 19. \\ &= 21 && \text{Add: } 19 + 2 = 21. \end{aligned}$$

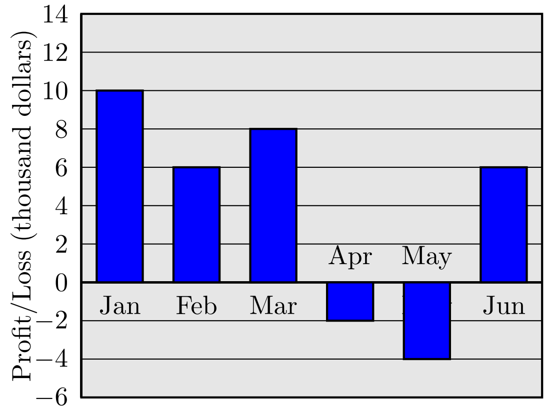
81. To find how much is in Gerry's account now, begin with the original deposit. Withdraws are represented by negative integers and deposits are represented by positive integers. Add together all the amounts withrawn and deposited.

$$\begin{aligned} &215 + (-40) + (-75) + (-20) + 185 && \text{Represent the deposits and withdraws as integers.} \\ &= \big[215 + 185\big] + \big[(-40) + (-75) + (-20)\big] && \text{Use commutative and associative properties} \\ & && \text{to rearrange integers.} \\ &= 400 + (-135) && \text{Add positives: } 215 + 185 = 400; \\ & && \text{Add negatives: } -40 + (-75) + (-20) = -135. \\ &= 265 && \text{Add integers with different signs.} \end{aligned}$$

Therefore, Gerry has \$265 in his account.

83. We read the following values from the bar chart.

Month	Jan	Feb	Mar	Apr	May	Jun
Profit/Loss	10	6	8	−2	−4	6



Positive integers represent profits; negative integers represent losses. To find the net profit or loss, sum the integers in the table.

$$\begin{aligned} 10+6+8+(-2)+(-4)+6 &= (10+6+8+6)+(-2+(-4)) && \text{Reorder and regroup.} \\ &= 30+(-6) && \text{Sum positives and negatives.} \\ &= 24 && \text{Add.} \end{aligned}$$

Because the result is positive and the profit and loss data is scaled in thousands of dollars, there was a net profit of \$24,000.

2.3 Subtracting Integers

1. Subtraction means "add the opposite," so change the difference into a sum.

$$\begin{aligned} 16-20 &= 16+(-20) && \text{Add the opposite.} \\ &= -4 && \text{Add.} \end{aligned}$$

3. Subtraction means "add the opposite," so change the difference into a sum.

$$\begin{aligned} 10-12 &= 10+(-12) && \text{Add the opposite.} \\ &= -2 && \text{Add.} \end{aligned}$$

5. Subtraction means "add the opposite," so change the difference into a sum.

$$\begin{aligned} 14 - 11 &= 14 + (-11) && \text{Add the opposite.} \\ &= 3 && \text{Add.} \end{aligned}$$

7. Subtraction means "add the opposite," so change the difference into a sum.

$$\begin{aligned} 7 - (-16) &= 7 + 16 && \text{Add the opposite.} \\ &= 23 && \text{Add.} \end{aligned}$$

9. Subtraction means "add the opposite," so change the difference into a sum.

$$\begin{aligned} -4 - (-9) &= -4 + 9 && \text{Add the opposite.} \\ &= 5 && \text{Add.} \end{aligned}$$

11. Subtraction means "add the opposite," so change the difference into a sum.

$$\begin{aligned} 8 - (-3) &= 8 + 3 && \text{Add the opposite.} \\ &= 11 && \text{Add.} \end{aligned}$$

13. Subtraction means "add the opposite," so change the difference into a sum.

$$\begin{aligned} 2 - 11 &= 2 + (-11) && \text{Add the opposite.} \\ &= -9 && \text{Add.} \end{aligned}$$

15. Subtraction means "add the opposite," so change the difference into a sum.

$$\begin{aligned} -8 - (-10) &= -8 + 10 && \text{Add the opposite.} \\ &= 2 && \text{Add.} \end{aligned}$$

17. Subtraction means "add the opposite," so change the difference into a sum.

$$\begin{aligned} 13 - (-1) &= 13 + 1 && \text{Add the opposite.} \\ &= 14 && \text{Add.} \end{aligned}$$

19. Subtraction means "add the opposite," so change the difference into a sum.

$$\begin{aligned} -4-(-2) &= -4+2 && \text{Add the opposite.} \\ &= -2 && \text{Add.} \end{aligned}$$

21. Subtraction means "add the opposite," so change the difference into a sum.

$$\begin{aligned} 7-(-8) &= 7+8 && \text{Add the opposite.} \\ &= 15 && \text{Add.} \end{aligned}$$

23. Subtraction means "add the opposite," so change the difference into a sum.

$$\begin{aligned} -3-(-10) &= -3+10 && \text{Add the opposite.} \\ &= 7 && \text{Add.} \end{aligned}$$

25. First, change the subtraction signs to "adding the opposite," then add, moving left to right.

$$\begin{aligned} 14-12-2 &= 14+(-12)+(-2) && \text{Add the opposite of 12, which is } -12. \\ & && \text{Add the opposite of 2, which is } -2. \\ &= 2+(-2) && \text{Add: } 14+(-12)=2. \\ &= 0 && \text{Add: } 2+(-2)=0. \end{aligned}$$

27. First, change the subtraction signs to "adding the opposite," then add, moving left to right.

$$\begin{aligned} -20-11-18 &= -20+(-11)+(-18) && \text{Add the opposite of 11, which is } -11. \\ & && \text{Add the opposite of 18, which is } -18. \\ &= -31+(-18) && \text{Add: } -20+(-11)=-31. \\ &= -49 && \text{Add: } -31+(-18)=-49. \end{aligned}$$

29. First, change the subtraction signs to "adding the opposite," then add, moving left to right.

$$\begin{aligned} 5-(-10)-20 &= 5+10+(-20) && \text{Add the opposite of } -10\text{, which is 10.} \\ & && \text{Add the opposite of 20, which is } -20. \\ &= 15+(-20) && \text{Add: } 5+10=15. \\ &= -5 && \text{Add: } 15+(-20)=-5. \end{aligned}$$

31. First, change the subtraction signs to "adding the opposite," then add, moving left to right.

$$\begin{aligned} -14 - 12 - 19 &= -14 + (-12) + (-19) && \text{Add the opposite of 12, which is } -12. \\ & && \text{Add the opposite of 19, which is } -19. \\ &= -26 + (-19) && \text{Add: } -14 + (-12) = -26. \\ &= -45 && \text{Add: } -26 + (-19) = -45. \end{aligned}$$

33. First, change the subtraction signs to "adding the opposite," then add, moving left to right.

$$\begin{aligned} -11 - (-7) - (-6) &= -11 + 7 + 6 && \text{Add the opposite of } -7\text{, which is 7.} \\ & && \text{Add the opposite of } -6\text{, which is 6.} \\ &= -4 + 6 && \text{Add: } -11 + 7 = -4. \\ &= 2 && \text{Add: } -4 + 6 = 2. \end{aligned}$$

35. First, evaluate what's inside the parentheses. Change the subtraction to "adding the opposite," then add.

$$\begin{aligned} -2 - (-6 - (-5)) &= -2 - (-6 + 5) && \text{Add the opposite of } -5\text{, or 5.} \\ &= -2 - (-1) && \text{Add: } -6 + 5 = -1. \\ &= -2 + 1 && \text{Add the opposite of } -1\text{, or 1.} \\ &= -1 && \text{Add: } -2 + 1 = -1. \end{aligned}$$

37. First, evaluate what's inside the parentheses. Change the subtraction to "adding the opposite," then add.

$$\begin{aligned} (-5 - (-8)) - (-3 - (-2)) &= (-5 + 8) - (-3 + 2) && \text{Add the opposite of } -8\text{, or 8.} \\ & && \text{Add the opposite of } -2\text{, or 2.} \\ &= 3 - (-1) && \text{Add: } -5 + 8 = 3. \\ & && \text{Add: } -3 + 2 = -1. \\ &= 3 + 1 && \text{Add the opposite of } -1\text{, or 1.} \\ &= 4 && \text{Add: } 3 + 1 = 4. \end{aligned}$$

39. First, evaluate what's inside the parentheses. Change the subtraction to "adding the opposite," then add.

$$\begin{aligned}
(6-(-9))-(3-(-6)) &= (6+9)-(3+6) && \text{Add the opposite of } -9\text{, or } 9. \\
& && \text{Add the opposite of } -6\text{, or } 6. \\
&= 15-9 && \text{Add: } 6+9=15. \\
& && \text{Add: } 3+6=9. \\
&= 15+(-9) && \text{Add the opposite of } 9\text{, or } -9. \\
&= 6 && \text{Add: } 15+(-9)=6.
\end{aligned}$$

41. First, evaluate what's inside the parentheses. Change the subtraction to "adding the opposite," then add.

$$\begin{aligned}
-1-(10-(-9)) &= -1-(10+9) && \text{Add the opposite of } -9\text{, or } 9. \\
&= -1-19 && \text{Add: } 10+9=19. \\
&= -1+(-19) && \text{Add the opposite of } 19\text{, or } -19. \\
&= -20 && \text{Add: } -1+(-19)=-20.
\end{aligned}$$

