Prealgebra Textbook

Second Edition

Chapter 4

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Chapter 4

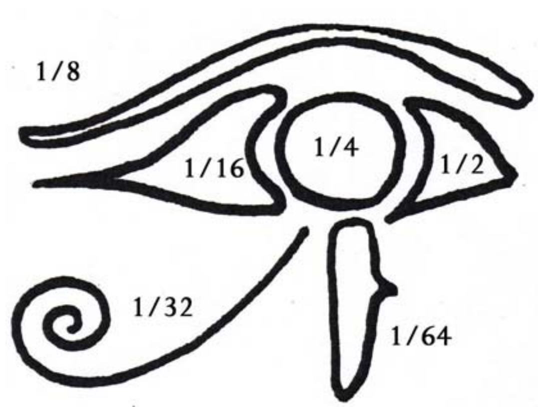
Fractions

Around 3000BC, Egyptians were carving hierglyphs into stone monuments to their kings and queens. Hierglyphs are pictures that represent objects and they were used for words and numbers.

Oddly, fractions were always written as sums of "unit fractions," fractions whose numerator is always 1. For instance, instead of writing 3/5 , they would write a sum of unit fractions.

$$\frac{3}{5} = \frac{1}{2} + \frac{1}{10}$$

Much of the ancient Egyptian math that we know of was in service to the agricultural and economic life of the people. in measuring dry goods such as grains, special glyphs were used to represent basic fractional amounts, glyphs that came together to represent the Eye of Horus.



Horus was a falcon-god whose father Osirus was murdered by his own brother Seth. When Horus attempted to avenge his father's death, Seth ripped out Horus' eye and cut it into six pieces, scattering them throughout Egypt.

Taking pity on Horus, Thot, the god of learning and magic, found the pieces and put them back together making Horus healthy and whole again.

Each piece of the Eye of Horus represents a different fraction of a hekat, or volume of grain. It was written that an apprentice scribe added the fractions one day and got

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}=\frac{63}{64}.$$

Asking where the missing 1/64 was, he was told that Thot would make up the difference to anyone "who sought and accepted is protection."

In this chapter, you'll learn how we use fractions.

4.1 Equivalent Fractions

In this section we deal with fractions, numbers or expressions of the form a/b.

Fractions. A number of the form

$$\frac{a}{b}$$

where a and b are numbers is called a *fraction*. The number a is called the *numerator* of the fraction, while the number b is called the *denominator* of the fraction.

Near the end of this section, we'll see that the numerator and denominator of a fraction can also be algebraic expressions, but for the moment we restrict our attention to fractions whose numerators and denominators are integers.

We start our study of fractions with the definition of *equivalent fractions*.

Equivalent Fractions. Two fractions are *equivalent* if they represent the same numerical value.

But how can we tell if two fractions represent the same number? Well, one technique involves some simple visualizations. Consider the image shown in Figure 4.1, where the shaded region represents 1/3 of the total area of the figure (one of three equal regions is shaded).

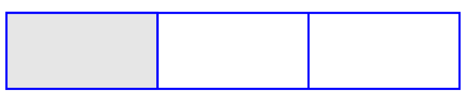


Figure 4.1: The shaded region is 1/3 of the whole region.

In Figure 4.2, we've shaded 2/6 of the entire region (two of six equal regions are shaded).

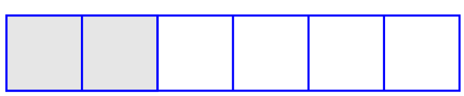


Figure 4.2: The shaded region is 2/6 of the whole region.

In Figure 4.3, we've shaded 4/12 of the entire region (four of twelve equal regions are shaded).

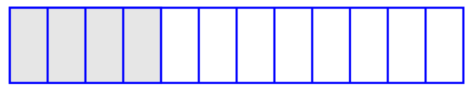


Figure 4.3: The shaded region is 4/12 of the whole region.

Let's take the diagrams from Figure 4.1, Figure 4.2, and Figure 4.3 and stack them one atop the other, as shown in Figure 4.4.

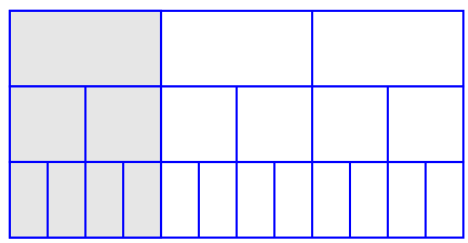


Figure 4.4: One of three equals two of six equals four of twelve.

Figure 4.4 provides solid visual evidence that the following fractions are equivalent.

$$\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$$

Key Observations

1. If we start with the fraction 1/3, then multiply both numerator and denominator by 2, we get the following result.

$$\begin{aligned} \frac{1}{3} &= \frac{1 \cdot 2}{3 \cdot 2} && \text{Multiply numerator and denominator by 2.} \\ &= \frac{2}{6} && \text{Simplify numerator and denominator..} \end{aligned}$$

   This is precisely the same thing that happens going from Figure 4.1 to 4.2, where we double the number of available boxes (going from 3 available to 6 available) and double the number of shaded boxes (going from 1 shaded to 2 shaded).

2. If we start with the fraction 1/3, then multiply both numerator and denominator by 4, we get the following result.

$$\begin{aligned} \frac{1}{3} &= \frac{1 \cdot 4}{3 \cdot 4} && \text{Multiply numerator and denominator by 4.} \\ &= \frac{4}{12} && \text{Simplify numerator and denominator.} \end{aligned}$$

This is precisely the same thing that happens going from Figure 4.1 to 4.3, where we multiply the number of available boxes by 4 (going from 3 available to 12 available) and multiply the number of shaded boxes by 4 (going from 1 shaded to 4 shaded).

The above discussion motivates the following fundamental result.

Creating Equivalent Fractions. If you start with a fraction, then multiply both its numerator and denominator by the same number, the resulting fraction is equivalent (has the same numerical value) to the original fraction. In symbols,

$$\frac{a}{b} = \frac{a \cdot x}{b \cdot x}.$$

Arguing in Reverse. Reversing the above argument also holds true.

1. If we start with the fraction $2/6$, then divide both numerator and denominator by 2, we get the following result.

$$\begin{aligned} \frac{2}{6} &= \frac{2 \div 2}{6 \div 2} && \text{Divide numerator and denominator by 2.} \\ &= \frac{1}{3} && \text{Simplify numerator and denominator.} \end{aligned}$$

   This is precisely the same thing that happens going backwards from Figure 4.2 to 4.1, where we divide the number of available boxes by 2 (going from 6 available to 3 available) and dividing the number of shaded boxes by 2 (going from 2 shaded to 1 shaded).

2. If we start with the fraction $4/12$, then divide both numerator and denominator by 4, we get the following result.

$$\begin{aligned} \frac{4}{12} &= \frac{4 \div 4}{12 \div 4} && \text{Multiply numerator and denominator by 4.} \\ &= \frac{1}{3} && \text{Simplify numerator and denominator.} \end{aligned}$$

   This is precisely the same thing that happens going backwards from Figure 4.3 to 4.1, where we divide the number of available boxes by 4 (going from 12 available to 3 available) and divide the number alignof shaded boxes by 4 (going from 4 shaded to 1 shaded).

The above discussion motivates the following fundamental result.

Creating Equivalent Fractions. If you start with a fraction, then divide both its numerator and denominator by the same number, the resulting fraction is equivalent (has the same numerical value) to the original fraction. In symbols,

$$\frac{a}{b} = \frac{a \div x}{b \div x}.$$

The Greatest Common Divisor

We need a little more terminology.

Divisor. If d and a are natural numbers, we say that "d divides a" if and only if when a is divided by d, the remainder is zero. In this case, we say that "d is a divisor of a."

For example, when 36 is divided by 4, the remainder is zero. In this case, we say that "4 is a divisor of 36." On the other hand, when 25 is divided by 4, the remainder is **not** zero. In this case, we say that "4 is **not** a divisor of 25."

Greatest Common Divisor. Let a and b be natural numbers. The *common divisors* of a and b are those natural numbers that divide both a and b. The *greatest common divisor* is the largest of these common divisors.

You Try It!

Find the greatest common divisor of 12 and 18

EXAMPLE 1. Find the greatest common divisor of 18 and 24.

Solution. First list the divisors of each number, the numbers that divide each number with zero remainder.

$$\begin{aligned} \text{Divisors of } 18 &: \ 1,\ 2,\ 3,\ 6,\ 9,\ \text{and } 18 \\ \text{Divisors of } 24 &: \ 1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ \text{and } 24 \end{aligned}$$

The common divisors are:

$$\text{Common Divisors}:\ 1,\ 2,\ 3,\ \text{and } 6$$

The greatest common divisor is the largest of the common divisors. That is,

$$\text{Greatest Common Divisor} = 6.$$

That is, the largest number that divides both 18 and 24 is the number 6.

Answer: 6

Reducing a Fraction to Lowest Terms

First, a definition.

Lowest Terms. A fraction is said to be *reduced to lowest terms* if the greatest common divisor of both numerator and denominator is 1.

Thus, for example, 2/3 is reduced to lowest terms because the greatest common divisior of 2 and 3 is 1. On the other hand, 4/6 is **not** reduced to lowest terms because the greatest common divisor of 4 and 6 is 2.

You Try It!

EXAMPLE 2. Reduce the fraction 18/24 to lowest terms.

Reduce the fraction 12/18 to lowest terms.

Solution. One technique that works well is dividing both numerator and denominator by the greatest common divisor of the numerator and denominator. In Example 1, we saw that the greatest common divisor of 18 and 24 is 6. We divide both numerator and denominator by 6 to get

$$\begin{aligned} \frac{18}{24} &= \frac{18 \div 6}{24 \div 6} && \text{Divide numerator and denominator by 6.} \\ &= \frac{3}{4} && \text{Simplify numerator and denominator.} \end{aligned}$$

Note that the greatest common divisor of 3 and 4 is now 1. Thus, 3/4 is reduced to lowest terms.

There is a second way we can show division of numerator and denominator by 6. First, factor both numerator and denominator as follows:

$$\frac{18}{24} = \frac{3 \cdot 6}{4 \cdot 6} \qquad \text{Factor out a 6.}$$

You can then show "division" of both numerator and denominator by 6 by "crossing out" or "canceling" a 6 in the numerator for a 6 in the denominator, like this:

$$\begin{aligned} &= \frac{3 \cdot \cancel{6}}{4 \cdot \cancel{6}} && \text{Cancel common factor.} \\ &= \frac{3}{4} \end{aligned}$$

Note that we get the same equivalent fraction, reduced to lowest terms, namely 3/4.

Answer: 2/3

□

Important Point. In Example 2 we saw that 6 was both a *divisor* and a *factor* of 18. The words *divisor* and *factor* are equivalent.

We used the following technique in our second solution in Example 2.

Cancellation Rule. If you express numerator and denominator as a **product**, then you may cancel common factors from the numerator and denominator. The result will be an equivalent fraction.

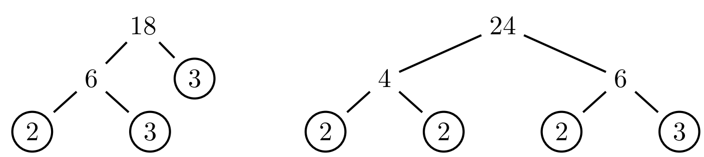
Because of the "Cancellation Rule," one of the most effective ways to reduce a fraction to lowest terms is to first find prime factorizations for both numerator and denominator, then cancel all common factors.

You Try It!

Reduce the fraction 28/35 to lowest terms.

EXAMPLE 3. Reduce the fraction 18/24 to lowest terms.

Solution. Use factor trees to prime factor numerator and denominator.



Once we've factored the numerator and denominator, we cancel common factors.

$$\begin{aligned}
\frac{18}{24} &= \frac{2\cdot 3\cdot 3}{2\cdot 2\cdot 2\cdot 3} && \text{Prime factor numerator and denominator.}\\
&= \frac{\cancel{2}\cdot \cancel{3}\cdot 3}{\cancel{2}\cdot 2\cdot 2\cdot \cancel{3}} && \text{Cancel common factors.}\\
&= \frac{3}{2\cdot 2} && \text{Remaining factors.}\\
&= \frac{3}{4} && \text{Simplify denominator.}
\end{aligned}$$

Thus, $18/24 = 3/4$.

Answer: 4/5

You Try It!

Reduce the fraction 36/60 to lowest terms.

EXAMPLE 4. Reduce the fraction 28/42 to lowest terms.

Solution. Use factor trees to prime factor numerator and denominator.

Now we can cancel common factors.

$$\begin{aligned}
\frac{28}{42} &= \frac{2\cdot 2\cdot 7}{2\cdot 3\cdot 7} && \text{Prime factor numerator and denominator.}\\
&= \frac{\cancel{2}\cdot 2\cdot \cancel{7}}{\cancel{2}\cdot 3\cdot \cancel{7}} && \text{Cancel common factors.}\\
&= \frac{2}{3}
\end{aligned}$$

Thus, $28/42 = 2/3$.

Answer: $3/5$

□

Reducing Fractions with Variables

We use exactly the same technique to reduce fractions whose numerators and denominators contain variables.

You Try It!

EXAMPLE 5. Reduce

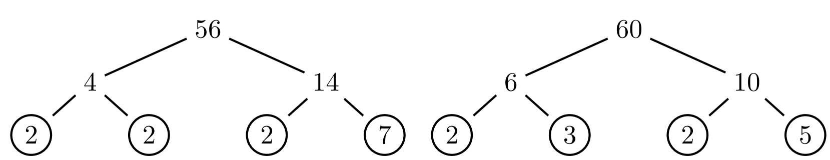
$$\frac{56x^2y}{60xy^2}$$

to lowest terms.

Reduce:

$$\frac{25a^3b}{40a^2b^3}$$

Solution. Use factor trees to factor the coefficients of numerator and denominator.



Now cancel common factors.

$$\begin{aligned}
\frac{56x^2y}{60xy^2} &= \frac{2\cdot 2\cdot 2\cdot 7\cdot x\cdot x\cdot y}{2\cdot 2\cdot 3\cdot 5\cdot x\cdot y\cdot y} && \text{Prime factor numerator and denominator.}\\
&= \frac{\cancel{2}\cdot \cancel{2}\cdot 2\cdot 7\cdot \cancel{x}\cdot x\cdot \cancel{y}}{\cancel{2}\cdot \cancel{2}\cdot 3\cdot 5\cdot \cancel{x}\cdot y\cdot \cancel{y}} && \text{Cancel common factors.}\\
&= \frac{2\cdot 7\cdot x}{3\cdot 5\cdot y} && \text{Remaining factors.}\\
&= \frac{14x}{15y} && \text{Simplify numerator and denominator.}
\end{aligned}$$

Thus, $56x^2y/(60xy^2) = 14x/(15y)$.

Answer: $\dfrac{5a}{8b^2}$

□

A Word on Mathematical Notation.

There are two types of mathematical notation: (1) inline mathematical notation, and (2) displayed mathematical notation.

Inline Mathematical Notation. The notation $14x/(15y)$ is called *inline mathematical notation*. When the same expression is centered on its own line, as in

$$\frac{14x}{15y},$$

this type of notation is called *displayed mathematical notation.*

When you work a problem by hand, using pencil and paper calculations, the preferred format is displayed notation, like the displayed notation used to simplify the given expression in Example 5. However, computers and calculators require that you enter your expressions using inline mathematical notation. Therefore, it is extremely important that you are equally competent with either mathematical notation: displayed or inline.

By the way, order of operations, when applied to the inline expression $14x/(15y)$, requires that we perform the multiplication inside the parentheses first. Then we must perform multiplications and divisions as they occur, as we move from left to right through the expression. This is why the inline notation $14x/(15y)$ is equivalent to the displayed notation

$$\frac{14x}{15y}.$$

However, the expression $14x/15y$ is a different beast. There are no parentheses, so we perform multiplication and division as they occur, moving left to right through the expression. Thus, we must first take the product of 14 and x, divide the result by 15, then multiply by y. In displayed notation, this result is equivalent to

$$\frac{14x}{15} \cdot y,$$

which is a different result.

Some readers might wonder why we did not use the notation $(14x)/(15y)$ to describe the solution in Example 5. After all, this inline notation is also equivalent to the displayed notation

$$\frac{14x}{15y}.$$

However, the point is that we don't need to, as order of operations already requires that we take the product of 14 and x before dividing by $15y$. If this is hurting your head, know that it's quite acceptable to use the equivalent notation $(14x)/(15y)$ instead of $14x/(15y)$. Both are correct.

Equivalent Fractions in Higher Terms

Sometimes the need arises to find an equivalent fraction with a different, larger denominator.

You Try It!

EXAMPLE 6. Express 3/5 as an equivalent fraction having denominator 20.

Solution. The key here is to remember that multiplying numerator and denominator by the same number produces an equivalent fraction. To get an equivalent fraction with a denominator of 20, we'll have to multiply numerator and denominator of 3/5 by 4.

$$\begin{aligned} \frac{3}{5} &= \frac{3 \cdot 4}{5 \cdot 4} && \text{Multiply numerator and denominator by 4.} \\ &= \frac{12}{20} && \text{Simplify numerator and denominator.} \end{aligned}$$

Therefore, 3/5 equals 12/20.

Express 2/3 as an equivalent fraction having denominator 21.

Answer: 14/21

You Try It!

EXAMPLE 7. Express 8 as an equivalent fraction having denominator 5.

Solution. The key here is to note that

$$8 = \frac{8}{1} \qquad \text{Understood denominator is 1.}$$

To get an equivalent fraction with a denominator of 5, we'll have to multiply numerator and denominator of 8/1 by 5.

$$\begin{aligned} &= \frac{8 \cdot 5}{1 \cdot 5} && \text{Multiply numerator and denominator by 5.} \\ &= \frac{40}{5} && \text{Simplify numerator and denominator.} \end{aligned}$$

Therefore, 8 equals 40/5.

Express 5 as an equivalent fraction having denominator 7.

Answer: 35/7

You Try It!

Express 3/8 as an equivalent fraction having denominator $24a$.

EXAMPLE 8. Express 2/9 as an equivalent fraction having denominator $18a$.

Solution. To get an equivalent fraction with a denominator of $18a$, we'll have to multiply numerator and denominator of 2/9 by $2a$.

$$\begin{aligned} \frac{2}{9} &= \frac{2 \cdot 2a}{9 \cdot 2a} && \text{Multiply numerator and denominator by } 2a. \\ &= \frac{4a}{18a} && \text{Simplify numerator and denominator.} \end{aligned}$$

Therefore, 2/9 equals $4a/(18a)$, or equivalently, $(4a)/(18a)$.

Answer: $\dfrac{9a}{24a}$

□

Negative Fractions

We have to also deal with fractions that are negative. First, let's discuss placement of the negative sign.

- Positive divided by negative is negative, so

$$\frac{3}{-5} = -\frac{3}{5}.$$

- But it is also true that negative divided by positive is negative. Thus,

$$\frac{-3}{5} = -\frac{3}{5}.$$

These two observations imply that all three of the following fractions are equivalent (the same number):

$$\frac{3}{-5} = -\frac{3}{5} = \frac{-3}{5}.$$

Note that there are three possible placements for the negative sign: (1) the denominator, (2) the fraction bar, or (3) the numerator. Any one of these placements produces an equivalent fraction.

Fractions and Negative Signs. Let a and b be any integers. All three of the following fractions are equivalent (same number):

$$\frac{a}{-b} = -\frac{a}{b} = \frac{-a}{b}.$$

Mathematicians prefer to place the negative sign either in the numerator or on the fraction bar. The use of a negative sign in the denominator is discouraged.

You Try It!

EXAMPLE 9. Reduce:

$$\frac{50x^3}{-75x^5}$$

to lowest terms.

Reduce:

$$\frac{14y^5}{-35y^3}$$

Solution. Prime factor numerator and denominator and cancel.

$$\begin{aligned}\frac{50x^3}{-75x^5} &= \frac{2 \cdot 5 \cdot 5 \cdot x \cdot x \cdot x}{-3 \cdot 5 \cdot 5 \cdot x \cdot x \cdot x \cdot x \cdot x} \\ &= \frac{2 \cdot \cancel{5} \cdot \cancel{5} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{-3 \cdot \cancel{5} \cdot \cancel{5} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x} \\ &= \frac{2}{-3 \cdot x \cdot x} \\ &= \frac{2}{-3x^2}\end{aligned}$$

However, it is preferred that there be no negative signs in the denominator, so let's place the negative sign on the fraction bar (the numerator would suit as well). Thus,

$$\frac{50x^3}{-75x^5} = -\frac{2}{3x^2}$$

Answer: $-\dfrac{2y^2}{5}$

□

We also have the following result.

Fractions and Negative Signs. Let a and b be any integers. Then,

$$\frac{-a}{-b} = \frac{a}{b}.$$

You Try It!

EXAMPLE 10. Reduce:

$$\frac{-12xy^2}{-18x^2y}$$

Reduce:

$$\frac{-21a^2b^3}{-56a^3b}$$

Solution. Unlike Example 9, some like to take care of the sign of the answer first.

$$\frac{-12xy^2}{-18x^2y} = \frac{12xy^2}{18x^2y}$$

Now we can factor numerator and denominator and cancel common factors.

$$\begin{aligned}
&= \frac{2 \cdot 2 \cdot 3 \cdot x \cdot y \cdot y}{2 \cdot 3 \cdot 3 \cdot x \cdot x \cdot y} \\
&= \frac{\cancel{2} \cdot 2 \cdot \cancel{3} \cdot \cancel{x} \cdot y \cdot \cancel{y}}{\cancel{2} \cdot \cancel{3} \cdot 3 \cdot \cancel{x} \cdot x \cdot \cancel{y}} \\
&= \frac{2y}{3x}
\end{aligned}$$

Thus,

$$\frac{-12xy^2}{-18x^2y} = \frac{2y}{3x}.$$

Answer: $\dfrac{3b^2}{8a}$

$\square$

Exercises

In Exercises 1-12, find the GCD of the given numbers.

1. 72, 8

2. 76, 52

3. 52, 20

4. 56, 96

5. 36, 63

6. 63, 21

7. 72, 44

8. 10, 40

9. 16, 56

10. 54, 66

11. 84, 24

12. 75, 45

In Exercises 13-28, reduce the given fraction to lowest terms.

13. $\dfrac{22}{98}$

14. $\dfrac{28}{56}$

15. $\dfrac{93}{15}$

16. $\dfrac{90}{39}$

17. $\dfrac{69}{21}$

18. $\dfrac{74}{62}$

19. $\dfrac{74}{12}$

20. $\dfrac{66}{10}$

21. $\dfrac{66}{57}$

22. $\dfrac{34}{30}$

23. $\dfrac{33}{99}$

24. $\dfrac{20}{58}$

25. $\dfrac{69}{24}$

26. $\dfrac{18}{96}$

27. $\dfrac{46}{44}$

28. $\dfrac{92}{24}$

29. Express 3 as an equivalent fraction having denominator 24.

30. Express 3 as an equivalent fraction having denominator 8.

31. Express $\dfrac{25}{19}$ as an equivalent fraction having denominator 57.

32. Express $\dfrac{29}{22}$ as an equivalent fraction having denominator 44.

33. Express 2 as an equivalent fraction having denominator 2.

34. Express 2 as an equivalent fraction having denominator 8.

35. Express $\frac{18}{19}$ as an equivalent fraction having denominator 95.

36. Express $\frac{17}{22}$ as an equivalent fraction having denominator 44.

37. Express $\frac{1}{3}$ as an equivalent fraction having denominator 24.

38. Express $\frac{15}{19}$ as an equivalent fraction having denominator 95.

39. Express 16 as an equivalent fraction having denominator 4.

40. Express 5 as an equivalent fraction having denominator 2.

In Exercises 41-56, reduce the given fraction to lowest terms.

41. $\frac{34}{-86}$

42. $\frac{-48}{14}$

43. $\frac{-72}{-92}$

44. $\frac{27}{-75}$

45. $\frac{-92}{82}$

46. $\frac{-44}{-62}$

47. $\frac{-21}{33}$

48. $\frac{57}{-99}$

49. $\frac{22}{-98}$

50. $\frac{-33}{69}$

51. $\frac{42}{-88}$

52. $\frac{-100}{48}$

53. $\frac{94}{-6}$

54. $\frac{-36}{-38}$

55. $\frac{10}{-86}$

56. $\frac{-100}{-46}$

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57. Express $\frac{3}{2}$ as an equivalent fraction having denominator $62n$.

58. Express $\frac{6}{25}$ as an equivalent fraction having denominator $50a$.

59. Express $\frac{13}{10}$ as an equivalent fraction having denominator $60m$.

60. Express $\frac{1}{16}$ as an equivalent fraction having denominator $80p$.

61. Express $\frac{3}{2}$ as an equivalent fraction having denominator $50n$.

62. Express $\frac{43}{38}$ as an equivalent fraction having denominator $76a$.

63. Express 11 as an equivalent fraction having denominator $4m$.

64. Express 13 as an equivalent fraction having denominator $6n$.

65. Express 3 as an equivalent fraction having denominator $10m$.

66. Express 10 as an equivalent fraction having denominator $8b$.

67. Express 6 as an equivalent fraction having denominator $5n$.

68. Express 16 as an equivalent fraction having denominator $2y$.

In Exercises 69-84, reduce the given fraction to lowest terms.

69. $\dfrac{82y^5}{-48y}$

70. $\dfrac{-40y^5}{-55y}$

71. $\dfrac{-77x^5}{44x^4}$

72. $\dfrac{-34x^6}{-80x}$

73. $\dfrac{-14y^5}{54y^2}$

74. $\dfrac{96y^4}{-40y^2}$

75. $\dfrac{42x}{81x^3}$

76. $\dfrac{26x^2}{32x^6}$

77. $\dfrac{-12x^5}{14x^6}$

78. $\dfrac{-28y^4}{72y^6}$

79. $\dfrac{-74x}{22x^2}$

80. $\dfrac{56x^2}{26x^3}$

81. $\dfrac{-12y^5}{98y^6}$

82. $\dfrac{96x^2}{14x^4}$

83. $\dfrac{18x^6}{-54x^2}$

84. $\dfrac{32x^6}{62x^2}$

In Exercises 85-100, reduce the given fraction to lowest terms.

85. $\dfrac{26y^2x^4}{-62y^6x^2}$

86. $\dfrac{6x^2y^3}{40x^3y^2}$

87. $\dfrac{-2y^6x^4}{-94y^2x^5}$

88. $\dfrac{90y^6x^3}{39y^3x^5}$

89. $\dfrac{30y^5x^5}{-26yx^4}$

90. $\dfrac{74x^6y^4}{-52xy^3}$

91. $\dfrac{36x^3y^2}{-98x^4y^5}$

92. $\dfrac{84x^3y}{16x^4y^2}$

93. $\dfrac{-8x^6y^3}{54x^3y^5}$

94. $\dfrac{70y^5x^2}{16y^4x^5}$

95. $\dfrac{34yx^6}{-58y^5x^4}$

96. $\dfrac{99y^2x^3}{88y^6x}$

97. $\dfrac{-36y^3x^5}{51y^2x}$

98. $\dfrac{44y^5x^5}{-88y^4x}$

99. $\dfrac{91y^3x^2}{-28y^5x^5}$

100. $\dfrac{-76y^2x}{-57y^5x^6}$

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101. **Hurricanes.** According to the National Atmospheric and Oceanic Administration, in 2008 there were 16 named storms, of which 8 grew into hurricanes and 5 were major.

i) What fraction of named storms grew into hurricanes? Reduce your answer to lowest terms.

ii) What fraction of named storms were major hurricanes? Reduce your answer to lowest terms.

iii) What fraction of hurricanes were major? Reduce your answer to lowest terms.

102. **Tigers.** Tigers are in critical decline because of human encroachment, the loss of more than nine-tenths of their habitat, and the growing trade in tiger skins and body parts. *Associated Press-Times-Standard 01/24/10 Pressure mounts to save the tiger.*

i) Write the loss of habitat as a fraction.

ii) Describe in words what the numerator and denominator of this fraction represent.

iii) If the fraction represents the loss of the whole original habitat, how much of the original habitat remains?

❧ ❧ ❧ Answers ☙ ☙ ☙

1. 8

3. 4

5. 9

7. 4

9. 8

11. 12

13. $\dfrac{11}{49}$

15. $\dfrac{31}{5}$

17. $\dfrac{23}{7}$

19. $\dfrac{37}{6}$

21. $\dfrac{22}{19}$

23. $\frac{1}{3}$

25. $\frac{23}{8}$

27. $\frac{23}{22}$

29. $\frac{72}{24}$

31. $\frac{75}{57}$

33. $\frac{4}{2}$

35. $\frac{90}{95}$

37. $\frac{8}{24}$

39. $\frac{64}{4}$

41. $-\frac{17}{43}$

43. $\frac{18}{23}$

45. $-\frac{46}{41}$

47. $-\frac{7}{11}$

49. $-\frac{11}{49}$

51. $-\frac{21}{44}$

53. $-\frac{47}{3}$

55. $-\frac{5}{43}$

57. $\frac{93n}{62n}$

59. $\frac{78m}{60m}$

61. $\frac{75n}{50n}$

63. $\frac{44m}{4m}$

65. $\frac{30m}{10m}$

67. $\frac{30n}{5n}$

69. $-\frac{41y^4}{24}$

71. $-\frac{7x}{4}$

73. $-\frac{7y^3}{27}$

75. $\frac{14}{27x^2}$

77. $-\frac{6}{7x}$

79. $-\frac{37}{11x}$

81. $-\frac{6}{49y}$

83. $-\frac{x^4}{3}$

85. $-\frac{13x^2}{31y^4}$

87. $\frac{y^4}{47x}$

89. $-\frac{15y^4x}{13}$

91. $-\dfrac{18}{49xy^3}$

93. $-\dfrac{4x^3}{27y^2}$

95. $-\dfrac{17x^2}{29y^4}$

97. $-\dfrac{12yx^4}{17}$

99. $-\dfrac{13}{4y^2x^3}$

101. i) $\dfrac{1}{2}$

ii) $\dfrac{5}{16}$

iii) $\dfrac{5}{8}$

4.2 Multiplying Fractions

Consider the image in Figure 4.5, where the vertical lines divide the rectangular region into three equal pieces. If we shade one of the three equal pieces, the shaded area represents 1/3 of the whole rectangular region.

Figure 4.5: The shaded region is 1/3 of the whole region.

We'd like to visualize taking 1/2 of 1/3. To do that, we draw an additional horizontal line which divides the shaded region in half horizontally. This is shown in Figure 4.6. The shaded region that represented 1/3 is now divided into two smaller rectangular regions, one of which is shaded with a different color. This region represents 1/2 of 1/3.

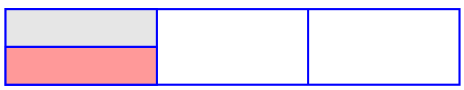


Figure 4.6: Shading 1/2 of 1/3.

Next, extend the horizontal line the full width of the rectangular region, as shown in Figure 4.7.

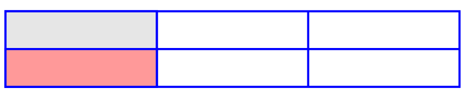


Figure 4.7: Shading 1/2 of 1/3.