43. First, evaluate what's inside the parentheses. Change the subtraction to "adding the opposite," then add.

$$\begin{aligned}
3-(-8-17) &= 3-(-8+(-17)) && \text{Add the opposite of } 17\text{, or } -17. \\
&= 3-(-25) && \text{Add: } -8+(-17)=-25. \\
&= 3+25 && \text{Add the opposite of } -25\text{, or } 25. \\
&= 28 && \text{Add: } 3+25=28.
\end{aligned}$$

45. First, evaluate what's inside the parentheses. Change the subtraction to "adding the opposite," then add.

$$\begin{aligned}
13-(16-(-1)) &= 13-(16+1) && \text{Add the opposite of } -1\text{, or } 1. \\
&= 13-17 && \text{Add: } 16+1=17. \\
&= 13+(-17) && \text{Add the opposite of } 17\text{, or } -17. \\
&= -4 && \text{Add: } 13+(-17)=-4.
\end{aligned}$$

47. First, evaluate what's inside the parentheses. Change the subtraction to "adding the opposite," then add.

$$\begin{aligned}(7-(-8))-(5-(-2)) &= (7+8)-(5+2) && \text{Add the opposite of } -8\text{, or } 8.\\ & && \text{Add the opposite of } -2\text{, or } 2.\\ &= 15-7 && \text{Add: } 7+8=15.\\ & && \text{Add: } 5+2=7.\\ &= 15+(-7) && \text{Add the opposite of } 7\text{, or } -7.\\ &= 8 && \text{Add: } 15+(-7)=8.\end{aligned}$$

49. First, evaluate what's inside the parentheses. Change the subtraction to "adding the opposite," then add.

$$\begin{aligned}(6-4)-(-8-2) &= (6+(-4))-(-8+(-2)) && \text{Add the opposite of } 4\text{, or } -4.\\ & && \text{Add the opposite of } 2\text{, or } -2.\\ &= 2-(-10) && \text{Add: } 6+(-4)=2.\\ & && \text{Add: } -8+(-2)=-10.\\ &= 2+10 && \text{Add the opposite of } -10\text{, or } 10.\\ &= 12 && \text{Add: } 2+10=12.\end{aligned}$$

51. The temperature change is found by subtracting the first temperature from the second temperature.

Change in Temperature	=	Latter Measurement	−	Former Measurement
	=	65° F	−	42° F
	=	23° F		

Hence, the change in temperature is 23° F.

53. The temperature change is found by subtracting the first temperature from the second temperature.

Change in Temperature	=	Latter Measurement	−	Former Measurement
	=	51° F	−	30° F
	=	21° F		

Hence, the change in temperature is 21° F.

55. To find the change in temperature, subtract the earlier temperature from the later temperature.

Change in Temperature		Latter Temperature		Former Temperature
	$=$	$-19°$ F	$-$	$(-2)°$ F
	$=$	$-19°$ F	$+$	$2°$ F
	$=$	$-17°$ F		

Hence, the change in temperature is $-17°$ F.

57. To find the distance the message must travel, subtract the lower elevation from the higher elevation.

$$\begin{aligned} 22,500 - (-1,600) &= 22,500 + 1,600 && \text{To subtract, add the opposite.} \\ &= 24,100 && \text{Add the integers.} \end{aligned}$$

The distance the message must travel is 24,100 feet.

59. To find how much lower the shore of the Dead Sea is from Death Valley, subtract the Dead Sea's lower elevation from Death Valley's higher elevation.

$$\begin{aligned} -282 - (-1,371) &= -282 + 1,371 && \text{To subtract, add the opposite.} \\ &= 1,089 && \text{Add the integers.} \end{aligned}$$

The Dead Sea at the Israel-Jordan border is -1,089 feet lower than Death Valley.

2.4 Subtracting Integers

1. The identity

$$(-2)\big[(-16)(13)\big] = \big[(-2)(-16)\big](13)$$

has the form

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c.$$

Hence, this is an example of the associative property of multiplication.

3. The identity

$$(-17)(-10) = (-10)(-17)$$

has the form

$$a \cdot b = b \cdot a.$$

Hence, this is an example of the commutative property of multiplication.

5. The identity
$$(4)(11) = (11)(4)$$
has the form
$$a \cdot b = b \cdot a.$$
Hence, this is an example of the commutative property of multiplication.

7. The identity
$$16 \cdot \bigl(8 + (-15)\bigr) = 16 \cdot 8 + 16 \cdot (-15)$$
has the form
$$a \cdot (b + c) = a \cdot b + a \cdot c.$$
Hence, this is an example of the distributive property.

9. The identity
$$(17)\bigl[(20)(11)\bigr] = \bigl[(17)(20)\bigr](11)$$
has the form
$$a \cdot (b \cdot c) = (a \cdot b) \cdot c.$$
Hence, this is an example of the associative property of multiplication.

11. The identity
$$-19 \cdot 1 = -19$$
has the form
$$a \cdot 1 = a.$$
Hence, this is an example of the multiplicative identity property of multiplication.

13. The identity
$$8 \cdot 1 = 8$$
has the form
$$a \cdot 1 = a.$$
Hence, this is an example of the multiplicative identity property of multiplication.

15. The identity
$$14 \cdot \bigl(-12 + 7\bigr) = 14 \cdot (-12) + 14 \cdot 7$$
has the form
$$a \cdot (b + c) = a \cdot b + a \cdot c.$$
Hence, this is an example of the distributive property.

17. When multiplying, like signs give a positive answer. Therefore,

$$4 \cdot 7 = 28.$$

19. When multiplying, unlike signs give a negative answer. Therefore,

$$3 \cdot (-3) = -9.$$

21. The property "multiplying by minus one" says that $(-1)a = -a$. Hence,

$$-1 \cdot 10 = -10.$$

23. The multiplicative property of zero says that $a \cdot 0 = 0$. Hence,

$$-1 \cdot 0 = 0.$$

25. The property "multiplying by minus one" says that $(-1)a = -a$. Hence,

$$\begin{aligned} -1 \cdot (-14) &= -(-14) && \text{Multiplication property of } -1\text{: } (-1)a = -a. \\ &= 14 && \text{The opposite of } -14 \text{ is } 14. \end{aligned}$$

27. The property "multiplying by minus one" says that $(-1)a = -a$. Hence,

$$\begin{aligned} -1 \cdot (-19) &= -(-19) && \text{Multiplication property of } -1\text{: } (-1)a = -a. \\ &= 19 && \text{The opposite of } -19 \text{ is } 19. \end{aligned}$$

29. The multiplicative property of zero says that $a \cdot 0 = 0$. Hence,

$$2 \cdot 0 = 0.$$

31. When multiplying, unlike signs give a negative answer. Therefore,

$$-3 \cdot 8 = -24.$$

33. When multiplying, like signs give a positive answer. Therefore,

$$7 \cdot 9 = 63.$$

35. The property "multiplying by minus one" says that $(-1)a = -a$. Hence,

$$-1 \cdot 5 = -5.$$

37. Order of operations demands that we work from left to right. *Remember, multiplying like signs gives a positive result, multiplying unlike signs gives a negative result.*

$$\begin{aligned}(-7)(-1)(3) &= (7)(3) && \text{Work left to right.}\\ & && \text{Like signs: } (-7)(-1) = 7.\\ &= 21 && \text{Like signs: : } (7)(3) = 21.\end{aligned}$$

39. Order of operations demands that we work from left to right. *Remember, multiplying like signs gives a positive result, multiplying unlike signs gives a negative result.*

$$\begin{aligned}(-7)(9)(10)(-10) &= (-63)(10)(-10) && \text{Work left to right.}\\ & && \text{Unlike signs: } (-7)(9) = -63.\\ &= (-630)(-10) && \text{Work left to right.}\\ & && \text{Unlike signs: : } (-63)(10) = -630.\\ &= (6300) && \text{Like signs: : } (-630)(-10) = 6300.\end{aligned}$$

41. Order of operations demands that we work from left to right. *Remember, multiplying like signs gives a positive result, multiplying unlike signs gives a negative result.*

$$\begin{aligned}(6)(5)(8) &= (30)(8) && \text{Work left to right.}\\ & && \text{Like signs: } (6)(5) = 30.\\ &= 240 && \text{Like signs: : } (30)(8) = 240.\end{aligned}$$

43. Order of operations demands that we work from left to right. *Remember, multiplying like signs gives a positive result, multiplying unlike signs gives a negative result.*

$$\begin{aligned}(-10)(4)(-3)(8) &= (-40)(-3)(8) && \text{Work left to right.}\\ & && \text{Unlike signs: } (-10)(4) = -40.\\ &= (120)(8) && \text{Work left to right.}\\ & && \text{Like signs: : } (-40)(-3) = 120.\\ &= (960) && \text{Like signs: : } (120)(8) = 960.\end{aligned}$$

45. Order of operations demands that we work from left to right. *Remember, multiplying like signs gives a positive result, multiplying unlike signs gives a negative result.*

$$\begin{aligned}(6)(-3)(-8) &= (-18)(-8) && \text{Work left to right.} \\ & && \text{Unlike signs: } (6)(-3) = -18. \\ &= 144 && \text{Like signs: : } (-18)(-8) = 144.\end{aligned}$$

47. Order of operations demands that we work from left to right. *Remember, multiplying like signs gives a positive result, multiplying unlike signs gives a negative result.*

$$\begin{aligned}(2)(1)(3)(4) &= (2)(3)(4) && \text{Work left to right.} \\ & && \text{Like signs: } (2)(1) = 2. \\ &= (6)(4) && \text{Work left to right.} \\ & && \text{Like signs: : } (2)(3) = 6. \\ &= (24) && \text{Like signs: : } (6)(4) = 24.\end{aligned}$$