Note that drawing the horizontal line, coupled with the three original vertical lines, has succeeded in dividing the full rectangular region into six smaller but equal pieces, only one of which (the one representing 1/2 of 1/3) is shaded in a new color. Hence, this newly shaded piece represents 1/6 of the whole region. The conclusion of our visual argument is the fact that 1/2 of 1/3 equals 1/6. In symbols,

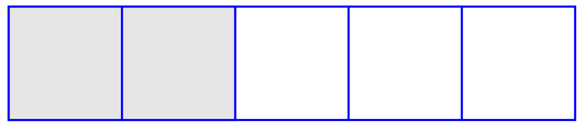
$$\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}.$$

You Try It!

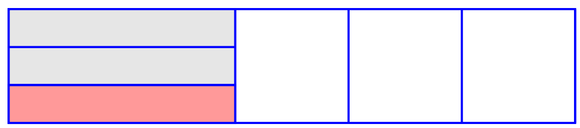
EXAMPLE 1. Create a visual argument showing that 1/3 of 2/5 is 2/15.

Create a visual argument showing that 1/2 of 1/4 is 1/8.

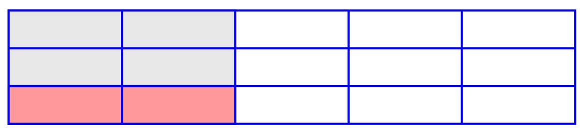
Solution. First, divide a rectangular region into five equal pieces and shade two of them. This represents the fraction 2/5.



Next, draw two horizontal lines that divide the shaded region into three equal pieces and shade 1 of the three equal pieces. This represents taking 1/3 of 2/5.



Next, extend the horizontal lines the full width of the region and return the original vertical line from the first image.



Note that the three horizontal lines, coupled with the five original vertical lines, have succeeded in dividing the whole region into 15 smaller but equal pieces, only two of which (the ones representing 1/3 of 2/5) are shaded in the new color. Hence, this newly shaded piece represents 2/15 of the whole region. The conclusion of this visual argument is the fact that 1/3 of 2/5 equals 2/15. In symbols,

$$\frac{1}{3} \cdot \frac{2}{5} = \frac{2}{15}.$$

Answer: 

□

Multiplication Rule

In Figure 4.7, we saw that 1/2 of 1/3 equals 1/6. Note what happens when we multiply the numerators and multiply the denominators of the fractions 1/2 and 1/3.

$$\begin{aligned} \frac{1}{2} \cdot \frac{1}{3} &= \frac{1 \cdot 1}{2 \cdot 3} && \text{Multiply numerators; multiply denominators.} \\ &= \frac{1}{6} && \text{Simplify numerators and denominators.} \end{aligned}$$

We get 1/6!

Could this be coincidence or luck? Let's try that again with the fractions from Example 1, where we saw that 1/3 of 2/5 equals 2/15. Again, multiply the numerators and denominators of 1/3 and 2/5.

$$\begin{aligned} \frac{1}{3} \cdot \frac{2}{5} &= \frac{1 \cdot 2}{3 \cdot 5} && \text{Multiply numerators; multiply denominators.} \\ &= \frac{2}{15} && \text{Simplify numerators and denominators.} \end{aligned}$$

Again, we get 2/15!

These two examples motivate the following definition.

Multiplication Rule. To find the product of the fractions a/b and c/d, multiply their numerators and denominators. In symbols,

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

You Try It!

EXAMPLE 2. Multiply 1/5 and 7/9.

Multiply:

$$\frac{1}{3} \cdot \frac{2}{5}$$

Solution. Multiply numerators and multiply denominators.

$$\begin{aligned} \frac{1}{5} \cdot \frac{7}{9} &= \frac{1 \cdot 7}{5 \cdot 9} && \text{Multiply numerators; multiply denominators.} \\ &= \frac{7}{45} && \text{Simplify numerators and denominators.} \end{aligned}$$

Multiply: $\frac{6}{15}$

□

You Try It!

EXAMPLE 3. Find the product of $-2/3$ and $7/9$.

Multiply:

$$-\frac{3}{5} \cdot \frac{2}{7}$$

Solution. The usual rules of signs apply to products. Unlike signs yield a negative result.

$$\begin{aligned} -\frac{2}{3} \cdot \frac{7}{9} &= -\frac{2 \cdot 7}{3 \cdot 9} && \text{Multiply numerators; multiply denominators.} \\ &= -\frac{14}{27} && \text{Simplify numerators and denominators.} \end{aligned}$$

It is not required that you physically show the middle step. If you want to do that mentally, then you can simply write

$$-\frac{2}{3} \cdot \frac{7}{9} = -\frac{14}{27}.$$

Answer: $-\frac{6}{35}$

□

Multiply and Reduce

After multiplying two fractions, make sure your answer is reduced to lowest terms (see Section 4.1).

You Try It!

Multiply:

$$\frac{3}{7} \cdot \frac{14}{9}$$

EXAMPLE 4. Multiply 3/4 times 8/9.

Solution. After multiplying, divide numerator and denominator by the greatest common divisor of the numerator and denominator.

$$\begin{aligned}
\frac{3}{4} \cdot \frac{8}{9} &= \frac{3 \cdot 8}{4 \cdot 9} && \text{Multiply numerators and denominators.} \\
&= \frac{24}{36} && \text{Simplify numerator and denominator.} \\
&= \frac{24 \div 12}{36 \div 12} && \text{Divide numerator and denominator by GCD.} \\
&= \frac{2}{3} && \text{Simplify numerator and denominator.}
\end{aligned}$$

Alternatively, after multiplying, you can prime factor both numerator and denominator, then cancel common factors.

$$\begin{aligned}
\frac{3}{4} \cdot \frac{8}{9} &= \frac{24}{36} && \text{Multiply numerators and denominators.} \\
&= \frac{2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 3 \cdot 3} && \text{Prime factor numerator and denominator.} \\
&= \frac{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{3}}{\cancel{2} \cdot \cancel{2} \cdot 3 \cdot \cancel{3}} && \text{Cancel common factors.} \\
&= \frac{2}{3}
\end{aligned}$$

Answer: $\frac{2}{3}$

□

You Try It!

Multiply:

$$-\frac{3x}{2} \cdot \frac{6}{21x^3}$$

EXAMPLE 5. Multiply $-7x/2$ and $5/(14x^2)$.

Solution. After multiplying, prime factor both numerator and denominator, then cancel common factors. Note that unlike signs yields a negative product.

$$\begin{aligned}
-\frac{7x}{2} \cdot \frac{5}{14x^2} &= -\frac{35x}{28x^2} && \text{Multiply numerators and denominators.} \\
&= -\frac{5 \cdot 7 \cdot x}{2 \cdot 2 \cdot 7 \cdot x \cdot x} && \text{Prime factor numerator and denominator.} \\
&= -\frac{5 \cdot \cancel{7} \cdot \cancel{x}}{2 \cdot 2 \cdot \cancel{7} \cdot \cancel{x} \cdot x} && \text{Cancel common factors.} \\
&= -\frac{5}{4x}
\end{aligned}$$

Answer: $-\frac{3}{7x^2}$

□

Multiply and Cancel or Cancel and Multiply

When you are working with larger numbers, it becomes a bit harder to multiply, factor, and cancel. Consider the following argument.

$$\begin{aligned}
\frac{18}{30}\cdot\frac{35}{6} &= \frac{630}{180} && \text{Multiply numerators; multiply denominators.}\\
&= \frac{2\cdot 3\cdot 3\cdot 5\cdot 7}{2\cdot 2\cdot 3\cdot 3\cdot 5} && \text{Prime factor numerators and denominators.}\\
&= \frac{\cancel{2}\cdot\cancel{3}\cdot\cancel{3}\cdot\cancel{5}\cdot 7}{2\cdot\cancel{2}\cdot\cancel{3}\cdot\cancel{3}\cdot\cancel{5}} && \text{Cancel common factors.}\\
&= \frac{7}{2} && \text{Remaining factors.}
\end{aligned}$$

There are a number of difficulties with this approach. First, you have to multiply large numbers, and secondly, you have to prime factor the even larger results.

One possible workaround is to not bother multiplying numerators and denominators, leaving them in factored form.

$$\frac{18}{30}\cdot\frac{35}{6} = \frac{18\cdot 35}{30\cdot 6} \qquad \text{Multiply numerators; multiply denominators.}$$

Finding the prime factorization of these smaller factors is easier.

$$= \frac{(2\cdot 3\cdot 3)\cdot(5\cdot 7)}{(2\cdot 3\cdot 5)\cdot(2\cdot 3)} \qquad \text{Prime factor.}$$

Now we can cancel common factors. Parentheses are no longer needed in the numerator and denominator because both contain a product of prime factors, so order and grouping do not matter.

$$\begin{aligned}
&= \frac{\cancel{2}\cdot\cancel{3}\cdot\cancel{3}\cdot\cancel{5}\cdot 7}{\cancel{2}\cdot\cancel{3}\cdot\cancel{5}\cdot 2\cdot\cancel{3}} && \text{Cancel common factors.}\\
&= \frac{7}{2} && \text{Remaining factors.}
\end{aligned}$$

Another approach is to factor numerators and denominators in place, cancel common factors, then multiply.

$$\begin{aligned}
\frac{18}{30}\cdot\frac{35}{6} &= \frac{2\cdot 3\cdot 3}{2\cdot 3\cdot 5}\cdot\frac{5\cdot 7}{2\cdot 3} && \text{Factor numerators and denominators.}\\
&= \frac{\cancel{2}\cdot\cancel{3}\cdot\cancel{3}}{\cancel{2}\cdot\cancel{3}\cdot\cancel{5}}\cdot\frac{\cancel{5}\cdot 7}{2\cdot\cancel{3}} && \text{Cancel common factors.}\\
&= \frac{7}{2} && \text{Remaining factors.}
\end{aligned}$$

Note that this yields exactly the same result, 7/2.

Cancellation Rule. When multiplying fractions, cancel common factors according to the following rule: "Cancel a factor in a numerator for an identical factor in a denominator."

You Try It!

Multiply:

$$\frac{6}{35} \cdot \frac{70}{36}$$

EXAMPLE 6. Find the product of 14/15 and 30/140.

Solution. Multiply numerators and multiply denominators. Prime factor, cancel common factors, then multiply.

$$\begin{aligned} \frac{14}{15} \cdot \frac{30}{140} &= \frac{14 \cdot 30}{15 \cdot 140} && \text{Multiply numerators; multiply denominators.} \\ &= \frac{(2 \cdot 7) \cdot (2 \cdot 3 \cdot 5)}{(3 \cdot 5) \cdot (2 \cdot 2 \cdot 5 \cdot 7)} && \text{Prime factor numerators and denominators.} \\ &= \frac{\cancel{2} \cdot \cancel{7} \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{5}}{\cancel{3} \cdot 5 \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{5} \cdot \cancel{7}} && \text{Cancel common factors.} \\ &= \frac{1}{5} && \text{Multiply.} \end{aligned}$$

Note: Everything in the numerator cancels because you've divided the numerator by itself. Hence, the answer has a 1 in its numerator.

Answer: $\dfrac{1}{3}$

□

When Everything Cancels. When all the factors in the numerator cancel, this means that you are dividing the numerator by itself. Hence, you are left with a 1 in the numerator. The same rule applies to the denominator. If everything in the denominator cancels, you're left with a 1 in the denominator.

You Try It!

Simplify:

$$\frac{6a}{15b} \cdot \left(-\frac{35b^2}{10a^2}\right)$$

EXAMPLE 7. Simplify the product: $-\dfrac{6x}{55y} \cdot \left(-\dfrac{110y^2}{105x^2}\right)$.

Solution. The product of two negatives is positive.

$$-\frac{6x}{55y} \cdot \left(-\frac{110y^2}{105x^2}\right) = \frac{6x}{55y} \cdot \frac{110y^2}{105x^2} \qquad \text{Like signs gives a positive.}$$

Prime factor numerators and denominators, then cancel common factors.

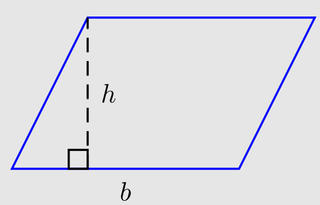
$$
\begin{aligned}
&= \frac{2\cdot 3\cdot x}{5\cdot 11\cdot y}\cdot\frac{2\cdot 5\cdot 11\cdot y\cdot y}{3\cdot 5\cdot 7\cdot x\cdot x} && \text{Prime factor numerators \& denominators.}\\
&= \frac{2\cdot \cancel{3}\cdot \cancel{x}}{5\cdot \cancel{11}\cdot \cancel{y}}\cdot\frac{2\cdot \cancel{5}\cdot \cancel{11}\cdot \cancel{y}\cdot y}{\cancel{3}\cdot \cancel{5}\cdot 7\cdot \cancel{x}\cdot x} && \text{Cancel common factors.}\\
&= \frac{2\cdot 2\cdot y}{5\cdot 7\cdot x} && \text{Remaining factors}\\
&= \frac{4y}{35x} && \text{Multiply numerators; multiply denominators.}
\end{aligned}
$$

Answer: $-\dfrac{21b}{5a}$

Parallelograms

In this section, we are going to learn how to find the area of a *parallelogram.* Let's begin with the definition of a parallelogram. Recall that a quadrilateral is a polygon having four sides. A parallelogram is a very special type of quadrilateral.

Parallelogram. A parallelogram is a quadrilateral whose opposite sides are parallel.



The side on which the parallelogram rests is called its *base* (labeled b in the figure) and the distance from its base to the opposite side is called its *height* (labeled h in the figure). Note that the altitude is *perpendicular* to the base (meets the base at a 90° angle).

Figure 4.8 shows a *rectangle* having length b and width h. Therefore, the area of the rectangle in Figure 4.8 is $A = bh$, which is found by taking the product of the length and width. Take a pair of scissors and cut a triangle from the right end of the rectangle as shown in Figure 4.9(a), then paste the cut triangle to the left end as shown in Figure 4.9(b). The result, seen in Figure 4.9(b) is a parallelogram having base b and height h.

Because we've *thrown no material away* in creating the parallelogram from the rectangle, the parallelogram has the same area as the original rectangle. That is, the area of the parallelogram is $A = bh$.

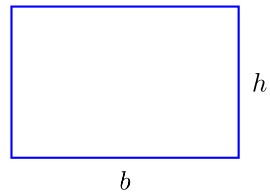
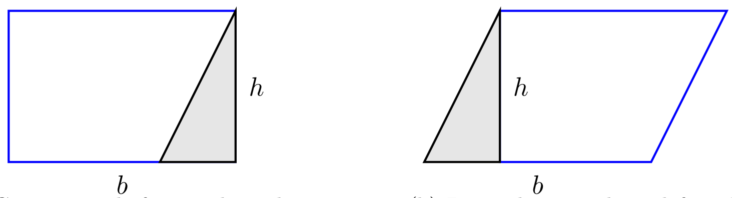


Figure 4.8: The area of the rectangle is $A = bh$.



(a) Cut a triangle from right end. (b) Paste the triangle on left end.

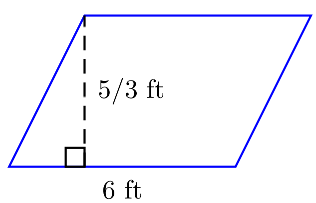
Figure 4.9: Creating a parallelogram from a rectangle.

Area of a Parallelogram. A parallelogram having base b and height h has area $A = bh$. That is, to find the area of a parallelogram, take the product of its base and height.

You Try It!

The base of a parallelogram measures 14 inches. The height is 8/7 of an inch. What is the area of the parallelogram?

EXAMPLE 8. Find the area of the parallelogram pictured below.



Solution. The area of the parallelogram is equal to the product of its base and height. That is,

$$
\begin{aligned}
A &= bh && \text{Area formula for parallelogram.} \\
&= (6\,\text{ft})\left(\frac{5}{3}\,\text{ft}\right) && \text{Substitute: } 6\,\text{ft for } b,\ 5/3\,\text{ft for } h. \\
&= \frac{30}{3}\,\text{ft}^2. && \text{Multiply numerators and denominators.} \\
&= 10\,\text{ft}^2. && \text{Divide.}
\end{aligned}
$$

Thus, the area of the parallelogram is 10 square feet.