49. In the expression $(-4)^4$, the exponent 4 tells us to write the base -4 four times as a factor. Thus,

$$(-4)^4 = (-4)(-4)(-4)(-4).$$

Now, the product of an even number of negative factors is positive.

$$(-4)^4 = 256$$

51. In the expression $(-5)^4$, the exponent 4 tells us to write the base -5 four times as a factor. Thus,

$$(-5)^4 = (-5)(-5)(-5)(-5).$$

Now, the product of an even number of negative factors is positive.

$$(-5)^4 = 625$$

53. In the expression $(-5)^2$, the exponent 2 tells us to write the base -5 two times as a factor. Thus,

$$(-5)^2 = (-5)(-5).$$

Now, the product of an even number of negative factors is positive.

$$(-5)^2 = 25$$

55. In the expression $(-6)^2$, the exponent 2 tells us to write the base -6 two times as a factor. Thus,

$$(-6)^2 = (-6)(-6).$$

Now, the product of an even number of negative factors is positive.

$$(-6)^2 = 36$$

57. In the expression $(-4)^5$, the exponent 5 tells us to write the base -4 five times as a factor. Thus,

$$(-4)^5 = (-4)(-4)(-4)(-4)(-4).$$

Now, the product of an odd number of negative factors is negative.

$$(-4)^5 = -1024$$

59. In the expression $(-5)^3$, the exponent 3 tells us to write the base -5 three times as a factor. Thus,

$$(-5)^3 = (-5)(-5)(-5).$$

Now, the product of an odd number of negative factors is negative.

$$(-5)^3 = -125$$

61. When dividing, like signs give a positive answer. Hence,

$$-16 \div (-8) = 2.$$

63. When dividing, unlike signs give a negative answer. Hence,

$$\frac{-8}{1} = -8.$$

65. Division by 0 is undefined. Thus,

$$\frac{-1}{0}$$

is undefined.

67. When dividing, unlike signs give a negative answer. Hence,

$$-3 \div 3 = -1.$$

69. When dividing, unlike signs give a negative answer. Hence,

$$\frac{56}{-28} = -2.$$

71. When zero is divided by a nonzero number, the answer is zero. That is,

$$0 \div 15 = 0.$$

73. When dividing, like signs give a positive answer. Hence,

$$\frac{63}{21} = 3.$$

75. When dividing, like signs give a positive answer. Hence,

$$\frac{78}{13} = 6.$$

77. When zero is divided by a nonzero number, the answer is zero. That is,

$$0 \div 5 = 0.$$

79. Division by 0 is undefined. Thus,

$$\frac{17}{0}$$

is undefined.

81. When dividing, unlike signs give a negative answer. Hence,

$$-45 \div 15 = -3.$$

83. When dividing, like signs give a positive answer. Hence,

$$12 \div 3 = 4.$$

85. The initial depth of the first diver is 25 feet below sea level, or -25. To find the final depth of the second diver, multiply the depth of the first diver by 5.

$$-25(5) = -125$$

The second diver's final depth is -125 feet, or, 125 feet below sea level.

2.5 Order of Operations

1. Order of operations demands that we do divisions before additions.

$$\begin{aligned} 7 - \frac{-14}{2} &= 7 - (-7) && \text{Divide: } -14/2 = -7. \\ &= 7 + 7 && \text{Subtract by adding the opposite.} \\ &= 14 && \text{Add: } 7 + 7 = 14. \end{aligned}$$

3. Order of operations demands that we do divisions before additions.

$$\begin{aligned} -7 - \frac{-18}{9} &= -7 - (-2) && \text{Divide: } -18/9 = -2. \\ &= -7 + 2 && \text{Subtract by adding the opposite.} \\ &= -5 && \text{Add: } -7 + 2 = -5. \end{aligned}$$

5. Order of operations demands that we apply the exponent first. The exponent 4 tells us to write the base 5 as a factor 4 times. That is,

$$-5^4 = -(5)(5)(5)(5)$$

Take the product of the factors, then negate.

$$-5^4 = -625$$

7.

$$\begin{aligned} 9 - 1(-7) &= 9 - (-7) && \text{Multiply first: } 1(-7) = -7. \\ &= 9 + 7 && \text{Add the opposite.} \\ &= 16 && \text{Add: } 9 + 7 = 16. \end{aligned}$$

9. Order of operations demands that we apply the exponent first. The exponent 3 tells us to write the base 6 as a factor 3 times. That is,

$$-6^3 = -(6)(6)(6)$$

Take the product of the factors, then negate.

$$-6^3 = -216$$

11. Order of operations demands that multiplication is applied first, then addition.

$$\begin{aligned} 3 + 9(4) &= 3 + 36 && \text{Multiply first: } 9(4) = 36. \\ &= 39 && \text{Add: } 3 + 36 = 39. \end{aligned}$$

13. Divisions and multiplications first, in the order that they appear, as you work from left to right.

$$\begin{aligned} 10 - 72 \div 6 \cdot 3 + 8 &= 10 - 12 \cdot 3 + 8 && \text{Divide: } 72 \div 6 = 12. \\ &= 10 - 36 + 8 && \text{Multiply: } 12 \cdot 3 = 36. \\ &= 10 + (-36) + 8 && \text{Subtract by adding the opposite.} \end{aligned}$$

Additions are done in order, working left to right.

$$\begin{aligned} &= -26 + 8 && \text{Add: } 10 + (-36) = -26. \\ &= -18 && \text{Add: } -26 + 8 = -18. \end{aligned}$$

15. Order of operations demands that we do divisions before additions.

$$\begin{aligned} 6 + \frac{14}{2} &= 6 + 7 && \text{Divide: } 14/2 = 7. \\ &= 13 && \text{Add: } 6 + 7 = 13. \end{aligned}$$

17. Order of operations demands that we apply the exponent first. The exponent 4 tells us to write the base 3 as a factor 4 times. That is,

$$-3^4 = -(3)(3)(3)(3)$$

Take the product of the factors, then negate.

$$-3^4 = -81$$

19. Divisions and multiplications first, in the order that they appear, as you work from left to right.

$$\begin{aligned} 3 - 24 \div 4 \cdot 3 + 4 &= 3 - 6 \cdot 3 + 4 && \text{Divide: } 24 \div 4 = 6. \\ &= 3 - 18 + 4 && \text{Multiply: } 6 \cdot 3 = 18. \\ &= 3 + (-18) + 4 && \text{Subtract by adding the opposite.} \end{aligned}$$

Additions are done in order, working left to right.

$$\begin{aligned} &= -15 + 4 && \text{Add: } 3 + (-18) = -15. \\ &= -11 && \text{Add: } -15 + 4 = -11. \end{aligned}$$

21. Division and multiplication must be done in the order that they appear, working from left to right.

$$\begin{aligned} 64 \div 4 \cdot 4 &= 16 \cdot 4 && \text{Divide: } 64 \div 4 = 16. \\ &= 64 && \text{Multiply: } 16 \cdot 4 = 64. \end{aligned}$$

23.

$$\begin{aligned} -2 - 3(-5) &= -2 - (-15) && \text{Multiply first: } 3(-5) = -15. \\ &= -2 + 15 && \text{Add the opposite.} \\ &= 13 && \text{Add: } -2 + 15 = 13. \end{aligned}$$

25. Division and multiplication must be done in the order that they appear, working from left to right.

$$\begin{aligned} 15 \div 1 \cdot 3 &= 15 \cdot 3 && \text{Divide: } 15 \div 1 = 15. \\ &= 45 && \text{Multiply: } 15 \cdot 3 = 45. \end{aligned}$$

27. Divisions and multiplications first, in the order that they appear, as you work from left to right.

$$\begin{aligned} 8 + 12 \div 6 \cdot 1 - 5 &= 8 + 2 \cdot 1 - 5 && \text{Divide: } 12 \div 6 = 2. \\ &= 8 + 2 - 5 && \text{Multiply: } 2 \cdot 1 = 2. \\ &= 8 + 2 + (-5) && \text{Subtract by adding the opposite.} \end{aligned}$$

Additions are done in order, working left to right.

$$\begin{aligned} &= 10 + (-5) && \text{Add: } 8 + 2 = 10. \\ &= 5 && \text{Add: } 10 + (-5) = 5. \end{aligned}$$

29. Division and multiplication must be done in the order that they appear, working from left to right.

$$\begin{aligned} 32 \div 4 \cdot 4 &= 8 \cdot 4 && \text{Divide: } 32 \div 4 = 8. \\ &= 32 && \text{Multiply: } 8 \cdot 4 = 32. \end{aligned}$$

31. Order of operations demands that we do divisions before additions.

$$\begin{aligned} -11 + \frac{16}{16} &= -11 + 1 && \text{Divide: } 16/16 = 1. \\ &= -10 && \text{Add: } -11 + 1 = -10. \end{aligned}$$

33. Order of operations demands that we apply the exponent first. The exponent 2 tells us to write the base 5 as a factor 2 times. That is,

$$-5^2 = -(5)(5)$$

Take the product of the factors, then negate.

$$-5^2 = -25$$

35. Order of operations demands that multiplication is applied first, then addition.

$$\begin{aligned} 10 + 12(-5) &= 10 + (-60) && \text{Multiply first: } 12(-5) = -60. \\ &= -50 && \text{Add: } 10 + (-60) = -50. \end{aligned}$$