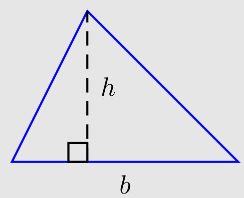
Answer: 16 square inches

□

Triangles

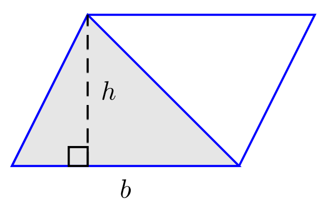
Let's turn our attention to learning how to find the area of a *triangle*.

Triangle. A triangle is a three-sided polygon. It is formed by plotting three points and connecting them with three line segments. Each of the three points is called a *vertex* of the triangle and each of the three line segments is called a *side* of the triangle.



The side on which the triangle rests is called its *base*, and the distance between its base and opposite vertex is called its *height* of *altitude*. The altitude is always *perpendicular* to the base; that is, it forms a 90° angle with the base.

It's easily seen that a triangle has half the area of a parallelogram.



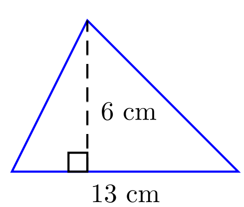
The parallelogram has area $A = bh$. Therefore, the triangle has one-half that area. That is, the area of the triangle is $A = (1/2)bh$.

Area of a Triangle. A triangle having base b and height h has area $A = (1/2)bh$. That is, to find the area of a triangle, take one-half the product of the base and height.

You Try It!

The base of a triangle measures 15 meters. The height is 12 meters. What is the area of the triangle?

EXAMPLE 9. Find the area of the triangle pictured below.



Solution. To find the area of the triangle, take one-half the product of the base and height.

$$\begin{aligned} A &= \frac{1}{2}bh && \text{Area of a triangle formula.} \\ &= \frac{1}{2}(13\,\text{cm})(6\,\text{cm}) && \text{Substitute: } 13\,\text{cm for } b,\ 6\,\text{cm for } h. \\ &= \frac{78\,\text{cm}^2}{2} && \text{Multiply numerators; multiply denominators.} \\ &= 39\,\text{cm}^2. && \text{Simplify.} \end{aligned}$$

Answer: 90 square meters

Therefore, the area of the triangle is 39 square centimeters.

Identifying the Base and Altitude. Sometimes it can be a bit difficult to determine the base and altitude (height) of a triangle. For example, consider the triangle in Figure 4.10(a). Let's say we choose the bottom edge of the triangle as the base and denote its length with the variable b, as shown in Figure 4.10(a).

(a) The base is easy.

(b) Adding the altitude (height).

Figure 4.10: Identifying the base and altitude (height) of a triangle.

The altitude (height) of the triangle is defined as the distance between the base of the triangle and its opposite vertex. To identify this altitude, we must first extend the base, as seen in the dashed extension in Figure 4.10(b), then drop a perpendicular dashed line from the opposite vertex to the extended base, also shown in Figure 4.10(b). This perpendicular is the altitude (height) of the triangle and we denote its length by h.

But we can go further. Any of the three sides of a triangle may be designated as the base of the triangle. Suppose, as shown in Figure 4.11(a), we identify a different side as the base, with length denoted by the variable b.

The altitude to this new base will be a segment from the opposite vertex, perpendicular to the base. Its length in Figure 4.11(b) is denoted by h.

In like manner, there is a third side of the triangle that could also be used as the base. The altitude to this third side is found by dropping a perpendicular from the vertex of the triangle directly opposite from this base. This would also require extending the base. We leave this to our readers to explore.

(a) A different base. (b) Adding the altitude (height).

Figure 4.11: Identifying the base and altitude (height) of a triangle.

Key Point. Any of the three sides of a triangle may be used as the base. The altitude is drawn by dropping a perpendicular from the opposite vertex to the chosen base. This sometimes requires that we extend the base. Regardless of which side we use for the base, the formula $A = bh/2$ will produce the same area result.

Exercises

1. Create a diagram, such as that shown in Figure 4.7, to show that 1/3 of 1/3 is 1/9.

2. Create a diagram, such as that shown in Figure 4.7, to show that 1/2 of 1/4 is 1/8.

3. Create a diagram, such as that shown in Figure 4.7, to show that 1/3 of 1/4 is 1/12.

4. Create a diagram, such as that shown in Figure 4.7, to show that 2/3 of 1/3 is 2/9.

In Exercises 1-28, multiply the fractions, and simplify your result.

5. $\frac{-21}{4} \cdot \frac{22}{19}$

6. $\frac{-4}{19} \cdot \frac{21}{8}$

7. $\frac{20}{11} \cdot \frac{-17}{22}$

8. $\frac{-9}{2} \cdot \frac{6}{7}$

9. $\frac{21}{8} \cdot \frac{-14}{15}$

10. $\frac{-17}{18} \cdot \frac{-3}{4}$

11. $\frac{-5}{11} \cdot \frac{7}{20}$

12. $\frac{-5}{2} \cdot \frac{-20}{19}$

13. $\frac{8}{13} \cdot \frac{-1}{6}$

14. $\frac{-12}{7} \cdot \frac{5}{9}$

15. $\frac{2}{15} \cdot \frac{-9}{8}$

16. $\frac{2}{11} \cdot \frac{-21}{8}$

17. $\frac{17}{12} \cdot \frac{3}{4}$

18. $\frac{7}{13} \cdot \frac{10}{21}$

19. $\frac{-6}{23} \cdot \frac{9}{10}$

20. $\frac{12}{11} \cdot \frac{-5}{2}$

21. $\frac{-23}{24} \cdot \frac{-6}{17}$

22. $\frac{4}{9} \cdot \frac{-21}{19}$

23. $\frac{24}{7} \cdot \frac{5}{2}$

24. $\frac{-20}{23} \cdot \frac{-1}{2}$

25. $\frac{1}{2} \cdot \frac{-8}{11}$

26. $\frac{-11}{18} \cdot \frac{-20}{3}$

27. $\frac{-24}{13} \cdot \frac{-7}{18}$

28. $\frac{21}{20} \cdot \frac{-4}{5}$

In Exercises 29-40, multiply the fractions, and simplify your result.

29. $\dfrac{-12y^3}{13} \cdot \dfrac{2}{9y^6}$

30. $\dfrac{-8x^3}{3} \cdot \dfrac{-6}{5x^5}$

31. $\dfrac{11y^3}{24} \cdot \dfrac{6}{5y^5}$

32. $\dfrac{11y}{18} \cdot \dfrac{21}{17y^6}$

33. $\dfrac{-8x^2}{21} \cdot \dfrac{-18}{19x}$

34. $\dfrac{2y^4}{11} \cdot \dfrac{-7}{18y}$

35. $\dfrac{13x^6}{15} \cdot \dfrac{9}{16x^2}$

36. $\dfrac{-22x^6}{15} \cdot \dfrac{17}{16x^3}$

37. $\dfrac{-6y^3}{5} \cdot \dfrac{-20}{7y^6}$

38. $\dfrac{-21y}{5} \cdot \dfrac{-8}{3y^2}$

39. $\dfrac{-3y^3}{4} \cdot \dfrac{23}{12y}$

40. $\dfrac{-16y^6}{15} \cdot \dfrac{-21}{13y^4}$

In Exercises 41-56, multiply the fractions, and simplify your result.

41. $\dfrac{13y^6}{20x^4} \cdot \dfrac{2x}{7y^2}$

42. $\dfrac{-8y^3}{13x^6} \cdot \dfrac{7x^2}{10y^2}$

43. $\dfrac{23y^4}{21x} \cdot \dfrac{-7x^6}{4y^2}$

44. $\dfrac{-2x^6}{9y^4} \cdot \dfrac{y^5}{20x}$

45. $\dfrac{11y^6}{12x^6} \cdot \dfrac{-2x^4}{7y^2}$

46. $\dfrac{16x^3}{13y^4} \cdot \dfrac{11y^2}{18x}$

47. $\dfrac{x^6}{21y^3} \cdot \dfrac{-7y^4}{9x^5}$

48. $\dfrac{-3y^3}{5x} \cdot \dfrac{14x^5}{15y^2}$

49. $\dfrac{19y^2}{18x} \cdot \dfrac{10x^3}{7y^3}$

50. $\dfrac{-20x}{9y^3} \cdot \dfrac{-y^6}{4x^3}$

51. $\dfrac{-4y^3}{5x^5} \cdot \dfrac{-10x}{21y^4}$

52. $\dfrac{11y^2}{14x^4} \cdot \dfrac{-22x}{21y^3}$

53. $\dfrac{-16x}{21y^2} \cdot \dfrac{-7y^3}{5x^2}$

54. $\dfrac{-4y}{5x} \cdot \dfrac{10x^3}{7y^6}$

55. $\dfrac{17x^3}{3y^6} \cdot \dfrac{-12y^2}{7x^4}$

56. $\dfrac{-6x^4}{11y^3} \cdot \dfrac{13y^2}{8x^5}$

In Exercises 57-62, find the area of the parallelogram having the given base and altitude.

57. base = 8 cm, altitude = 7 cm

58. base = 2 cm, altitude = 11 cm

59. base = 6 cm, altitude = 13 cm

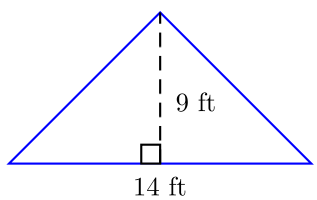
60. base = 2 cm, altitude = 6 cm

61. base = 18 cm, altitude = 14 cm

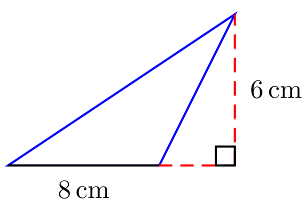
62. base = 20 cm, altitude = 2 cm

In Exercises 63-68, find the area of the triangle shown in the figure. (Note: Figures are not drawn to scale.)

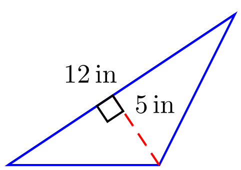
63.



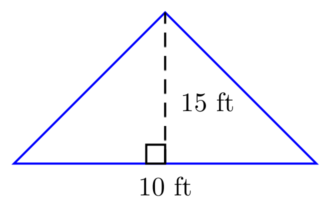
64.



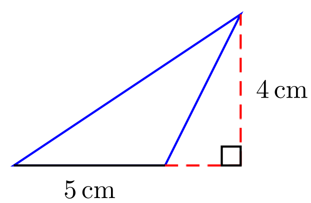
65.



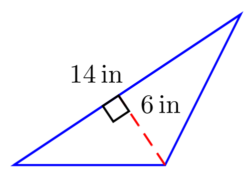
66.



67.



68.



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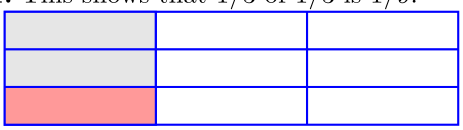
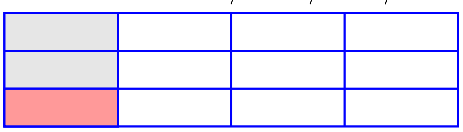
69. **Weight on the Moon.** On the moon, you would only weigh 1/6 of what you weigh on earth. If you weigh 138 pounds on earth, what would your weight on the moon be?

Answers

1. This shows that 1/3 of 1/3 is 1/9.



3. This shows that 1/3 of 1/4 is 1/12.



5. $-\dfrac{231}{38}$

7. $-\dfrac{170}{121}$

9. $-\dfrac{49}{20}$

11. $-\dfrac{7}{44}$

13. $-\dfrac{4}{39}$

15. $-\dfrac{3}{20}$

17. $\dfrac{17}{16}$

19. $-\dfrac{27}{115}$

21. $\dfrac{23}{68}$

23. $\dfrac{60}{7}$

25. $-\dfrac{4}{11}$

27. $\dfrac{28}{39}$

29. $-\dfrac{8}{39y^3}$

31. $\dfrac{11}{20y^2}$

33. $\dfrac{48x}{133}$

35. $\dfrac{39x^4}{80}$

37. $\dfrac{24}{7y^3}$

39. $-\dfrac{23y^2}{16}$

41. $\dfrac{13y^4}{70x^3}$

43. $-\dfrac{23y^2x^5}{12}$

45. $-\dfrac{11y^4}{42x^2}$

47. $-\dfrac{xy}{27}$

49. $\dfrac{95x^2}{63y}$

51. $\dfrac{8}{21yx^4}$

53. $\dfrac{16y}{15x}$

55. $-\dfrac{68}{7xy^4}$

57. $56\,\mathrm{cm}^2$

59. $78\,\mathrm{cm}^2$

61. $252\,\mathrm{cm}^2$

63. $63\,\mathrm{ft}^2$

65. $30\,\mathrm{in}^2$

67. $10\,\mathrm{cm}^2$

69. 23 pounds

4.3 Dividing Fractions

Suppose that you have four pizzas and each of the pizzas has been sliced into eight equal slices. Therefore, each slice of pizza represents 1/8 of a whole pizza.

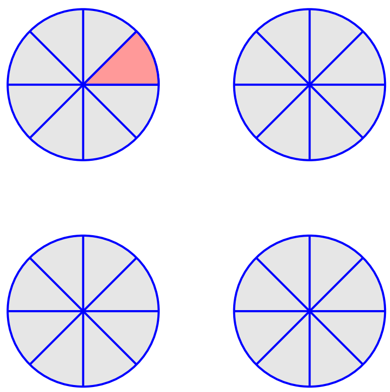


Figure 4.12: One slice of pizza is 1/8 of one whole pizza.

Now for the question: How many one-eighths are there in four? This is a division statement. To find how many one-eighths there are in 4, divide 4 by 1/8. That is,

$$\text{Number of one-eighths in four} = 4 \div \frac{1}{8}.$$

On the other hand, to find the number of one-eights in four, Figure 4.12 clearly demonstrates that this is equivalent to asking how many slices of pizza are there in four pizzas. Since there are 8 slices per pizza and four pizzas,

$$\text{Number of pizza slices} = 4 \cdot 8.$$

The conclusion is the fact that $4 \div (1/8)$ is equivalent to $4 \cdot 8$. That is,

$$\begin{aligned} 4 \div \frac{1}{8} &= 4 \cdot 8 \\ &= 32. \end{aligned}$$

Therefore, we conclude that there are 32 one-eighths in 4.

Reciprocals

The number 1 is still the *multiplicative identity* for fractions.

Multiplicative Identity Property. Let a/b be any fraction. Then,

$$\frac{a}{b} \cdot 1 = \frac{a}{b} \quad \text{and} \quad 1 \cdot \frac{a}{b} = \frac{a}{b}.$$

The number 1 is called the *multiplicative identity* because the identical number is returned when you multiply by 1.

Next, if we invert 3/4, that is, if we turn 3/4 upside down, we get 4/3. Note what happens when we multiply 3/4 by 4/3.

$$\begin{aligned} \frac{3}{4} \cdot \frac{4}{3} &= \frac{3 \cdot 4}{4 \cdot 3} && \text{Multiply numerators; multiply denominators.} \\ &= \frac{12}{12} && \text{Simplify numerators and denominators.} \\ &= 1 && \text{Divide.} \end{aligned}$$

The number 4/3 is called the *multiplicative inverse* or *reciprocal* of 3/4. The product of reciprocals is always 1.

Multiplicative Inverse Property. Let a/b be any fraction. The number b/a is called the *multiplicative inverse* or *reciprocal* of a/b. The product of reciprocals is 1.

$$\frac{a}{b} \cdot \frac{b}{a} = 1$$

Note: To find the multiplicative inverse (reciprocal) of a number, simply invert the number (turn it upside down).