37. Divisions and multiplications first, in the order that they appear, as you work from left to right.

$$\begin{aligned} 2 + 6 \div 1 \cdot 6 - 1 &= 2 + 6 \cdot 6 - 1 && \text{Divide: } 6 \div 1 = 6. \\ &= 2 + 36 - 1 && \text{Multiply: } 6 \cdot 6 = 36. \\ &= 2 + 36 + (-1) && \text{Subtract by adding the opposite.} \end{aligned}$$

Additions are done in order, working left to right.

$$\begin{aligned} &= 38 + (-1) && \text{Add: } 2 + 36 = 38. \\ &= 37 && \text{Add: } 38 + (-1) = 37. \end{aligned}$$

39. Division and multiplication must be done in the order that they appear, working from left to right.

$$\begin{aligned} 40 \div 5 \cdot 4 &= 8 \cdot 4 && \text{Divide: } 40 \div 5 = 8. \\ &= 32 && \text{Multiply: } 8 \cdot 4 = 32. \end{aligned}$$

41. Absolute value bars are like grouping symbols, only stronger. We must simplify the expression inside the absolute value bars first.

$$\begin{aligned} -11 + |-1 - (-6)^2| &= -11 + |-1 - 36| && \text{Inside the abolute value bars,} \\ & && \text{Apply exponent: } (-6)^2 = 36. \\ &= -11 + |-1 + (-36)| && \text{Subtract: Add the opposite.} \\ &= -11 + |-37| && \text{Add: } -1 + (-36) = -37. \\ &= -11 + 37 && \text{Take absolute value: } |-37| = 37. \\ &= 26 && \text{Add: } -11 + 37 = 26. \end{aligned}$$

43. Absolute values act like grouping symbols, only stronger. Therefore, we must simplify the expression inside the absolute value first.

$$\begin{aligned}
|0(-4)| - 4(-4) &= |0| - 4(-4) && \text{Inside absolute value bars,} \\
& && \text{Multiply: } 0(-4) = 0. \\
&= 0 - 4(-4) && \text{Take absolute value: } |0| = 0. \\
&= 0 - (-16) && \text{Multiply: } 4(-4) = -16. \\
&= 0 + 16 && \text{Subtract: Add the opposite.} \\
&= 16 && \text{Add: } 0 + 16 = 16.
\end{aligned}$$

45. The parenthetical expression must be simplified first. Inside the parentheses, order of operations demands that we multiply first.

$$\begin{aligned}
(2 + 3 \cdot 4) - 8 &= (2 + 12) - 8 && \text{Multiply: } 3 \cdot 4 = 12. \\
&= 14 - 8 && \text{Add: } 2 + 12 = 14. \\
&= 6 && \text{Subtract: } 14 - 8 = 6.
\end{aligned}$$

47. The parenthetical expression must be simplified first. Inside the parentheses, order of operations demands that we multiply first.

$$\begin{aligned}
(8 - 1 \cdot 12) + 4 &= (8 - 12) + 4 && \text{Multiply: } 1 \cdot 12 = 12. \\
&= -4 + 4 && \text{Subtract: } 8 - 12 = -4. \\
&= 0 && \text{Add: } -4 + 4 = 0.
\end{aligned}$$

49. The parenthetical expression must be simplified first. Inside the parentheses, order of operations demands that we multiply first.

$$\begin{aligned}
(6 + 10 \cdot 4) - 6 &= (6 + 40) - 6 && \text{Multiply: } 10 \cdot 4 = 40. \\
&= 46 - 6 && \text{Add: } 6 + 40 = 46. \\
&= 40 && \text{Subtract: } 46 - 6 = 40.
\end{aligned}$$

51.

$$\begin{aligned}
10 + (6 - 4)^3 - 3 &= 10 + 2^3 - 3 && \text{Subtract: } 6 - 4 = 2. \\
&= 10 + 8 - 3 && \text{Apply the exponent: } 2^3 = 8. \\
&= 10 + 8 + (-3) && \text{Subtract: Add the opposite.} \\
&= 18 + (-3) && \text{Add: } 10 + 8 = 18. \\
&= 15 && \text{Add: } 18 + (-3) = 15.
\end{aligned}$$

53. We must first simplify the expressions inside the parentheses. Then we can apply the exponents and after that, subtract.

$$\begin{aligned}
(6-8)^2-(4-7)^2 &= (-2)^2-(-3)^3 && \text{Subtract: } 6-8=-2 \text{ and } 4-7=-3.\\
&= 4-(-27) && \text{Square: } (-2)^2=4.\\
& && \text{Cube: } (-3)^3=-27.\\
&= 4+27 && \text{Subtract: Add the opposite.}\\
&= 31 && \text{Add: } 4+27=31.
\end{aligned}$$

55. Absolute values act like grouping symbols, only stronger. Therefore, we must simplify the expression inside the absolute value first.

$$\begin{aligned}
|0(-10)|+4(-4) &= |0|+4(-4) && \text{Inside absolute value bars, Multiply: } 0(-10)=0.\\
&= 0+4(-4) && \text{Take absolute value: } |0|=0.\\
&= 0+(-16) && \text{Multiply: } 4(-4)=-16.\\
&= -16 && \text{Add: } 0+(-16)=-16.
\end{aligned}$$

57. Absolute values act like grouping symbols, only stronger. Therefore, we must simplify the expression inside the absolute value first.

$$\begin{aligned}
|8(-1)|-8(-7) &= |-8|-8(-7) && \text{Inside absolute value bars,}\\
& && \text{Multiply: } 8(-1)=-8.\\
&= 8-8(-7) && \text{Take absolute value: } |-8|=8.\\
&= 8-(-56) && \text{Multiply: } 8(-7)=-56.\\
&= 8+56 && \text{Subtract: Add the opposite.}\\
&= 64 && \text{Add: } 8+56=64.
\end{aligned}$$

59.

$$\begin{aligned}
3+(3-8)^2-7 &= 3+(-5)^2-7 && \text{Subtract: } 3-8=-5.\\
&= 3+25-7 && \text{Apply the exponent: } (-5)^2=25.\\
&= 3+25+(-7) && \text{Subtract: Add the opposite.}\\
&= 28+(-7) && \text{Add: } 3+25=28.\\
&= 21 && \text{Add: } 28+(-7)=21.
\end{aligned}$$

61. We must first simplify the expressions inside the parentheses. Then we can apply the exponents and after that, subtract.

$$\begin{aligned}
(4-2)^2-(7-2)^2 &= 2^2-5^3 && \text{Subtract: } 4-2=2 \text{ and } 7-2=5.\\
&= 4-125 && \text{Square: } 2^2=4.\\
& && \text{Cube: } 5^3=125.\\
&= 4+(-125) && \text{Subtract: Add the opposite.}\\
&= -121 && \text{Add: } 4+(-125)=-121.
\end{aligned}$$

63. Absolute value bars are like grouping symbols, only stronger. We must simplify the expression inside the absolute value bars first.

$$\begin{aligned}
8-|-25-(-4)^2| &= 8-|-25-16| && \text{Inside the abolute value bars,}\\
& && \text{Apply exponent: } (-4)^2=16.\\
&= 8-|-25+(-16)| && \text{Subtract: Add the opposite.}\\
&= 8-|-41| && \text{Add: } -25+(-16)=-41.\\
&= 8-41 && \text{Take absolute value: } |-41|=41.\\
&= 8+(-41) && \text{Subtract: Add the opposite.}\\
&= -33 && \text{Add: } 8+(-41)=-33.
\end{aligned}$$

65. Absolute value bars are like grouping symbols, only stronger. We must simplify the expression inside the absolute value bars first.

$$\begin{aligned}
-4-|30-(-5)^2| &= -4-|30-25| && \text{Inside the abolute value bars,}\\
& && \text{Apply exponent: } (-5)^2=25.\\
&= -4-|30+(-25)| && \text{Subtract: Add the opposite.}\\
&= -4-|5| && \text{Add: } 30+(-25)=5.\\
&= -4-5 && \text{Take absolute value: } |5|=5.\\
&= -4+(-5) && \text{Subtract: Add the opposite.}\\
&= -9 && \text{Add: } -4+(-5)=-9.
\end{aligned}$$

67. We must first simplify the expressions inside the parentheses. Then we can apply the exponents and after that, subtract.

$$\begin{aligned}
(8-7)^2-(2-6)^2 &= 1^2-(-4)^3 && \text{Subtract: } 8-7=1 \text{ and } 2-6=-4.\\
&= 1-(-64) && \text{Square: } 1^2=1.\\
& && \text{Cube: } (-4)^3=-64.\\
&= 1+64 && \text{Subtract: Add the opposite.}\\
&= 65 && \text{Add: } 1+64=65.
\end{aligned}$$

69. Simplify the expression inside the parentheses first, then apply the exponent, then add and subtract, moving left to right.

$$\begin{aligned} 4-(3-6)^3+4 &= 4-(-3)^3+4 && \text{Subtract: } 3-6=-3. \\ &= 4-(-27)+4 && \text{Apply the exponent: } (-3)^3=-27. \\ &= 4+27+4 && \text{Subtract: Add the opposite.} \\ &= 31+4 && \text{Add: } 4+27=31. \\ &= 35 && \text{Add: } 31+4=35. \end{aligned}$$

71. Absolute value bars are like grouping symbols, only stronger. We must simplify the expression inside the absolute value bars first.

$$\begin{aligned} -3+|-22-5^2| &= -3+|-22-25| && \text{Inside the abolute value bars,} \\ & && \text{Apply exponent: } 5^2=25. \\ &= -3+|-22+(-25)| && \text{Subtract: Add the opposite.} \\ &= -3+|-47| && \text{Add: } -22+(-25)=-47. \\ &= -3+47 && \text{Take absolute value: } |-47|=47. \\ &= 44 && \text{Add: } -3+47=44. \end{aligned}$$

73. The parenthetical expression must be simplified first. Inside the parentheses, order of operations demands that we multiply first.

$$\begin{aligned} (3-4\cdot 1)+6 &= (3-4)+6 && \text{Multiply: } 4\cdot 1=4. \\ &= -1+6 && \text{Subtract: } 3-4=-1. \\ &= 5 && \text{Add: } -1+6=5. \end{aligned}$$

75. Simplify the expression inside the parentheses first, then apply the exponent, then add and subtract, moving left to right.

$$\begin{aligned} 1-(1-5)^2+11 &= 1-(-4)^2+11 && \text{Subtract: } 1-5=-4. \\ &= 1-16+11 && \text{Apply the exponent: } (-4)^2=16. \\ &= 1+(-16)+11 && \text{Subtract: Add the opposite.} \\ &= -15+11 && \text{Add: } 1+(-16)=-15. \\ &= -4 && \text{Add: } -15+11=-4. \end{aligned}$$

77. We must first simplify the expressions inside the parentheses. Then we can apply the exponents and after that, subtract.

$$
\begin{aligned}
(2-6)^2-(8-6)^2 &= (-4)^2-2^3 && \text{Subtract: } 2-6=-4 \text{ and } 8-6=2.\\
&= 16-8 && \text{Square: } (-4)^2=16.\\
& && \text{Cube: } 2^3=8.\\
&= 16+(-8) && \text{Subtract: Add the opposite.}\\
&= 8 && \text{Add: } 16+(-8)=8.
\end{aligned}
$$

79. Absolute values act like grouping symbols, only stronger. Therefore, we must simplify the expression inside the absolute value first.

$$
\begin{aligned}
|9(-3)|+12(-2) &= |-27|+12(-2) && \text{Inside absolute value bars,}\\
& && \text{Multiply: } 9(-3)=-27.\\
&= 27+12(-2) && \text{Take absolute value: } |-27|=27.\\
&= 27+(-24) && \text{Multiply: } 12(-2)=-24.\\
&= 3 && \text{Add: } 27+(-24)=3.
\end{aligned}
$$

81. We must simplify numerator first. This requires that we first multiply.

$$
\begin{aligned}
\frac{4(-10)-5}{-9} &= \frac{-40-5}{-9} && \text{Multiply: } 4(-10)=-40.\\
&= \frac{-40+(-5)}{-9} && \text{Subtract: Add the opposite.}\\
&= \frac{-45}{-9} && \text{Add: } -40+(-5)=-45.\\
&= 5 && \text{Divide: } -45/(-9)=5.
\end{aligned}
$$

83. First, simplify numerator and denominator. In the numerator, exponents first, then subtract. In the denominator, multiply, then subtract.

$$
\begin{aligned}
\frac{10^2-4^2}{2\cdot 6-10} &= \frac{100-16}{12-10} && \text{Numerator: } 10^2=100,\ 4^2=16.\\
& && \text{Denominator: } 2\cdot 6=12.\\
&= \frac{84}{2} && \text{Numerator: } 100-16=84.\\
& && \text{Denominator: } 12-10=2.\\
&= 42 && \text{Divide: } 84/2=42.
\end{aligned}
$$

85. First, simplify numerator and denominator. In the numerator, exponents first, then subtract. In the denominator, multiply, then subtract.

$$\frac{3^2+6^2}{5-1\cdot 8} = \frac{9+36}{5-8}$$ Numerator: $3^2 = 9$, $6^2 = 36$.

Denominator: $1 \cdot 8 = 8$.

$$= \frac{45}{-3}$$ Numerator: $9+36 = 45$.

Denominator: $5-8 = -3$.

$$= -15$$ Divide: $45/(-3) = -15$.

87. With a fractional expression, we must simplify numerator and denominator first, then divide.

$$\frac{-8-4}{7-13} = \frac{-8+(-4)}{7+(-13)}$$ In numerator and denominator,

add the opposite.

$$= \frac{-12}{-6}$$ Numerator: $-8+(-4) = -12$.

Denominator: $7+(-13) = -6$.

$$= 2$$ Divide: $-12/(-6) = 2$.

89. First, simplify numerator and denominator. In the numerator, exponents first, then subtract. In the denominator, multiply, then subtract.

$$\frac{2^2+6^2}{11-4\cdot 4} = \frac{4+36}{11-16}$$ Numerator: $2^2 = 4$, $6^2 = 36$.

Denominator: $4 \cdot 4 = 16$.

$$= \frac{40}{-5}$$ Numerator: $4+36 = 40$.

Denominator: $11-16 = -5$.

$$= -8$$ Divide: $40/(-5) = -8$.

91. First, simplify numerator and denominator. In the numerator, exponents first, then subtract. In the denominator, multiply, then subtract.

$$\frac{1^2-5^2}{9\cdot 1-5} = \frac{1-25}{9-5}$$ Numerator: $1^2 = 1$, $5^2 = 25$.

Denominator: $9 \cdot 1 = 9$.

$$= \frac{-24}{4}$$ Numerator: $1-25 = -24$.

Denominator: $9-5 = 4$.

$$= -6$$ Divide: $-24/4 = -6$.

93. First, simplify numerator and denominator. In the numerator, exponents first, then subtract. In the denominator, multiply, then subtract.

$$\frac{4^2 - 8^2}{6 \cdot 3 - 2} = \frac{16 - 64}{18 - 2}$$ Numerator: $4^2 = 16$, $8^2 = 64$.

Denominator: $6 \cdot 3 = 18$.

$$= \frac{-48}{16}$$ Numerator: $16 - 64 = -48$.

Denominator: $18 - 2 = 16$.

$$= -3$$ Divide: $-48/16 = -3$.

95. First, simplify numerator and denominator. In the numerator, exponents first, then subtract. In the denominator, multiply, then subtract.

$$\frac{10^2 + 2^2}{7 - 2 \cdot 10} = \frac{100 + 4}{7 - 20}$$ Numerator: $10^2 = 100$, $2^2 = 4$.

Denominator: $2 \cdot 10 = 20$.

$$= \frac{104}{-13}$$ Numerator: $100 + 4 = 104$.

Denominator: $7 - 20 = -13$.

$$= -8$$ Divide: $104/(-13) = -8$.

97. With a fractional expression, we must simplify numerator and denominator first, then divide.

$$\frac{16 - (-2)}{19 - 1} = \frac{16 + 2}{19 + (-1)}$$ In numerator and denominator,

add the opposite.

$$= \frac{18}{18}$$ Numerator: $16 + 2 = 18$.

Denominator: $19 + (-1) = 18$.

$$= 1$$ Divide: $18/18 = 1$.

99. With a fractional expression, we must simplify numerator and denominator first, then divide.

$$\frac{15 - (-15)}{13 - (-17)} = \frac{15 + 15}{13 + 17}$$ In numerator and denominator,

add the opposite.

$$= \frac{30}{30}$$ Numerator: $15 + 15 = 30$.

Denominator: $13 + 17 = 30$.

$$= 1$$ Divide: $30/30 = 1$.

101. We must simplify numerator first. This requires that we first multiply.

$$\begin{aligned} \frac{4\cdot 5-(-19)}{3} &= \frac{20-(-19)}{3} && \text{Multiply: } 4\cdot 5 = 20. \\ &= \frac{20+19}{3} && \text{Subtract: Add the opposite.} \\ &= \frac{39}{3} && \text{Add: } 20+19=39. \\ &= 13 && \text{Divide: } 39/3 = 13. \end{aligned}$$

103. We must simplify numerator first. This requires that we first multiply.

$$\begin{aligned} \frac{-6\cdot 9-(-4)}{2} &= \frac{-54-(-4)}{2} && \text{Multiply: } -6\cdot 9 = -54. \\ &= \frac{-54+4}{2} && \text{Subtract: Add the opposite.} \\ &= \frac{-50}{2} && \text{Add: } -54+4=-50. \\ &= -25 && \text{Divide: } -50/2 = -25. \end{aligned}$$

2.6 Solving Equations Involving Integers

1. To see if -11 is a solution of $2x+3=-19$, we substitute -11 for x in the equation and check to see if this results in a true or false statement.

$$\begin{aligned} 2x+3 &= -19 && \text{Original equation.} \\ 2(-11)+3 &= -19 && \text{Substitute } x=-11. \\ -22+3 &= -19 && \text{On the left, multiply: } 2(-11)=-22. \\ -19 &= -19 && \text{On the left, add: } -22+3=-19. \end{aligned}$$

This last statement is a true statement. Therefore, -11 is a solution of the equation $2x+3=-19$.

3. To see if 6 is a solution of $3x+1=19$, we substitute 6 for x in the equation and check to see if this results in a true or false statement.

$$\begin{aligned} 3x+1 &= 19 && \text{Original equation.} \\ 3(6)+1 &= 19 && \text{Substitute } x=6. \\ 18+1 &= 19 && \text{On the left, multiply: } 3(6)=18. \\ 19 &= 19 && \text{On the left, add: } 18+1=19. \end{aligned}$$

This last statement is a true statement. Therefore, 6 is a solution of the equation $3x+1=19$.

5. To see if 12 is a solution of $4x + 5 = -8$, we substitute 12 for x in the equation and check to see if this results in a true or false statement.

$$\begin{aligned} 4x + 5 &= -8 && \text{Original equation.} \\ 4(12) + 5 &= -8 && \text{Substitute } x = 12. \\ 48 + 5 &= -8 && \text{On the left, multiply: } 4(12) = 48. \\ 53 &= -8 && \text{On the left, add: } 48 + 5 = 53. \end{aligned}$$

This last statement is a false statement. Therefore, 12 is **not** a solution of the equation $4x + 5 = -8$.