For example, the number 1/8 is the multiplicative inverse (reciprocal) of 8 because

$$8 \cdot \frac{1}{8} = 1.$$

Note that 8 can be thought of as 8/1. Invert this number (turn it upside down) to find its multiplicative inverse (reciprocal) 1/8.

You Try It!

Find the reciprocals of: (a) $-3/7$ and (b) 15

EXAMPLE 1. Find the multiplicative inverses (reciprocals) of: (a) 2/3, (b) $-3/5$, and (c) -12.

Solution.

a) Because

$$\frac{2}{3} \cdot \frac{3}{2} = 1,$$

the multiplicative inverse (reciprocal) of 2/3 is 3/2.

b) Because

$$-\frac{3}{5} \cdot \left(-\frac{5}{3}\right) = 1,$$

the multiplicative inverse (reciprocal) of $-3/5$ is $-5/3$. Again, note that we simply inverted the number $-3/5$ to get its reciprocal $-5/3$.

c) Because

$$-12 \cdot \left(-\frac{1}{12}\right) = 1,$$

the multiplicative inverse (reciprocal) of -12 is $-1/12$. Again, note that we simply inverted the number -12 (understood to equal $-12/1$) to get its reciprocal $-1/12$.

Answer: (a) $-7/3$, (b) $1/15$

□

Division

Recall that we computed the number of one-eighths in four by doing this calculation:

$$\begin{aligned} 4 \div \frac{1}{8} &= 4 \cdot 8 \\ &= 32. \end{aligned}$$

Note how we inverted the divisor (second number), then changed the division to multiplication. This motivates the following definition of division.

Division Definition. If a/b and c/d are any fractions, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}.$$

That is, we invert the divisor (second number) and change the division to multiplication. *Note: We like to use the phrase "invert and multiply" as a memory aid for this definition.*

You Try It!

EXAMPLE 2. Divide $1/2$ by $3/5$.

Divide:

$$\frac{2}{3} \div \frac{10}{3}$$

Solution. To divide $1/2$ by $3/5$, invert the divisor (second number), then multiply.

$$\begin{aligned} \frac{1}{2} \div \frac{3}{5} &= \frac{1}{2} \cdot \frac{5}{3} && \text{Invert the divisor (second number).} \\ &= \frac{5}{6} && \text{Multiply.} \end{aligned}$$

Answer: $1/5$

□

You Try It!

Divide:

$$\frac{15}{7} \div 5$$

EXAMPLE 3. Simplify the following expressions: (a) $3 \div \frac{2}{3}$ and (b) $\frac{4}{5} \div 5$.

Solution. In each case, invert the divisor (second number), then multiply.

a) Note that 3 is understood to be 3/1.

$$\begin{aligned} 3 \div \frac{2}{3} &= \frac{3}{1} \cdot \frac{3}{2} && \text{Invert the divisor (second number).} \\ &= \frac{9}{2} && \text{Multiply numerators; multiply denominators.} \end{aligned}$$

b) Note that 5 is understood to be 5/1.

$$\begin{aligned} \frac{4}{5} \div 5 &= \frac{4}{5} \cdot \frac{1}{5} && \text{Invert the divisor (second number).} \\ &= \frac{4}{25} && \text{Multiply numerators; multiply denominators.} \end{aligned}$$

Answer: $\frac{3}{7}$

□

After inverting, you may need to factor and cancel, as we learned to do in Section 4.2.

You Try It!

Divide:

$$\frac{6}{15} \div \left(-\frac{42}{35}\right)$$

EXAMPLE 4. Divide $-6/35$ by $33/55$.

Solution. Invert, multiply, factor, and cancel common factors.

$$\begin{aligned} -\frac{6}{35} \div \frac{33}{55} &= -\frac{6}{35} \cdot \frac{55}{33} && \text{Invert the divisor (second number).} \\ &= -\frac{6 \cdot 55}{35 \cdot 33} && \text{Multiply numerators; multiply denominators.} \\ &= -\frac{(2 \cdot 3) \cdot (5 \cdot 11)}{(5 \cdot 7) \cdot (3 \cdot 11)} && \text{Factor numerators and denominators.} \\ &= -\frac{2 \cdot \cancel{3} \cdot \cancel{5} \cdot \cancel{11}}{\cancel{5} \cdot 7 \cdot \cancel{3} \cdot \cancel{11}} && \text{Cancel common factors.} \\ &= -\frac{2}{7} && \text{Remaining factors.} \end{aligned}$$

Answer: $-1/3$

Note that unlike signs produce a negative answer.

□

Of course, you can also choose to factor numerators and denominators in place, then cancel common factors.

You Try It!

EXAMPLE 5. Divide $-6/x$ by $-12/x^2$.

Divide:

$$-\frac{12}{a} \div \left(-\frac{15}{a^3}\right)$$

Solution. Invert, factor numerators and denominators, cancel common factors, then multiply.

$$\begin{aligned}
-\frac{6}{x} \div \left(-\frac{12}{x^2}\right) &= -\frac{6}{x} \cdot \left(-\frac{x^2}{12}\right) && \text{Invert second number.}\\
&= -\frac{2 \cdot 3}{x} \cdot -\frac{x \cdot x}{2 \cdot 2 \cdot 3} && \text{Factor numerators and denominators.}\\
&= -\frac{\cancel{2} \cdot \cancel{3}}{\cancel{x}} \cdot -\frac{\cancel{x} \cdot x}{2 \cdot \cancel{2} \cdot \cancel{3}} && \text{Cancel common factors.}\\
&= \frac{x}{2} && \text{Multiply.}
\end{aligned}$$

Note that like signs produce a positive answer.

Answer: $-\dfrac{4a^2}{5}$

□

❧ ❧ ❧ Exercises ☙ ☙ ☙

In Exercises 1-16, find the reciprocal of the given number.

1. $-16/5$

2. $-3/20$

3. -17

4. -16

5. $15/16$

6. $7/9$

7. 30

8. 28

9. -46

10. -50

11. $-9/19$

12. $-4/7$

13. $3/17$

14. $3/5$

15. 11

16. 48

In Exercises 17-32, determine which property of multiplication is depicted by the given identity.

17. $\frac{2}{9} \cdot \frac{9}{2} = 1$

18. $\frac{12}{19} \cdot \frac{19}{12} = 1$

19. $\frac{-19}{12} \cdot 1 = \frac{-19}{12}$

20. $\frac{-19}{8} \cdot 1 = \frac{-19}{8}$

21. $-6 \cdot \left(-\frac{1}{6}\right) = 1$

22. $-19 \cdot \left(-\frac{1}{19}\right) = 1$

23. $\frac{-16}{11} \cdot 1 = \frac{-16}{11}$

24. $\frac{-7}{6} \cdot 1 = \frac{-7}{6}$

25. $-\frac{4}{1} \cdot \left(-\frac{1}{4}\right) = 1$

26. $-\frac{9}{10} \cdot \left(-\frac{10}{9}\right) = 1$

27. $\frac{8}{1} \cdot 1 = \frac{8}{1}$

28. $\frac{13}{15} \cdot 1 = \frac{13}{15}$

29. $14 \cdot \frac{1}{14} = 1$

30. $4 \cdot \frac{1}{4} = 1$

31. $\frac{13}{8} \cdot 1 = \frac{13}{8}$

32. $\frac{1}{13} \cdot 1 = \frac{1}{13}$

In Exercises 33-56, divide the fractions, and simplify your result.

33. $\dfrac{8}{23} \div \dfrac{-6}{11}$

34. $\dfrac{-10}{21} \div \dfrac{-6}{5}$

35. $\dfrac{18}{19} \div \dfrac{-16}{23}$

36. $\dfrac{13}{10} \div \dfrac{17}{18}$

37. $\dfrac{4}{21} \div \dfrac{-6}{5}$

38. $\dfrac{2}{9} \div \dfrac{-12}{19}$

39. $\dfrac{-1}{9} \div \dfrac{8}{3}$

40. $\dfrac{1}{2} \div \dfrac{-15}{8}$

41. $\dfrac{-21}{11} \div \dfrac{3}{10}$

42. $\dfrac{7}{24} \div \dfrac{-23}{2}$

43. $\dfrac{-12}{7} \div \dfrac{2}{3}$

44. $\dfrac{-9}{16} \div \dfrac{6}{7}$

45. $\dfrac{2}{19} \div \dfrac{24}{23}$

46. $\dfrac{7}{3} \div \dfrac{-10}{21}$

47. $\dfrac{-9}{5} \div \dfrac{-24}{19}$

48. $\dfrac{14}{17} \div \dfrac{-22}{21}$

49. $\dfrac{18}{11} \div \dfrac{14}{9}$

50. $\dfrac{5}{6} \div \dfrac{20}{19}$

51. $\dfrac{13}{18} \div \dfrac{4}{9}$

52. $\dfrac{-3}{2} \div \dfrac{-7}{12}$

53. $\dfrac{11}{2} \div \dfrac{-21}{10}$

54. $\dfrac{-9}{2} \div \dfrac{-13}{22}$

55. $\dfrac{3}{10} \div \dfrac{12}{5}$

56. $\dfrac{-22}{7} \div \dfrac{-18}{17}$

In Exercises 57-68, divide the fractions, and simplify your result.

57. $\dfrac{20}{17} \div 5$

58. $\dfrac{21}{8} \div 7$

59. $-7 \div \dfrac{21}{20}$

60. $-3 \div \dfrac{12}{17}$

61. $\dfrac{8}{21} \div 2$

62. $\dfrac{-3}{4} \div (-6)$

63. $8 \div \dfrac{-10}{17}$

64. $-6 \div \dfrac{20}{21}$

65. $-8 \div \dfrac{18}{5}$

66. $6 \div \dfrac{-21}{8}$

67. $\dfrac{3}{4} \div (-9)$

68. $\dfrac{2}{9} \div (-8)$

In Exercises 69-80, divide the fractions, and simplify your result.

69. $\dfrac{11x^2}{12} \div \dfrac{8x^4}{3}$

70. $\dfrac{-4x^2}{3} \div \dfrac{11x^6}{6}$

71. $\dfrac{17y}{9} \div \dfrac{10y^6}{3}$

72. $\dfrac{-5y}{12} \div \dfrac{-3y^5}{2}$

73. $\dfrac{-22x^4}{13} \div \dfrac{12x}{11}$

74. $\dfrac{-9y^6}{4} \div \dfrac{24y^5}{13}$

75. $\dfrac{-3x^4}{10} \div \dfrac{-4x}{5}$

76. $\dfrac{18y^4}{11} \div \dfrac{4y^2}{7}$

77. $\dfrac{-15y^2}{14} \div \dfrac{-10y^5}{13}$

78. $\dfrac{3x}{20} \div \dfrac{2x^3}{5}$

79. $\dfrac{-15x^5}{13} \div \dfrac{20x^2}{19}$

80. $\dfrac{18y^6}{7} \div \dfrac{14y^4}{9}$

In Exercises 81-96, divide the fractions, and simplify your result.

81. $\dfrac{11y^4}{14x^2} \div \dfrac{-9y^2}{7x^3}$

82. $\dfrac{-5x^2}{12y^3} \div \dfrac{-22x}{21y^5}$

83. $\dfrac{10x^4}{3y^4} \div \dfrac{7x^5}{24y^2}$

84. $\dfrac{20x^3}{11y^5} \div \dfrac{5x^5}{6y^3}$

85. $\dfrac{22y^4}{21x^5} \div \dfrac{-5y^2}{6x^4}$

86. $\dfrac{-7y^5}{8x^6} \div \dfrac{21y}{5x^5}$

87. $\dfrac{-22x^4}{21y^3} \div \dfrac{-17x^3}{3y^4}$

88. $\dfrac{-7y^4}{4x} \div \dfrac{-15y}{22x^4}$

89. $\dfrac{-16y^2}{3x^3} \div \dfrac{2y^6}{11x^5}$

90. $\dfrac{-20x}{21y^2} \div \dfrac{-22x^5}{y^6}$

91. $\dfrac{-x}{12y^4} \div \dfrac{-23x^3}{16y^3}$

92. $\dfrac{20x^2}{17y^3} \div \dfrac{8x^3}{15y}$

93. $\dfrac{y^2}{4x} \div \dfrac{-9y^5}{8x^3}$

94. $\dfrac{-10y^4}{13x^2} \div \dfrac{-5y^6}{6x^3}$

95. $\dfrac{-18x^6}{13y^4} \div \dfrac{3x}{y^2}$

96. $\dfrac{20x^4}{9y^6} \div \dfrac{14x^2}{17y^4}$

❧ ❧ ❧ Answers ☙ ☙ ☙

1. $-\frac{5}{16}$

3. $-\frac{1}{17}$

5. $\frac{16}{15}$

7. $\frac{1}{30}$

9. $-\frac{1}{46}$

11. $-\frac{19}{9}$

13. $\frac{17}{3}$

15. $\frac{1}{11}$

17. multiplicative inverse property

19. multiplicative identity property

21. multiplicative inverse property

23. multiplicative identity property

25. multiplicative inverse property

27. multiplicative identity property

29. multiplicative inverse property

31. multiplicative identity property

33. $-\frac{44}{69}$

35. $-\frac{207}{152}$

37. $-\frac{10}{63}$

39. $-\frac{1}{24}$

41. $-\frac{70}{11}$

43. $-\frac{18}{7}$

45. $\frac{23}{228}$

47. $\frac{57}{40}$

49. $\frac{81}{77}$

51. $\frac{13}{8}$

53. $-\frac{55}{21}$

55. $\frac{1}{8}$

57. $\frac{4}{17}$

59. $-\frac{20}{3}$

61. $\frac{4}{21}$

63. $-\frac{68}{5}$

65. $-\frac{20}{9}$

67. $-\frac{1}{12}$

69. $\dfrac{11}{32x^2}$

71. $\dfrac{17}{30y^5}$

73. $-\dfrac{121x^3}{78}$

75. $\dfrac{3x^3}{8}$

77. $\dfrac{39}{28y^3}$

79. $-\dfrac{57x^3}{52}$

81. $-\dfrac{11y^2x}{18}$

83. $\dfrac{80}{7xy^2}$

85. $-\dfrac{44y^2}{35x}$

87. $\dfrac{22xy}{119}$

89. $-\dfrac{88x^2}{3y^4}$

91. $\dfrac{4}{69x^2y}$

93. $-\dfrac{2x^2}{9y^3}$

95. $-\dfrac{6x^5}{13y^2}$

4.4 Adding and Subtracting Fractions

Paul and Tony order a pizza which has been cut into eight equal slices. Thus, each slice is 1/8 of the whole pizza. Paul eats two slices (shaded in light gray in Figure 4.13), or 2/8 of the whole pizza. Tony eats three slices (shaded in light red (or a darker shade of gray in black-and-white printing) in Figure 4.13), or 3/8 of the whole pizza.

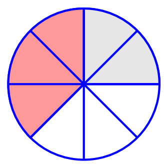


Figure 4.13: Paul eats two slices (2/8) and Tony eats three slices (3/8).

It should be clear that together Paul and Tony eat five slices, or 5/8 of the whole pizza. This reflects the fact that

$$\frac{2}{8}+\frac{3}{8}=\frac{5}{8}.$$

This demonstrates how to add two fractions with a common (same) denominator. Keep the common denominator and add the numerators. That is,

$$\begin{aligned} \frac{2}{8}+\frac{3}{8} &= \frac{2+3}{8} && \text{Keep denominator; add numerators.} \\ &= \frac{5}{8} && \text{Simplify numerator.} \end{aligned}$$

Adding Fractions with Common Denominators. Let a/c and b/c be two fractions with a common (same) denominator. Their sum is defined as

$$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}.$$

That is, to add two fractions having common denominators, keep the common denominator and add their numerators.

A similar rule holds for subtraction.

Subtracting Fractions with Common Denominators. Let a/c and b/c be two fractions with a common (same) denominator. Their difference is defined as

$$\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}.$$

That is, to subtract two fractions having common denominators, keep the common denominator and subtract their numerators.

You Try It!

Add:
$$\frac{1}{8}+\frac{2}{8}$$

EXAMPLE 1. Find the sum of 4/9 and 3/9.

Solution. Keep the common denominator and add the numerators.

$$\begin{aligned}\frac{4}{9}+\frac{3}{9} &= \frac{4+3}{9} && \text{Keep denominator; add numerators.}\\ &= \frac{7}{9} && \text{Simplify numerator.}\end{aligned}$$

Answer: 3/8

□

You Try It!

Subtract:
$$\frac{11}{12}-\frac{7}{12}$$

EXAMPLE 2. Subtract 5/16 from 13/16.

Solution. Keep the common denominator and subtract the numerators.

$$\begin{aligned}\frac{13}{16}-\frac{5}{16} &= \frac{13-5}{16} && \text{Keep denominator; subtract numerators.}\\ &= \frac{8}{16} && \text{Simplify numerator.}\end{aligned}$$

Of course, as we learned in Section 4.1, we should always reduce our final answer to lowest terms. One way to accomplish that in this case is to divide numerator and denominator by 8, the greatest common divisor of 8 and 16.

$$\begin{aligned} &= \frac{8\div 8}{16\div 8} && \text{Divide numerator and denominator by 8.}\\ &= \frac{1}{2} && \text{Simplify numerator and denominator.}\end{aligned}$$

Answer: 1/3

□

You Try It!

Subtract:
$$-\frac{5}{y}-\left(-\frac{2}{y}\right)$$

EXAMPLE 3. Simplify: $\dfrac{3}{x}-\left(-\dfrac{7}{x}\right)$.

Solution. Both fractions share a common denominator.

$$\begin{aligned} \frac{3}{x} - \left(-\frac{7}{x}\right) &= \frac{3}{x} + \frac{7}{x} && \text{Add the opposite.} \\ &= \frac{3+7}{x} && \text{Keep denominator, add numerators.} \\ &= \frac{10}{x} && \text{Simplify.} \end{aligned}$$

Answer: $-3/y$

Adding Fractions with Different Denominators

Consider the sum

$$\frac{4}{9} + \frac{1}{6}.$$

We cannot add these fractions because they do not have a common denominator. So, what to do?

Goal. In order to add two fractions with different denominators, we need to:

1. Find a common denominator for the given fractions.
2. Make fractions with the common denominator that are equivalent to the original fractions.

If we accomplish the two items in the "Goal," we will be able to find the sum of the given fractions.

So, how to start? We need to find a common denominator, but not just any common denominator. Let's agree that we want to keep the numbers as small as possible and find a *least common denominator*.

Least Common Denominator. The *least common denominator* (LCD) for a set of fractions is the smallest number divisible by each of the denominators of the given fractions.

Consider again the sum we wish to find:

$$\frac{4}{9} + \frac{1}{6}.$$

The denominators are 9 and 6. We wish to find a least common denominator, the smallest number that is divisible by both 9 and 6. A number of candidates come to mind: 36, 54, and 72 are all divisible by 9 and 6, to name a few. But the smallest number that is divisible by both 9 and 6 is 18. This is the least common denominator for 9 and 6.

We now proceed to the second item in "Goal." We need to make fractions having 18 as a denominator that are equivalent to 4/9 and 1/6. In the case of 4/9, if we multiply both numerator and denominator by 2, we get

$$\begin{aligned} \frac{4}{9} &= \frac{4 \cdot 2}{9 \cdot 2} && \text{Multiply numerator and denominator by 2.} \\ &= \frac{8}{18}. && \text{Simplify numerator and denominator.} \end{aligned}$$

In the case of 1/6, if we multiply both numerator and denominator by 3, we get

$$\begin{aligned} \frac{1}{6} &= \frac{1 \cdot 3}{6 \cdot 3} && \text{Multiply numerator and denominator by 3.} \\ &= \frac{3}{18}. && \text{Simplify numerator and denominator.} \end{aligned}$$

Typically, we'll arrange our work as follows.

$$\begin{aligned} \frac{4}{9} + \frac{1}{6} &= \frac{4 \cdot 2}{9 \cdot 2} + \frac{1 \cdot 3}{6 \cdot 3} && \text{Equivalent fractions with LCD} = 18. \\ &= \frac{8}{18} + \frac{3}{18} && \text{Simplify numerators and denominators.} \\ &= \frac{8+3}{18} && \text{Keep common denominator; add numerators.} \\ &= \frac{11}{18} && \text{Simplify numerator.} \end{aligned}$$

Let's summarize the procedure.

Adding or Subtracting Fractions with Different Denominators.

1. Find the LCD, the smallest number divisible by all the denominators of the given fractions.

2. Create fractions using the LCD as the denominator that are equivalent to the original fractions.

3. Add or subtract the resulting equivalent fractions. Simplify, including reducing the final answer to lowest terms.

You Try It!

EXAMPLE 4. Simplify: $\dfrac{3}{5} - \dfrac{2}{3}$.

Subtract:

$$\frac{3}{4} - \frac{7}{5}$$

Solution. The smallest number divisible by both 5 and 3 is 15.

$$\begin{aligned}
\frac{3}{5} - \frac{2}{3} &= \frac{3 \cdot 3}{5 \cdot 3} - \frac{2 \cdot 5}{3 \cdot 5} && \text{Equivalent fractions with LCD} = 15.\\
&= \frac{9}{15} - \frac{10}{15} && \text{Simplify numerators and denominators.}\\
&= \frac{9 - 10}{15} && \text{Keep LCD; subtract numerators.}\\
&= \frac{-1}{15} && \text{Simplify numerator.}
\end{aligned}$$

Although this answer is perfectly acceptable, negative divided by positive gives us a negative answer, so we could also write

$$= -\frac{1}{15}.$$

Answer: $-13/20$

□

You Try It!

EXAMPLE 5. Simplify: $-\dfrac{1}{4} - \dfrac{5}{6}$.

Subtract:

$$-\frac{3}{8} - \frac{1}{12}$$

Solution. The smallest number divisible by both 4 and 6 is 12.

$$\begin{aligned}
-\frac{1}{4} - \frac{5}{6} &= -\frac{1 \cdot 3}{4 \cdot 3} - \frac{5 \cdot 2}{6 \cdot 2} && \text{Equivalent fractions with LCD} = 12.\\
&= -\frac{3}{12} - \frac{10}{12} && \text{Simplify numerators and denominators.}\\
&= \frac{-3 - 10}{12} && \text{Keep LCD; subtract numerators.}\\
&= \frac{-13}{12} && \text{Simplify numerator.}
\end{aligned}$$

Answer: $-11/24$

□

You Try It!

Add:

$$\frac{5}{z} + \frac{2}{3}$$

EXAMPLE 6. Simplify: $\dfrac{5}{x} + \dfrac{3}{4}$.

Solution. The smallest number divisible by both 4 and x is $4x$.

$$\begin{aligned}
\frac{5}{x} + \frac{3}{4} &= \frac{5 \cdot 4}{x \cdot 4} + \frac{3 \cdot x}{4 \cdot x} && \text{Equivalent fractions with LCD} = 4x. \\
&= \frac{20}{4x} + \frac{3x}{4x} && \text{Simplify numerators and denominators.} \\
&= \frac{20 + 3x}{4x} && \text{Keep LCD; add numerators.}
\end{aligned}$$

Answer: $\dfrac{15 + 2z}{3z}$

□

You Try It!

Simplify:

$$\frac{3}{7} - \frac{y}{4}$$

EXAMPLE 7. Simplify: $\dfrac{2}{3} - \dfrac{x}{5}$.

Solution. The smallest number divisible by both 3 and 5 is 15.

$$\begin{aligned}
\frac{2}{3} - \frac{x}{5} &= \frac{2 \cdot 5}{3 \cdot 5} - \frac{x \cdot 3}{5 \cdot 3} && \text{Equivalent fractions with LCD} = 15. \\
&= \frac{10}{15} - \frac{3x}{15} && \text{Simplify numerators and denominators.} \\
&= \frac{10 - 3x}{15} && \text{Keep LCD; subtract numerators.}
\end{aligned}$$

Answer: $\dfrac{12 - 7y}{28}$

□

Least Common Multiple

First we define the *multiple* of a number.

Multiples. The multiples of a number d are $1d$, $2d$, $3d$, $4d$, etc. That is, the multiples of d are the numbers nd, where n is a natural number.

For example, the multiples of 8 are $1 \cdot 8$, $2 \cdot 8$, $3 \cdot 8$, $4 \cdot 8$, etc., or equivalently, 8, 16, 24, 32, etc.

Least Common Multiple. The *least common multiple* (LCM) of a set of numbers is the smallest number that is a multiple of each number of the given set. The procedure for finding an LCM follows:

1. List all of the multiples of each number in the given set of numbers.
2. List the multiples that are in common.
3. Pick the least of the multiples that are in common.

You Try It!

EXAMPLE 8. Find the least common multiple (LCM) of 12 and 16.

Find the least common denominator of 6 and 9.

Solution. List the multiples of 12 and 16.

$$\begin{aligned} \text{Multiples of 12:} &\quad 12,\ 24,\ 36,\ 48,\ 60,\ 72,\ 84,\ 96, \ldots \\ \text{Multiples of 16:} &\quad 16,\ 32,\ 48,\ 64,\ 80,\ 96,\ 112, \ldots \end{aligned}$$

Pick the common multiples.

$$\text{Common Multiples:} \quad 48,\ 96, \ldots$$

The LCM is the least of the common multiples.

$$\text{LCM}(12{,}16) = 48$$

Answer: 18

Important Observation. The *least common denominator* is the *least common multiple* of the denominators.

For example, suppose your problem is $5/12 + 5/16$. The LCD is the smallest number divisible by both 12 and 16. That number is 48, which is also the LCM of 12 and 16. Therefore, the procedure for finding the LCM can also be used to find the LCD.

Least Common Multiple Using Prime Factorization

You can also find the LCM using prime factorization.

LCM by Prime Factorization. To find an LCM for a set of numbers, follow this procedure:

1. Write down the prime factorization for each number in compact form using exponents.
2. The LCM is found by writing down every factor that appears in step 1 to the highest power of that factor that appears.

You Try It!

Use prime factorization to find the least common denominator of 18 and 24.

EXAMPLE 9. Use prime factorization to find the least common multiple (LCM) of 12 and 16.

Solution. Prime factor 12 and 16.

$$12 = 2 \cdot 2 \cdot 3$$
$$16 = 2 \cdot 2 \cdot 2 \cdot 2$$

Write the prime factorizations in compact form using exponents.

$$12 = 2^2 \cdot 3^1$$
$$16 = 2^4$$

To find the LCM, write down each factor that appears to the highest power of that factor that appears. The factors that appear are 2 and 3. The highest power of 2 that appears is 2^4. The highest power of 3 that appears is 3^1.

$$\text{LCM} = 2^4 \cdot 3^1 \qquad \text{Keep highest power of each factor.}$$

Now we expand this last expression to get our LCM.

$$= 16 \cdot 3 \qquad \text{Expand: } 2^4 = 16 \text{ and } 3^1 = 3.$$
$$= 48. \qquad \text{Multiply.}$$

Note that this answer is identical to the LCM found in Example 8 that was found by listing multiples and choosing the smallest multiple in common.

Answer: 72

□

You Try It!

Simplify:

$$\frac{5}{24} + \frac{5}{36}$$

EXAMPLE 10. Simplify: $\dfrac{5}{28} + \dfrac{11}{42}$.

Solution. Prime factor the denominators in compact form using exponents.

$$\begin{aligned}28 &= 2 \cdot 2 \cdot 7 = 2^2 \cdot 7\\ 42 &= 2 \cdot 3 \cdot 7 = 2^1 \cdot 3^1 \cdot 7^1\end{aligned}$$

To find the LCD, write down each factor that appears to the highest power of that factor that appears. The factors that appear are 2, 3, and 7. The highest power of 2 that appears is 2^2. The highest power of 3 that appears is 3^1. The highest power of 7 that appears is 7^1.

$$\begin{aligned}\text{LCM} &= 2^2 \cdot 3^1 \cdot 7^1 && \text{Keep highest power of each factor.}\\ &= 4 \cdot 3 \cdot 7 && \text{Expand: } 2^2 = 4,\ 3^1 = 3,\ 7^1 = 7.\\ &= 84 && \text{Multiply.}\end{aligned}$$

Create equivalent fractions with the new LCD, then add.

$$\begin{aligned}\frac{5}{28} + \frac{11}{42} &= \frac{5 \cdot 3}{28 \cdot 3} + \frac{11 \cdot 2}{42 \cdot 2} && \text{Equivalent fractions with LCD=84.}\\ &= \frac{15}{84} + \frac{22}{84} && \text{Simplify numerators and denominators.}\\ &= \frac{37}{84} && \text{Keep LCD; add numerators.}\end{aligned}$$

Answer: 25/72

□

You Try It!

Simplify:

$$-\frac{5}{24} - \frac{11}{36}$$

EXAMPLE 11. Simplify: $-\dfrac{11}{24} - \dfrac{1}{18}$.

Solution. Prime factor the denominators in compact form using exponents.

$$\begin{aligned}24 &= 2 \cdot 2 \cdot 2 \cdot 3 = 2^3 \cdot 3^1\\ 18 &= 2 \cdot 3 \cdot 3 = 2^1 \cdot 3^2\end{aligned}$$

To find the LCD, write down each factor that appears to the highest power of that factor that appears. The factors that appear are 2 and 3. The highest power of 2 that appears is 2^3. The highest power of 3 that appears is 3^2.

$$\begin{aligned}\text{LCM} &= 2^3 \cdot 3^2 && \text{Keep highest power of each factor.}\\ &= 8 \cdot 9 && \text{Expand: } 2^3 = 8 \text{ and } 3^2 = 9.\\ &= 72. && \text{Multiply.}\end{aligned}$$

Create equivalent fractions with the new LCD, then subtract.

$$\begin{aligned} -\frac{11}{24} - \frac{1}{18} &= -\frac{11 \cdot 3}{24 \cdot 3} - \frac{1 \cdot 4}{18 \cdot 4} && \text{Equivalent fractions with LCD=72.} \\ &= -\frac{33}{72} - \frac{4}{72} && \text{Simplify numerators and denominators.} \\ &= \frac{-33 - 4}{72} && \text{Keep LCD; subtract numerators.} \\ &= \frac{-37}{72} && \text{Simplify numerator.} \end{aligned}$$

Of course, negative divided by positive yields a negative answer, so we can also write our answer in the form

$$-\frac{11}{24} - \frac{1}{18} = -\frac{37}{72}.$$

Answer: $-37/72$

□

Comparing Fractions

The simplest way to compare fractions is to create equivalent fractions.

You Try It!

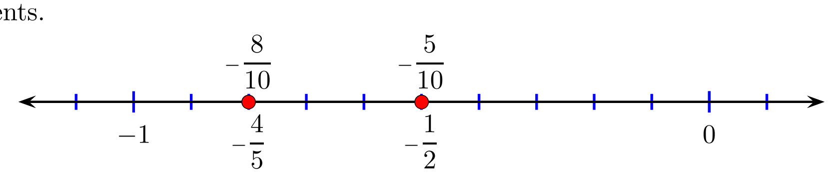
Compare $-3/8$ and $-1/2$.

EXAMPLE 12. Arrange the fractions $-1/2$ and $-4/5$ on a number line, then compare them by using the appropriate inequality symbol.