7. To see if 15 is a solution of $2x + 6 = -9$, we substitute 15 for x in the equation and check to see if this results in a true or false statement.

$$\begin{aligned} 2x + 6 &= -9 && \text{Original equation.} \\ 2(15) + 6 &= -9 && \text{Substitute } x = 15. \\ 30 + 6 &= -9 && \text{On the left, multiply: } 2(15) = 30. \\ 36 &= -9 && \text{On the left, add: } 30 + 6 = 36. \end{aligned}$$

This last statement is a false statement. Therefore, 15 is **not** a solution of the equation $2x + 6 = -9$.

9. To see if -15 is a solution of $-3x + 6 = -17$, we substitute -15 for x in the equation and check to see if this results in a true or false statement.

$$\begin{aligned} -3x + 6 &= -17 && \text{Original equation.} \\ -3(-15) + 6 &= -17 && \text{Substitute } x = -15. \\ 45 + 6 &= -17 && \text{On the left, multiply: } -3(-15) = 45. \\ 51 &= -17 && \text{On the left, add: } 45 + 6 = 51. \end{aligned}$$

This last statement is a false statement. Therefore, -15 is **not** a solution of the equation $-3x + 6 = -17$.

11. To see if -6 is a solution of $-2x + 3 = 15$, we substitute -6 for x in the equation and check to see if this results in a true or false statement.

$$\begin{aligned} -2x + 3 &= 15 && \text{Original equation.} \\ -2(-6) + 3 &= 15 && \text{Substitute } x = -6. \\ 12 + 3 &= 15 && \text{On the left, multiply: } -2(-6) = 12. \\ 15 &= 15 && \text{On the left, add: } 12 + 3 = 15. \end{aligned}$$

This last statement is a true statement. Therefore, -6 is a solution of the equation $-2x + 3 = 15$.

13. To undo the effect of subtracting 13, add −13 from both sides of the equation.

$x - 13 = 11$	Original equation.
$x - 13 + 13 = 11 + 13$	Add 13 to both sides.
$x = 24$	On the left, simplify. On the right: $11 + 13 = 24$.

15. To undo the effect of subtracting 3, add −3 from both sides of the equation.

$x - 3 = 6$	Original equation.
$x - 3 + 3 = 6 + 3$	Add 3 to both sides.
$x = 9$	On the left, simplify. On the right: $6 + 3 = 9$.

17. To undo the effect of adding 10, subtract 10 from both sides of the equation.

$x + 10 = 17$	Original equation.
$x + 10 - 10 = 17 - 10$	Subtract 10 from both sides.
$x = 17 + (-10)$	On the left, simplify. On the right, add the opposite.
$x = 7$	Add: $17 + (-10) = 7$.

19. To undo the effect of subtracting 6, add −6 from both sides of the equation.

$x - 6 = 1$	Original equation.
$x - 6 + 6 = 1 + 6$	Add 6 to both sides.
$x = 7$	On the left, simplify. On the right: $1 + 6 = 7$.

21. To undo the effect of subtracting 15, add −15 from both sides of the equation.

$x - 15 = -12$	Original equation.
$x - 15 + 15 = -12 + 15$	Add 15 to both sides.
$x = 3$	On the left, simplify. On the right: $-12 + 15 = 3$.

23. To undo the effect of adding 11, subtract 11 from both sides of the equation.

$x + 11 = -19$	Original equation.
$x + 11 - 11 = -19 - 11$	Subtract 11 from both sides.
$x = -19 + (-11)$	On the left, simplify. On the right, add the opposite.
$x = -30$	Add: $-19 + (-11) = -30$.

25. To undo the effect of adding 2, subtract 2 from both sides of the equation.

$x + 2 = 1$	Original equation.
$x + 2 - 2 = 1 - 2$	Subtract 2 from both sides.
$x = 1 + (-2)$	On the left, simplify. On the right, add the opposite.
$x = -1$	Add: $1 + (-2) = -1$.

27. To undo the effect of adding 5, subtract 5 from both sides of the equation.

$x + 5 = -5$	Original equation.
$x + 5 - 5 = -5 - 5$	Subtract 5 from both sides.
$x = -5 + (-5)$	On the left, simplify. On the right, add the opposite.
$x = -10$	Add: $-5 + (-5) = -10$.

29. To undo the effect of multiplying by -1, divide both sides of the equation by -1.

$-x = -20$	Original equation.
$\dfrac{-x}{-1} = \dfrac{-20}{-1}$	Divide boths sides by -1.
$x = 20$	On the left, simplify. On the right, divide: $-20/(-1) = 20$.

31. To undo the effect of dividing by -7, multiply both sides of the equation by -7.

$\dfrac{x}{-7} = 10$	Original equation.
$-7\left(\dfrac{x}{-7}\right) = -7\,(10)$	Multiply both sides by -7.
$x = -70$	On the left, simplify. On the right, multiply: $-7(10) = -70$.

33. To undo the effect of dividing by -10, multiply both sides of the equation by -10.

$\dfrac{x}{-10} = 12$	Original equation.
$-10\left(\dfrac{x}{-10}\right) = -10\,(12)$	Multiply both sides by -10.
$x = -120$	On the left, simplify. On the right, multiply: $-10(12) = -120$.

35. To undo the effect of dividing by 9, multiply both sides of the equation by 9.

$\dfrac{x}{9} = -16$	Original equation.
$9\left(\dfrac{x}{9}\right) = 9\,(-16)$	Multiply both sides by 9.
$x = -144$	On the left, simplify. On the right, multiply: $9(-16) = -144$.

37. To undo the effect of multiplying by -10, divide both sides of the equation by -10.

$-10x = 20$	Original equation.
$\dfrac{-10x}{-10} = \dfrac{20}{-10}$	Divide boths sides by -10.
$x = -2$	On the left, simplify. On the right, divide: $20/(-10) = -2$.

39. To undo the effect of multiplying by 14, divide both sides of the equation by 14.

$14x = 84$	Original equation.
$\dfrac{14x}{14} = \dfrac{84}{14}$	Divide boths sides by 14.
$x = 6$	On the left, simplify. On the right, divide: $84/14 = 6$.

41. To undo the effect of multiplying by -2, divide both sides of the equation by -2.

$-2x = 28$	Original equation.
$\dfrac{-2x}{-2} = \dfrac{28}{-2}$	Divide boths sides by -2.
$x = -14$	On the left, simplify. On the right, divide: $28/(-2) = -14$.

43. To undo the effect of dividing by -10, multiply both sides of the equation by -10.

$$\frac{x}{-10} = 15 \quad \text{Original equation.}$$

$$-10\left(\frac{x}{-10}\right) = -10\,(15) \quad \text{Multiply both sides by } -10.$$

$$x = -150 \quad \text{On the left, simplify. On the right, multiply: } -10(15) = -150.$$

45. On the left, order of operations demands that we first multiply by -4, then subtract 4. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add 4 to both sides of the equation, then divide both sides of the resulting equation by -4.

$$-4x - 4 = 16 \quad \text{Original equation.}$$

$$-4x - 4 + 4 = 16 + 4 \quad \text{Subtract } -4 \text{ from both sides.}$$

$$-4x = 20 \quad \text{On the left, simplify. On the right, add: } 16 + 4 = 20.$$

$$\frac{-4x}{-4} = \frac{20}{-4} \quad \text{Divide both sides by } -4.$$

$$x = -5 \quad \text{On the left, simplify. On the right, } 20/(-4) = -5.$$

47. On the left, order of operations demands that we first multiply by 4, then subtract 4. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add 4 to both sides of the equation, then divide both sides of the resulting equation by 4.

$$4x - 4 = 76 \quad \text{Original equation.}$$

$$4x - 4 + 4 = 76 + 4 \quad \text{Subtract } -4 \text{ from both sides.}$$

$$4x = 80 \quad \text{On the left, simplify. On the right, add: } 76 + 4 = 80.$$

$$\frac{4x}{4} = \frac{80}{4} \quad \text{Divide both sides by 4.}$$

$$x = 20 \quad \text{On the left, simplify. On the right, } 80/4 = 20.$$

49. On the left, order of operations demands that we first multiply by 5, then subtract 14. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add 14 to both sides of the equation, then divide both sides of the resulting equation by 5.

$5x - 14 = -79$	Original equation.
$5x - 14 + 14 = -79 + 14$	Subtract -14 from both sides.
$5x = -65$	On the left, simplify. On the right, add: $-79 + 14 = -65$.
$\dfrac{5x}{5} = \dfrac{-65}{5}$	Divide both sides by 5.
$x = -13$	On the left, simplify. On the right, $-65/5 = -13$.

51. On the left, order of operations demands that we first multiply by -10, then subtract 16. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add 16 to both sides of the equation, then divide both sides of the resulting equation by -10.

$-10x - 16 = 24$	Original equation.
$-10x - 16 + 16 = 24 + 16$	Subtract -16 from both sides.
$-10x = 40$	On the left, simplify. On the right, add: $24 + 16 = 40$.
$\dfrac{-10x}{-10} = \dfrac{40}{-10}$	Divide both sides by -10.
$x = -4$	On the left, simplify. On the right, $40/(-10) = -4$.

53. On the left, order of operations demands that we first multiply by 9, then add 5. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract 5 from both sides of the equation, then divide both sides of the resulting equation by 9.