Solution. The least common denominator for 2 and 5 is the number 10. First, make equivalent fractions with a LCD equal to 10.

$$\begin{aligned} -\frac{1}{2} &= -\frac{1 \cdot 5}{2 \cdot 5} = -\frac{5}{10} \\ -\frac{4}{5} &= -\frac{4 \cdot 2}{5 \cdot 2} = -\frac{8}{10} \end{aligned}$$

To plot tenths, subdivide the interval between -1 and 0 into ten equal increments.



Because $-4/5$ lies to the left of $-1/2$, we have that $-4/5$ is *less than* $-1/2$, so we write

$$-\frac{4}{5} < -\frac{1}{2}.$$

Answer: $-\frac{1}{2} < -\frac{3}{8}$

□

❧ ❧ ❧ Exercises ☙ ☙ ☙

In Exercises 1-10, list the multiples the given numbers, then list the common multiples. Select the LCM from the list of common multiples.

1. 9 and 15

2. 15 and 20

3. 20 and 8

4. 15 and 6

5. 16 and 20

6. 6 and 10

7. 20 and 12

8. 12 and 8

9. 10 and 6

10. 10 and 12

In Exercises 11-20, for the given numbers, calculate the LCM using prime factorization.

11. 54 and 12

12. 108 and 24

13. 18 and 24

14. 36 and 54

15. 72 and 108

16. 108 and 72

17. 36 and 24

18. 18 and 12

19. 12 and 18

20. 12 and 54

In Exercises 21-32, add or subtract the fractions, as indicated, and simplify your result.

21. $\frac{7}{12} - \frac{1}{12}$

22. $\frac{3}{7} - \frac{5}{7}$

23. $\frac{1}{9} + \frac{1}{9}$

24. $\frac{1}{7} + \frac{3}{7}$

25. $\frac{1}{5} - \frac{4}{5}$

26. $\frac{3}{5} - \frac{2}{5}$

27. $\frac{3}{7} - \frac{4}{7}$

28. $\frac{6}{7} - \frac{2}{7}$

29. $\frac{4}{11} + \frac{9}{11}$

30. $\frac{10}{11} + \frac{4}{11}$

31. $\frac{3}{11} + \frac{4}{11}$

32. $\frac{3}{7} + \frac{2}{7}$

In Exercises 33-56, add or subtract the fractions, as indicated, and simplify your result.

33. $\frac{1}{6}-\frac{1}{8}$

34. $\frac{7}{9}-\frac{2}{3}$

35. $\frac{1}{5}+\frac{2}{3}$

36. $\frac{7}{9}+\frac{2}{3}$

37. $\frac{2}{3}+\frac{5}{8}$

38. $\frac{3}{7}+\frac{5}{9}$

39. $\frac{4}{7}-\frac{5}{9}$

40. $\frac{3}{5}-\frac{7}{8}$

41. $\frac{2}{3}-\frac{3}{8}$

42. $\frac{2}{5}-\frac{1}{8}$

43. $\frac{6}{7}-\frac{1}{6}$

44. $\frac{1}{2}-\frac{1}{4}$

45. $\frac{1}{6}+\frac{2}{3}$

46. $\frac{4}{9}+\frac{7}{8}$

47. $\frac{7}{9}+\frac{1}{8}$

48. $\frac{1}{6}+\frac{1}{7}$

49. $\frac{1}{3}+\frac{1}{7}$

50. $\frac{5}{6}+\frac{1}{4}$

51. $\frac{1}{2}-\frac{2}{7}$

52. $\frac{1}{3}-\frac{1}{8}$

53. $\frac{5}{6}-\frac{4}{5}$

54. $\frac{1}{2}-\frac{1}{9}$

55. $\frac{1}{3}+\frac{1}{8}$

56. $\frac{1}{6}+\frac{7}{9}$

In Exercises 57-68, add or subtract the fractions, as indicated, by first using prime factorization to find the least common denominator.

57. $\frac{7}{36}+\frac{11}{54}$

58. $\frac{7}{54}+\frac{7}{24}$

59. $\frac{7}{18}-\frac{5}{12}$

60. $\frac{5}{54}-\frac{7}{12}$

61. $\frac{7}{36}+\frac{7}{54}$

62. $\frac{5}{72}+\frac{5}{108}$

63. $\frac{7}{24}-\frac{5}{36}$

64. $\frac{11}{54}+\frac{7}{72}$

65. $\frac{11}{12}+\frac{5}{18}$

66. $\frac{11}{24}+\frac{11}{108}$

67. $\frac{11}{54}-\frac{5}{24}$

68. $\frac{7}{54}-\frac{5}{24}$

In Exercises 69-80, add or subtract the fractions, as indicated, and simplify your result.

69. $-\frac{3}{7}+\left(-\frac{3}{7}\right)$

70. $-\frac{5}{9}+\left(-\frac{1}{9}\right)$

71. $\frac{7}{9}-\left(-\frac{1}{9}\right)$

72. $\frac{8}{9}-\left(-\frac{4}{9}\right)$

73. $\frac{7}{9}+\left(-\frac{2}{9}\right)$

74. $\frac{2}{3}+\left(-\frac{1}{3}\right)$

75. $-\frac{3}{5}-\frac{4}{5}$

76. $-\frac{7}{9}-\frac{1}{9}$

77. $-\frac{7}{8}+\frac{1}{8}$

78. $-\frac{2}{3}+\frac{1}{3}$

79. $-\frac{1}{3}-\left(-\frac{2}{3}\right)$

80. $-\frac{7}{8}-\left(-\frac{5}{8}\right)$

In Exercises 81-104, add or subtract the fractions, as indicated, and simplify your result.

81. $-\frac{2}{7}+\frac{4}{5}$

82. $-\frac{1}{4}+\frac{2}{7}$

83. $-\frac{1}{4}-\left(-\frac{4}{9}\right)$

84. $-\frac{3}{4}-\left(-\frac{1}{8}\right)$

85. $-\frac{2}{7}+\frac{3}{4}$

86. $-\frac{1}{3}+\frac{5}{8}$

87. $-\frac{4}{9}-\frac{1}{3}$

88. $-\frac{5}{6}-\frac{1}{3}$

89. $-\frac{5}{7}-\left(-\frac{1}{5}\right)$

90. $-\frac{6}{7}-\left(-\frac{1}{8}\right)$

91. $\frac{1}{9}+\left(-\frac{1}{3}\right)$

92. $\frac{1}{8}+\left(-\frac{1}{2}\right)$

93. $\frac{2}{3}+\left(-\frac{1}{9}\right)$

94. $\frac{3}{4}+\left(-\frac{2}{3}\right)$

95. $-\frac{1}{2}+\left(-\frac{6}{7}\right)$

96. $-\frac{4}{5}+\left(-\frac{1}{2}\right)$

97. $-\frac{1}{2}+\left(-\frac{3}{4}\right)$

98. $-\frac{3}{5}+\left(-\frac{1}{2}\right)$

99. $-\frac{1}{4}-\frac{1}{2}$

100. $-\frac{8}{9}-\frac{2}{3}$

101. $\dfrac{5}{8}-\left(-\dfrac{3}{4}\right)$

102. $\dfrac{3}{4}-\left(-\dfrac{3}{8}\right)$

103. $\dfrac{1}{8}-\left(-\dfrac{1}{3}\right)$

104. $\dfrac{1}{2}-\left(-\dfrac{4}{9}\right)$

In Exercises 105-120, add or subtract the fractions, as indicated, and write your answer is lowest terms.

105. $\dfrac{1}{2}+\dfrac{3q}{5}$

106. $\dfrac{4}{7}-\dfrac{b}{3}$

107. $\dfrac{4}{9}-\dfrac{3a}{4}$

108. $\dfrac{4}{9}-\dfrac{b}{2}$

109. $\dfrac{2}{s}+\dfrac{1}{3}$

110. $\dfrac{2}{s}+\dfrac{3}{7}$

111. $\dfrac{1}{3}-\dfrac{7}{b}$

112. $\dfrac{1}{2}-\dfrac{9}{s}$

113. $\dfrac{4b}{7}+\dfrac{2}{3}$

114. $\dfrac{2a}{5}+\dfrac{5}{8}$

115. $\dfrac{2}{3}-\dfrac{9}{t}$

116. $\dfrac{4}{7}-\dfrac{1}{y}$

117. $\dfrac{9}{s}+\dfrac{7}{8}$

118. $\dfrac{6}{t}-\dfrac{1}{9}$

119. $\dfrac{7b}{8}-\dfrac{5}{9}$

120. $\dfrac{3p}{4}-\dfrac{1}{8}$

In Exercises 121-132, determine which of the two given statements is true.

121. $-\dfrac{2}{3}<-\dfrac{8}{7}$ or $-\dfrac{2}{3}>-\dfrac{8}{7}$

122. $-\dfrac{1}{7}<-\dfrac{8}{9}$ or $-\dfrac{1}{7}>-\dfrac{8}{9}$

123. $\dfrac{6}{7}<\dfrac{7}{3}$ or $\dfrac{6}{7}>\dfrac{7}{3}$

124. $\dfrac{1}{2}<\dfrac{2}{7}$ or $\dfrac{1}{2}>\dfrac{2}{7}$

125. $-\dfrac{9}{4}<-\dfrac{2}{3}$ or $-\dfrac{9}{4}>-\dfrac{2}{3}$

126. $-\dfrac{3}{7}<-\dfrac{9}{2}$ or $-\dfrac{3}{7}>-\dfrac{9}{2}$

127. $\dfrac{5}{7}<\dfrac{5}{9}$ or $\dfrac{5}{7}>\dfrac{5}{9}$

128. $\dfrac{1}{2}<\dfrac{1}{3}$ or $\dfrac{1}{2}>\dfrac{1}{3}$

129. $-\dfrac{7}{2}<-\dfrac{1}{5}$ or $-\dfrac{7}{2}>-\dfrac{1}{5}$

130. $-\dfrac{3}{4}<-\dfrac{5}{9}$ or $-\dfrac{3}{4}>-\dfrac{5}{9}$

131. $\dfrac{5}{9}<\dfrac{6}{5}$ or $\dfrac{5}{9}>\dfrac{6}{5}$

132. $\dfrac{3}{2}<\dfrac{7}{9}$ or $\dfrac{3}{2}>\dfrac{7}{9}$

Answers

1. 45

3. 40

5. 80

7. 60

9. 30

11. 108

13. 72

15. 216

17. 72

19. 36

21. $\frac{1}{2}$

23. $\frac{2}{9}$

25. $-\frac{3}{5}$

27. $-\frac{1}{7}$

29. $\frac{13}{11}$

31. $\frac{7}{11}$

33. $\frac{1}{24}$

35. $\frac{13}{15}$

37. $\frac{31}{24}$

39. $\frac{1}{63}$

41. $\frac{7}{24}$

43. $\frac{29}{42}$

45. $\frac{5}{6}$

47. $\frac{65}{72}$

49. $\frac{10}{21}$

51. $\frac{3}{14}$

53. $\frac{1}{30}$

55. $\frac{11}{24}$

57. $\frac{43}{108}$

59. $-\frac{1}{36}$

61. $\frac{35}{108}$

63. $\frac{11}{72}$

65. $\frac{43}{36}$

67. $-\frac{1}{216}$

69. $-\frac{6}{7}$

71. $\frac{8}{9}$

73. $\frac{5}{9}$

75. $-\frac{7}{5}$

77. $-\frac{3}{4}$

79. $\frac{1}{3}$

81. $\frac{18}{35}$

83. $\frac{7}{36}$

85. $\frac{13}{28}$

87. $-\frac{7}{9}$

89. $-\frac{18}{35}$

91. $-\frac{2}{9}$

93. $\frac{5}{9}$

95. $-\frac{19}{14}$

97. $-\frac{5}{4}$

99. $-\frac{3}{4}$

101. $\frac{11}{8}$

103. $\frac{11}{24}$

105. $\frac{5+6q}{10}$

107. $\frac{16-27a}{36}$

109. $\frac{6+s}{3s}$

111. $\frac{b-21}{3b}$

113. $\frac{12b+14}{21}$

115. $\frac{2t-27}{3t}$

117. $\frac{72+7s}{8s}$

119. $\frac{63b-40}{72}$

121. $-\frac{2}{3} > -\frac{8}{7}$

123. $\frac{6}{7} < \frac{7}{3}$

125. $-\frac{9}{4} < -\frac{2}{3}$

127. $\frac{5}{7} > \frac{5}{9}$

129. $-\frac{7}{2} < -\frac{1}{5}$

131. $\frac{5}{9} < \frac{6}{5}$

4.5 Multiplying and Dividing Mixed Fractions

We begin with definitions of *proper* and *improper* fractions.

Proper and Improper Fractions. A *proper fraction* is a fraction whose numerator is smaller than its denominator. An *improper fraction* is a fraction whose numerator is larger than its denominator.

For example,

$$\frac{2}{3}, \; -\frac{23}{39}, \quad \text{and} \quad \frac{119}{127}$$

are all examples of proper fractions. On the other hand,

$$\frac{4}{3}, \; -\frac{317}{123}, \quad \text{and} \quad -\frac{233}{101}$$

are all examples of improper fractions.

A *mixed fraction*[1] is part whole number, part fraction.

Mixed Fractions. The number

$$5\frac{3}{4}$$

is called a *mixed fraction.* It is defined to mean

$$5\frac{3}{4} = 5 + \frac{3}{4}.$$

In the mixed fraction $5\frac{3}{4}$, the 5 is the *whole number part* and the $3/4$ is the *fractional part.*

Changing Mixed Fractions to Improper Fractions

We have all the tools required to change a mixed fraction into an improper fraction. We begin with an example.

You Try It!

EXAMPLE 1. Change the mixed fraction $4\frac{7}{8}$ into an improper fraction.

Change $5\frac{3}{4}$ to an improper fraction.

[1] A mixed fractions is sometimes called a *mixed number.*

Solution. We employ the definition of a mixed fraction, make an equivalent fraction for the whole number part, then add.

$$\begin{aligned}
4\frac{7}{8} &= 4 + \frac{7}{8} && \text{By definition.}\\
&= \frac{4 \cdot 8}{8} + \frac{7}{8} && \text{Equivalent fraction with LCD} = 8.\\
&= \frac{4 \cdot 8 + 7}{8} && \text{Add numerators over common denominator.}\\
&= \frac{39}{8} && \text{Simplify the numerator.}
\end{aligned}$$

Answer: 23/4

Thus, $4\frac{7}{8}$ is equal to 39/8.

There is a quick technique you can use to change a mixed fraction into an improper fraction.

Quick Way to Change a Mixed Fraction to an Improper Fraction. To change a mixed fraction to an improper fraction, multiply the whole number part by the denominator, add the numerator, then place the result over the denominator.

Thus, to quickly change $4\frac{7}{8}$ to an improper fraction, multiply the whole number 4 by the denominator 8, add the numerator 7, then place the result over the denominator. In symbols, this would look like this:

$$4\frac{7}{8} = \frac{4 \cdot 8 + 7}{8}.$$

This is precisely what the third step in Example 1 looks like; we're just eliminating a lot of the work.

You Try It!

Change $7\frac{3}{8}$ to an improper fraction.

EXAMPLE 2. Change $4\frac{2}{3}$ to an improper fraction.

Solution. Take $4\frac{2}{3}$, multiply the whole number part by the denominator, add the numerator, then put the result over the denominator.

$$4\frac{2}{3} = \frac{4 \cdot 3 + 2}{3}$$

Thus, the result is

$$4\frac{2}{3} = \frac{14}{3}.$$

Answer: 59/8

It is very easy to do the intermediate step in Example 2 mentally, allowing you to skip the intermediate step and go directly from the mixed fraction to the improper fraction without writing down a single bit of work.

You Try It!

EXAMPLE 3. Without writing down any work, use mental arithmetic to change $-2\frac{3}{5}$ to an improper fraction.

Change $-3\frac{5}{12}$ to an improper fraction.