$9x + 5 = -85$	Original equation.
$9x + 5 - 5 = -85 - 5$	Subtract 5 from both sides.
$9x = -85 + (-5)$	On the left, simplify. On the right, add the opposite.
$9x = -90$	Add: $-85 + (-5) = -90$.
$\dfrac{9x}{9} = \dfrac{-90}{9}$	Divide both sides by 9.
$x = -10$	On the left, simplify. On the right, $-90/9 = -10$.

55. On the left, order of operations demands that we first multiply by 7, then add 15. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract 15 from both sides of the equation, then divide both sides of the resulting equation by 7.

$7x + 15 = -55$	Original equation.
$7x + 15 - 15 = -55 - 15$	Subtract 15 from both sides.
$7x = -55 + (-15)$	On the left, simplify. On the right, add the opposite.
$7x = -70$	Add: $-55 + (-15) = -70$.
$\dfrac{7x}{7} = \dfrac{-70}{7}$	Divide both sides by 7.
$x = -10$	On the left, simplify. On the right, $-70/7 = -10$.

57. On the left, order of operations demands that we first multiply by -1, then add 8. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract 8 from both sides of the equation, then divide both sides of the resulting equation by -1.

$-x + 8 = 13$	Original equation.
$-x + 8 - 8 = 13 - 8$	Subtract 8 from both sides.
$-1x = 13 + (-8)$	On the left, simplify. On the right, add the opposite.
$-1x = 5$	Add: $13 + (-8) = 5$.
$\dfrac{-1x}{-1} = \dfrac{5}{-1}$	Divide both sides by -1.
$x = -5$	On the left, simplify. On the right, $5/(-1) = -5$.

59. On the left, order of operations demands that we first multiply by 12, then subtract 15. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add 15 to both sides of the equation, then divide both sides of the resulting equation by 12.

$12x - 15 = -3$	Original equation.
$12x - 15 + 15 = -3 + 15$	Subtract -15 from both sides.
$12x = 12$	On the left, simplify. On the right, add: $-3 + 15 = 12$.
$\dfrac{12x}{12} = \dfrac{12}{12}$	Divide both sides by 12.
$x = 1$	On the left, simplify. On the right, $12/12 = 1$.

61. On the left, order of operations demands that we first multiply by 4, then subtract 12. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add 12 to both sides of the equation, then divide both sides of the resulting equation by 4.

$$4x - 12 = -56 \quad \text{Original equation.}$$
$$4x - 12 + 12 = -56 + 12 \quad \text{Subtract } -12 \text{ from both sides.}$$
$$4x = -44 \quad \text{On the left, simplify. On the right, add: } -56 + 12 = -44.$$
$$\frac{4x}{4} = \frac{-44}{4} \quad \text{Divide both sides by 4.}$$
$$x = -11 \quad \text{On the left, simplify. On the right, } -44/4 = -11.$$

63. On the left, order of operations demands that we first multiply by 19, then add 18. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract 18 from both sides of the equation, then divide both sides of the resulting equation by 19.

$$19x + 18 = 113 \quad \text{Original equation.}$$
$$19x + 18 - 18 = 113 - 18 \quad \text{Subtract 18 from both sides.}$$
$$19x = 113 + (-18) \quad \text{On the left, simplify. On the right, add the opposite.}$$
$$19x = 95 \quad \text{Add: } 113 + (-18) = 95.$$
$$\frac{19x}{19} = \frac{95}{19} \quad \text{Divide both sides by 19.}$$
$$x = 5 \quad \text{On the left, simplify. On the right, } 95/19 = 5.$$

65. On the left, order of operations demands that we first multiply by -14, then add 12. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract 12 from both sides of the equation, then divide both sides of the resulting equation by -14.

$$-14x + 12 = -2 \quad \text{Original equation.}$$
$$-14x + 12 - 12 = -2 - 12 \quad \text{Subtract 12 from both sides.}$$
$$-14x = -2 + (-12) \quad \text{On the left, simplify. On the right, add the opposite.}$$
$$-14x = -14 \quad \text{Add: } -2 + (-12) = -14.$$
$$\frac{-14x}{-14} = \frac{-14}{-14} \quad \text{Divide both sides by } -14.$$
$$x = 1 \quad \text{On the left, simplify. On the right, } -14/(-14) = 1.$$

67. On the left, order of operations demands that we first multiply by 14, then add 16. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract 16 from both sides of the equation, then divide both sides of the resulting equation by 14.

$14x + 16 = 44$	Original equation.
$14x + 16 - 16 = 44 - 16$	Subtract 16 from both sides.
$14x = 44 + (-16)$	On the left, simplify. On the right, add the opposite.
$14x = 28$	Add: $44 + (-16) = 28$.
$\dfrac{14x}{14} = \dfrac{28}{14}$	Divide both sides by 14.
$x = 2$	On the left, simplify. On the right, $28/14 = 2$.

69. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.
2. *Set up an Equation.* "Two less than eight times certain number is -74" becomes:

eight times a certain number	less	Two	is	-74
$8x$	$-$	2	$=$	-74

3. *Solve the Equation.* On the left, order of operations requires that we first multiply x by 8, then subtract 2. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will (1) add 2 from both sides of the equation, then (2) divide both sides of the resulting equation by 8.

$8x - 2 = -74$	Original equation.
$8x - 2 + 2 = -74 + 2$	Add 2 to both sides.
$8x = -72$	Simplify both sides.
$\dfrac{8x}{8} = \dfrac{-72}{8}$	Divide both sides by 8.
$x = -9$	Simplify both sides.

4. *Answer the Question.* The unknown number is -9.
5. *Look Back.* Does the answer satisfy the problem constraints? Two less than 8 times -9 is 2 less than -72, or -74. So the solution is correct.

71. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* "Eight more than two times certain number is 0" becomes:

Eight	more than	two times a certain number	is	0
8	$+$	$2x$	$=$	0

3. *Solve the Equation.* On the left, order of operations requires that we first multiply x by 2, then add 8. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will (1) subtract 8 from both sides of the equation, then (2) divide both sides of the resulting equation by 2.

$$\begin{aligned} 8 + 2x &= 0 && \text{Original equation.} \\ 8 + 2x - 8 &= 0 - 8 && \text{Subtract 8 from both sides.} \\ 2x &= -8 && \text{Simplify both sides.} \\ \frac{2x}{2} &= \frac{-8}{2} && \text{Divide both sides by 2.} \\ x &= -4 && \text{Simplify both sides.} \end{aligned}$$

4. *Answer the Question.* The unknown number is -4.

5. *Look Back.* Does the answer satisfy the problem constraints? Eight more than 2 times -4 is 8 more than -8, or 0. So the solution is correct.

73. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* "The number -6 is 2 more than an unknown number" becomes:

-6	is	2	more than	unknown number
-6	$=$	2	$+$	x

3. *Solve the Equation.* To "undo" adding 2, subtract 2 from both sides of the equation.

$$\begin{aligned} -6 &= 2 + x && \text{Original equation.} \\ -6 - 2 &= 2 + x - 2 && \text{Subtract 2 from both sides.} \\ -8 &= x && \text{Simplify both sides.} \end{aligned}$$

4. *Answer the Question.* The unknown number is −8.

5. *Look Back.* Does the answer satisfy the problem constraints? Well, 2 more than −8 is −6, so the answer is correct.

75. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* "Three more than eight times certain number is −29" becomes:

Three	more than	eight times a certain number	is	−29
3	+	$8x$	=	−29

3. *Solve the Equation.* On the left, order of operations requires that we first multiply x by 8, then add 3. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will (1) subtract 3 from both sides of the equation, then (2) divide both sides of the resulting equation by 8.

$$\begin{aligned} 3 + 8x &= -29 && \text{Original equation.} \\ 3 + 8x - 3 &= -29 - 3 && \text{Subtract 3 from both sides.} \\ 8x &= -32 && \text{Simplify both sides.} \\ \frac{8x}{8} &= \frac{-32}{8} && \text{Divide both sides by 8.} \\ x &= -4 && \text{Simplify both sides.} \end{aligned}$$

4. *Answer the Question.* The unknown number is −4.

5. *Look Back.* Does the answer satisfy the problem constraints? Three more than 8 times −4 is 3 more than −32, or −29. So the solution is correct.

77. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, the unknown is the score on Alan's fourth exam. Let x represent Alan's score on his fourth exam.

2. *Set up an Equation.* To find the average of four exam scores, sum the four scores, then divide by 4.

Sum of four exam scores	divided by	4	equals	Average score
$(79 + 61 + 54 + x)$	$\div$	4	$=$	71

This last result can be simplified by summing the three known exam scores.

$$\frac{194 + x}{4} = 71$$

3. *Solve the Equation.* To "undo" the effect of dividing by 4, multiply both sides of the equation by 4.

$$\begin{aligned} \frac{194 + x}{4} &= 71 && \text{Original equation.} \\ 4\left(\frac{194 + x}{4}\right) &= 4(71) && \text{Multiply both sides by 4.} \\ x + 194 &= 284 && \text{Simplify both sides.} \end{aligned}$$

To "undo" the effect of adding 194, subtract 194 from both sides of the equation.

$$\begin{aligned} x + 194 - 194 &= 284 - 194 && \text{Subtract 194 from both sides.} \\ x &= 90 && \text{Simplify both sides.} \end{aligned}$$

4. *Answer the Question.* The fourth exam score is 90.

5. *Look Back.* Add the four exam scores, 79, 61, 54, and 90, to get 284. Divide this sum by 4 to get 71, which is the required average. Hence, our solution is correct.

79. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* One interpretation of "the quotient of -2 and x is 5 is:

-2	divided by	x	equals	5
-2	$\div$	x	$=$	5

This can be written as follows:

$$\frac{-2}{x} = 5$$

3. *Solve the Equation.* To "undo" dividing by x, multiply both sides of the equation by x.

$$\begin{aligned} \frac{-2}{x} &= 5 && \text{Original equation.} \\ x\left(\frac{-2}{x}\right) &= x(5) && \text{Multiply both sides by } x. \\ -2 &= 5x && \text{Simplify both sides.} \\ \frac{-2}{5} &= \frac{5x}{5} && \text{Divide both sides by 5.} \\ -\frac{2}{5} &= x && \text{Simplify.} \end{aligned}$$

However, $-2/5$ is not an integer. This is the wrong interpretation of the "quotient." Another interpretation of the quotient of -2 and x is:

x	divided by	-2	equals	5
x	$\div$	-2	$=$	5

This can be written as follows:

$$\frac{x}{-2} = 5$$

4. *Solve the Equation.* To "undo" dividing by -2, multiply both sides of the equation by -2.

$$\begin{aligned} \frac{x}{-2} &= 5 && \text{Original equation.} \\ -2\left(\frac{x}{-2}\right) &= -2(5) && \text{Multiply both sides by } -2. \\ x &= -10 && \text{Simplify both sides.} \end{aligned}$$

This result is an integer.

5. *Answer the Question.* The unknown number is -10.

6. *Look Back.* Does the answer satisfy the problem constraints? Well, the quotient of -10 and -2 is 5, so our answer is correct.

81. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* One interpretation of "the quotient of -8 and x is 9 is:

-8	divided by	x	equals	9
-8	$\div$	x	$=$	9

This can be written as follows:

$$\frac{-8}{x} = 9$$

3. *Solve the Equation.* To "undo" dividing by x, multiply both sides of the equation by x.

$$\begin{aligned}
\frac{-8}{x} &= 9 && \text{Original equation.} \\
x\left(\frac{-8}{x}\right) &= x(9) && \text{Multiply both sides by } x. \\
-8 &= 9x && \text{Simplify both sides.} \\
\frac{-8}{9} &= \frac{9x}{9} && \text{Divide both sides by 9.} \\
-\frac{8}{9} &= x && \text{Simplify.}
\end{aligned}$$

However, $-8/9$ is not an integer. This is the wrong interpretation of the "quotient." Another interpretation of the quotient of -8 and x is:

x	divided by	-8	equals	9
x	$\div$	-8	$=$	9

This can be written as follows:

$$\frac{x}{-8} = 9$$

4. *Solve the Equation.* To "undo" dividing by −8, multiply both sides of the equation by −8.

$$\begin{aligned} \frac{x}{-8} &= 9 && \text{Original equation.} \\ -8\left(\frac{x}{-8}\right) &= -8(9) && \text{Multiply both sides by } -8. \\ x &= -72 && \text{Simplify both sides.} \end{aligned}$$

   This result is an integer.

5. *Answer the Question.* The unknown number is −72.

6. *Look Back.* Does the answer satisfy the problem constraints? Well, the quotient of −72 and −8 is 9, so our answer is correct.

83. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* "The number −5 is 8 more than an unknown number" becomes:

−5	is	8	more than	unknown number
-5	$=$	8	$+$	x

3. *Solve the Equation.* To "undo" adding 8, subtract 8 from both sides of the equation.

$$\begin{aligned} -5 &= 8 + x && \text{Original equation.} \\ -5 - 8 &= 8 + x - 8 && \text{Subtract 8 from both sides.} \\ -13 &= x && \text{Simplify both sides.} \end{aligned}$$

4. *Answer the Question.* The unknown number is −13.

5. *Look Back.* Does the answer satisfy the problem constraints? Well, 8 more than −13 is −5, so the answer is correct.

85. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, the unknown is the original balance in the student's account. Let B represent this original balance.

2. *Set up an Equation.* A positive integer represents a healthy balance, while a negative number represents an account that is overdrawn. After the student's deposit, the account is still overdrawn by \$70. We will say that this balance is −\$70. Thus,

Original Balance	plus	Student Deposit	equals	Current Balance
B	$+$	\$260	$=$	−\$70

3. *Solve the Equation.* To "undo" the addition, subtract 260 from both sides of the equation.

$$\begin{aligned} B + 260 &= -70 && \text{Original equation.} \\ B + 260 - 260 &= -70 - 260 && \text{Subtract 260 from both sides.} \\ B &= -330 && \text{Simplify both sides.} \end{aligned}$$

4. *Answer the Question.* The original balance was overdrawn to the tune of 330 dollars.

5. *Look Back.* If the original balance was overdrawn by \$330, then we let −\$330 represent this balance. The student makes a deposit of \$260. Add this to the original balance to get −\$330 + \$120 = −\$70, the correct current balance.

87. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, the unknown is the original balance in the student's account. Let B represent this original balance.

2. *Set up an Equation.* A positive integer represents a healthy balance, while a negative number represents an account that is overdrawn. After the student's deposit, the account is still overdrawn by \$90. We will say that this balance is −\$90. Thus,

Original Balance	plus	Student Deposit	equals	Current Balance
B	$+$	\$360	$=$	−\$90

3. *Solve the Equation.* To "undo" the addition, subtract 360 from both sides of the equation.

$$\begin{aligned} B + 360 &= -90 && \text{Original equation.} \\ B + 360 - 360 &= -90 - 360 && \text{Subtract 360 from both sides.} \\ B &= -450 && \text{Simplify both sides.} \end{aligned}$$

4. *Answer the Question.* The original balance was overdrawn to the tune of 450 dollars.

5. *Look Back.* If the original balance was overdrawn by \$450, then we let −\$450 represent this balance. The student makes a deposit of \$360. Add this to the original balance to get $-\$450 + \$120 = -\$90$, the correct current balance.

89. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* "The number −10 is −5 times larger than an unknown number" becomes:

−10	is	−5	times	unknown number
-10	$=$	-5	$\cdot$	x

3. *Solve the Equation.* To "undo" multiplying by −5, divide both sides of the equation by −5.

$$\begin{aligned} -10 &= -5x && \text{Original equation.} \\ \frac{-10}{-5} &= \frac{-5x}{-5} && \text{Divide both sides by } -5. \\ 2 &= x && \text{Simplify both sides.} \end{aligned}$$

4. *Answer the Question.* The unknown number is 2.

5. *Look Back.* Does the answer satisfy the problem constraints? Well, −5 times 2 is −10, so the answer is correct.

91. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* "The number −15 is −5 times larger than an unknown number" becomes:

−15	is	−5	times	unknown number
-15	$=$	-5	$\cdot$	x

3. *Solve the Equation.* To "undo" multiplying by -5, divide both sides of the equation by -5.

$$
\begin{aligned}
-15 &= -5x && \text{Original equation.}\\
\frac{-15}{-5} &= \frac{-5x}{-5} && \text{Divide both sides by } -5.\\
3 &= x && \text{Simplify both sides.}
\end{aligned}
$$

4. *Answer the Question.* The unknown number is 3.

5. *Look Back.* Does the answer satisfy the problem constraints? Well, -5 times 3 is -15, so the answer is correct.

93. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* "Two less than nine times certain number is 7" becomes:

nine times a certain number	less	Two	is	7
$9x$	$-$	2	$=$	7

3. *Solve the Equation.* On the left, order of operations requires that we first multiply x by 9, then subtract 2. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will (1) add 2 from both sides of the equation, then (2) divide both sides of the resulting equation by 9.

$$
\begin{aligned}
9x - 2 &= 7 && \text{Original equation.}\\
9x - 2 + 2 &= 7 + 2 && \text{Add 2 to both sides.}\\
9x &= 9 && \text{Simplify both sides.}\\
\frac{9x}{9} &= \frac{9}{9} && \text{Divide both sides by 9.}\\
x &= 1 && \text{Simplify both sides.}
\end{aligned}
$$

4. *Answer the Question.* The unknown number is 1.

5. *Look Back.* Does the answer satisfy the problem constraints? Two less than 9 times 1 is 2 less than 9, or 7. So the solution is correct.

95. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, the unknown is the score on Mark's fourth exam. Let x represent Mark's score on his fourth exam.

2. *Set up an Equation.* To find the average of four exam scores, sum the four scores, then divide by 4.

Sum of four exam scores	divided by	4	equals	Average score
$(79 + 84 + 71 + x)$	$\div$	4	$=$	74

This last result can be simplified by summing the three known exam scores.

$$\frac{234 + x}{4} = 74$$

3. *Solve the Equation.* To "undo" the effect of dividing by 4, multiply both sides of the equation by 4.

$$\begin{aligned} \frac{234 + x}{4} &= 74 && \text{Original equation.} \\ 4\left(\frac{234 + x}{4}\right) &= 4(74) && \text{Multiply both sides by 4.} \\ x + 234 &= 296 && \text{Simplify both sides.} \end{aligned}$$

To "undo" the effect of adding 234, subtract 234 from both sides of the equation.

$$\begin{aligned} x + 234 - 234 &= 296 - 234 && \text{Subtract 234 from both sides.} \\ x &= 62 && \text{Simplify both sides.} \end{aligned}$$

4. *Answer the Question.* The fourth exam score is 62.

5. *Look Back.* Add the four exam scores, 79, 84, 71, and 62, to get 296. Divide this sum by 4 to get 74, which is the required average. Hence, our solution is correct.