Solution. To change $-2\frac{3}{5}$ to an improper fraction, ignore the minus sign, proceed as before, then prefix the minus sign to the resulting improper fraction. So, multiply 5 times 2 and add 3. Put the result 13 over the denominator 5, then prefix the resulting improper fraction with a minus sign. That is,

$$-2\frac{3}{5} = -\frac{13}{5}.$$

Answer: $-41/12$

□

Changing Improper Fractions to Mixed Fractions

The first step in changing the improper fraction $27/5$ to a mixed fraction is to write the improper fraction as a sum.

$$\frac{27}{5} = \frac{25}{5} + \frac{2}{5} \tag{4.1}$$

Simplifying equation 4.1, we get

$$\begin{aligned} \frac{27}{5} &= 5 + \frac{2}{5} \\ &= 5\frac{2}{5}. \end{aligned}$$

Comment. You can't just choose any sum. The sum used in equation 4.1 is constructed so that the first fraction will equal a whole number and the second fraction is proper. Any other sum will fail to produce the correct mixed fraction. For example, the sum

$$\frac{27}{5} = \frac{23}{5} + \frac{4}{5}$$

is useless, because $23/5$ is not a whole number. Likewise, the sum

$$\frac{27}{5} = \frac{20}{5} + \frac{7}{5}$$

is no good. Even though $20/5 = 4$ is a whole number, the second fraction $7/5$ is still improper.

You Try It!

Change 25/7 to a mixed fraction.

EXAMPLE 4. Change 25/9 to a mixed fraction.

Solution. Break 25/9 into the appropriate sum.

$$\begin{aligned}\frac{25}{9} &= \frac{18}{9} + \frac{7}{9} \\ &= 2 + \frac{7}{9} \\ &= 2\frac{7}{9}\end{aligned}$$

Answer: $3\frac{4}{7}$

Comment. A pattern is emerging.

- In the case of 27/5, note that 27 divided by 5 is equal to 5 with a remainder of 2. Compare this with the mixed fraction result: $27/5 = 5\frac{2}{5}$.
- In the case of Example 4, note that 25 divided by 9 is 2 with a remainder of 7. Compare this with the mixed fraction result: $25/9 = 2\frac{7}{9}$.

These observations motivate the following technique.

Quick Way to Change an Improper Fraction to a Mixed Fraction. To change an improper fraction to a mixed fraction, divide the numerator by the denominator. The quotient will be the whole number part of the mixed fraction. If you place the remainder over the denominator, this will be the fractional part of the mixed fraction.

You Try It!

Change 38/9 to a mixed fraction.

EXAMPLE 5. Change 37/8 to a mixed fraction.

Solution. 37 divided by 8 is 4, with a remainder of 5. That is:

$$\begin{array}{r} 4 \\ 8\overline{)37} \\ \underline{32} \\ 5 \end{array}$$

The quotient becomes the whole number part and we put the remainder over the divisor. Thus,

$$\frac{37}{8} = 4\frac{5}{8}.$$

Note: You can check your result with the "Quick Way to Change a Mixed Fraction to an Improper Fraction." 8 times 4 plus 5 is 37. Put this over 8 to get 37/8.

Answer: $4\frac{2}{9}$

□

You Try It!

EXAMPLE 6. Change $-43/5$ to a mixed fraction.

Change $-27/8$ to a mixed fraction.

Solution. Ignore the minus sign and proceed in the same manner as in Example 5. 43 divided by 5 is 8, with a remainder of 3.

$$\begin{array}{r} 8 \\ 5\overline{)43} \\ \underline{40} \\ 3 \end{array}$$

The quotient is the whole number part, then we put the remainder over the divisor. Finally, prefix the minus sign.

$$-\frac{43}{5} = -8\frac{3}{5}.$$

Answer: $-3\frac{3}{8}$

□

Multiplying and Dividing Mixed Fractions

You have all the tools needed to multiply and divide mixed fractions. First, change the mixed fractions to improper fractions, then multiply or divide as you did in previous sections.

You Try It!

EXAMPLE 7. Simplify: $-2\frac{1}{12} \cdot 2\frac{4}{5}$.

Simplify:

$$-3\frac{3}{4} \cdot 2\frac{2}{5}$$

Solution. Change to improper fractions, factor, cancel, and simplify.

$$\begin{aligned}
-2\frac{1}{12} \cdot 2\frac{4}{5} &= -\frac{25}{12} \cdot \frac{14}{5} && \text{Change to improper fractions.} \\
&= -\frac{25 \cdot 14}{12 \cdot 5} && \text{Multiply numerators; multiply denominators.} \\
& && \text{Unlike signs; product is negative.} \\
&= -\frac{(5 \cdot 5) \cdot (2 \cdot 7)}{(2 \cdot 2 \cdot 3) \cdot (5)} && \text{Prime factor.} \\
&= -\frac{\cancel{5} \cdot 5 \cdot \cancel{2} \cdot 7}{\cancel{2} \cdot 2 \cdot 3 \cdot \cancel{5}} && \text{Cancel common factors.} \\
&= -\frac{35}{6} && \text{Multiply numerators and denominators.}
\end{aligned}$$

This is a perfectly good answer, but if you want a mixed fraction answer, 35 divided by 6 is 5, with a remainder of 5. Hence,

$$-2\frac{1}{12} \cdot 2\frac{4}{5} = -5\frac{5}{6}.$$

Answer: -9

You Try It!

Simplify:

$$-2\frac{4}{9} \cdot 3\frac{2}{3}$$

Answer: $-2/3$

EXAMPLE 8. Simplify: $-4\frac{4}{5} \div 5\frac{3}{5}$.

Solution. Change to improper fractions, invert and multiply, factor, cancel, and simplify.

$$\begin{aligned}
-4\frac{4}{5} \div 5\frac{3}{5} &= -\frac{24}{5} \div \frac{28}{5} && \text{Change to improper fractions.} \\
&= -\frac{24}{5} \cdot \frac{5}{28} && \text{Invert and multiply.} \\
&= -\frac{2 \cdot 2 \cdot 2 \cdot 3}{5} \cdot \frac{5}{2 \cdot 2 \cdot 7} && \text{Prime factor.} \\
&= -\frac{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot 3}{\cancel{5}} \cdot \frac{\cancel{5}}{\cancel{2} \cdot \cancel{2} \cdot 7} && \text{Cancel common factors.} \\
&= -\frac{6}{7} && \text{Multiply numerators and denominators.}
\end{aligned}$$

Exercises

In Exercises 1-12, convert the mixed fraction to an improper fraction.

1. $2\frac{1}{3}$

2. $1\frac{8}{11}$

3. $1\frac{1}{19}$

4. $-1\frac{1}{5}$

5. $-1\frac{3}{7}$

6. $1\frac{3}{17}$

7. $1\frac{1}{9}$

8. $1\frac{5}{11}$

9. $-1\frac{1}{2}$

10. $-1\frac{5}{8}$

11. $1\frac{1}{3}$

12. $-1\frac{5}{7}$

In Exercises 13-24, convert the improper fraction to a mixed fraction.

13. $\frac{13}{7}$

14. $-\frac{17}{9}$

15. $-\frac{13}{5}$

16. $-\frac{10}{3}$

17. $-\frac{16}{5}$

18. $\frac{16}{13}$

19. $\frac{9}{8}$

20. $\frac{16}{5}$

21. $-\frac{6}{5}$

22. $-\frac{17}{10}$

23. $-\frac{3}{2}$

24. $-\frac{7}{4}$

In Exercises 25-48, multiply the numbers and express your answer as a mixed fraction.

25. $1\frac{1}{7} \cdot 2\frac{1}{2}$

26. $1\frac{1}{8} \cdot 1\frac{1}{6}$

27. $4 \cdot 1\frac{1}{6}$

28. $1\frac{7}{10} \cdot 4$

29. $\left(-1\frac{1}{12}\right)\left(3\frac{3}{4}\right)$

30. $\left(-3\frac{1}{2}\right)\left(3\frac{1}{3}\right)$

31. $7\frac{1}{2}\cdot 1\frac{1}{13}$

32. $2\frac{1}{4}\cdot 1\frac{5}{11}$

33. $\left(1\frac{2}{13}\right)\left(-4\frac{2}{3}\right)$

34. $\left(1\frac{1}{14}\right)\left(-2\frac{2}{5}\right)$

35. $\left(1\frac{3}{7}\right)\left(-3\frac{3}{4}\right)$

36. $\left(1\frac{4}{5}\right)\left(-3\frac{3}{4}\right)$

37. $9\cdot\left(-1\frac{2}{15}\right)$

38. $4\cdot\left(-2\frac{5}{6}\right)$

39. $\left(-2\frac{1}{8}\right)(-6)$

40. $(-9)\left(-3\frac{1}{6}\right)$

41. $\left(-4\frac{1}{2}\right)\left(-2\frac{2}{5}\right)$

42. $\left(-1\frac{3}{7}\right)\left(-3\frac{3}{4}\right)$

43. $\left(-2\frac{1}{6}\right)\cdot 4$

44. $(-6)\cdot\left(1\frac{1}{9}\right)$

45. $\left(-1\frac{4}{15}\right)\left(2\frac{1}{2}\right)$

46. $\left(-1\frac{1}{5}\right)\left(1\frac{5}{9}\right)$

47. $\left(-2\frac{1}{2}\right)\left(-1\frac{7}{11}\right)$

48. $\left(-1\frac{7}{11}\right)\left(-1\frac{7}{12}\right)$

In Exercises 49-72, divide the mixed fractions and express your answer as a mixed fraction.

49. $8\div 2\frac{2}{9}$

50. $4\frac{2}{3}\div 4$

51. $\left(-3\frac{1}{2}\right)\div\left(1\frac{1}{16}\right)$

52. $\left(-1\frac{2}{5}\right)\div\left(1\frac{1}{15}\right)$

53. $6\frac{1}{2}\div 1\frac{7}{12}$

54. $5\frac{1}{2}\div 1\frac{9}{10}$

55. $(-4)\div\left(1\frac{5}{9}\right)$

56. $\left(-4\frac{2}{3}\right)\div 4$

57. $\left(-5\frac{2}{3}\right)\div\left(-2\frac{1}{6}\right)$

58. $\left(-2\frac{1}{2}\right)\div\left(-2\frac{2}{9}\right)$

59. $\left(-6\frac{1}{2}\right)\div\left(4\frac{1}{4}\right)$

60. $\left(-1\frac{1}{6}\right)\div\left(1\frac{1}{8}\right)$

61. $(-6)\div\left(-1\frac{3}{11}\right)$

62. $\left(-6\frac{2}{3}\right)\div(-6)$

63. $\left(4\frac{2}{3}\right) \div (-4)$

64. $\left(6\frac{2}{3}\right) \div (-6)$

65. $\left(1\frac{3}{4}\right) \div \left(-1\frac{1}{12}\right)$

66. $\left(2\frac{4}{7}\right) \div \left(-1\frac{1}{5}\right)$

67. $5\frac{2}{3} \div 1\frac{1}{9}$

68. $1\frac{2}{3} \div 1\frac{2}{9}$

69. $\left(-7\frac{1}{2}\right) \div \left(-2\frac{2}{5}\right)$

70. $\left(-5\frac{1}{3}\right) \div \left(-2\frac{5}{6}\right)$

71. $\left(3\frac{2}{3}\right) \div \left(-1\frac{1}{9}\right)$

72. $\left(8\frac{1}{2}\right) \div \left(-1\frac{3}{4}\right)$

---

73. **Small Lots.** How many quarter-acre lots can be made from $6\frac{1}{2}$ acres of land?

74. **Big Field.** A field was formed from $17\frac{1}{2}$ half-acre lots. How many acres was the resulting field ?

75. **Jewelry.** To make some jewelry, a bar of silver $4\frac{1}{2}$ inches long was cut into pieces $\frac{1}{12}$ inch long. How many pieces were made?

76. **Muffins.** This recipe will make 6 muffins: 1 cup milk, $1\frac{2}{3}$ cups flour, 2 eggs, 1/2 teaspoon salt, $1\frac{1}{2}$ teaspoons baking powder. Write the recipe for six dozen muffins.

❧ ❧ ❧ Answers ☙ ☙ ☙

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1. $\frac{7}{3}$

3. $\frac{20}{19}$

5. $-\frac{10}{7}$

7. $\frac{10}{9}$

9. $-\frac{3}{2}$

11. $\frac{4}{3}$

13. $1\frac{6}{7}$

15. $-2\frac{3}{5}$

17. $-3\frac{1}{5}$

19. $1\frac{1}{8}$

21. $-1\frac{1}{5}$

23. $-1\frac{1}{2}$

25. $2\frac{6}{7}$

27. $4\frac{2}{3}$

29. $-4\dfrac{1}{16}$

31. $8\dfrac{1}{13}$

33. $-5\dfrac{5}{13}$

35. $-5\dfrac{5}{14}$

37. $-10\dfrac{1}{5}$

39. $12\dfrac{3}{4}$

41. $10\dfrac{4}{5}$

43. $-8\dfrac{2}{3}$

45. $-3\dfrac{1}{6}$

47. $4\dfrac{1}{11}$

49. $3\dfrac{3}{5}$

51. $-3\dfrac{5}{17}$

53. $4\dfrac{2}{19}$

55. $-2\dfrac{4}{7}$

57. $2\dfrac{8}{13}$

59. $-1\dfrac{9}{17}$

61. $4\dfrac{5}{7}$

63. $-1\dfrac{1}{6}$

65. $-1\dfrac{8}{13}$

67. $5\dfrac{1}{10}$

69. $3\dfrac{1}{8}$

71. $-3\dfrac{3}{10}$

73. 26 quarter-acre lots

75. 54 pieces

4.6 Adding and Subtracting Mixed Fractions

In this section, we will learn how to add and subtract mixed fractions.

Adding Mixed Fractions

We can use tools we've already developed to add two or more mixed fractions.

You Try It!

EXAMPLE 1. Simplify: $2\frac{7}{8} + 1\frac{3}{4}$.

Simplify:

$$3\frac{2}{3} + 4\frac{1}{8}$$

Solution. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then add.

$$\begin{aligned}
2\,\frac{7}{8} + 1\,\frac{3}{4} &= \frac{23}{8} + \frac{7}{4} && \text{Change to improper fractions.}\\
&= \frac{23}{8} + \frac{7 \cdot 2}{4 \cdot 2} && \text{Equivalent fractions with LCD} = 8.\\
&= \frac{23}{8} + \frac{14}{8} && \text{Simplify numerators and denominators.}\\
&= \frac{37}{8} && \text{Add numerators over common denominator.}
\end{aligned}$$

Although this answer is perfectly acceptable, let's change the answer to a mixed fraction: 37 divided by 8 is 4, with a remainder of 5. Thus,

$$2\,\frac{7}{8} + 1\,\frac{3}{4} = 4\,\frac{5}{8}.$$

Answer: $7\frac{19}{24}$

□

You Try It!

EXAMPLE 2. Simplify: $3\frac{1}{4} + 2\frac{1}{3}$.

Simplify:

$$8\frac{1}{2} + 2\frac{2}{3}$$

Solution. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then add.

$$\begin{aligned}
3\,\frac{1}{4} + 2\,\frac{1}{3} &= \frac{13}{4} + \frac{7}{3} && \text{Change to improper fractions.}\\
&= \frac{13 \cdot 3}{4 \cdot 3} + \frac{7 \cdot 4}{3 \cdot 4} && \text{Equivalent fractions with LCD} = 12.\\
&= \frac{39}{12} + \frac{28}{12} && \text{Simplify numerators and denominators.}\\
&= \frac{67}{12} && \text{Add numerators over common denominator.}
\end{aligned}$$

Although this answer is perfectly acceptable, let's change the answer to a mixed fraction: 67 divided by 12 is 5, with a remainder of 7. Thus,

$$3\,\frac{1}{4} + 2\,\frac{1}{3} = 5\,\frac{7}{12}.$$

Answer: $11\frac{1}{6}$

Mixed Fraction Approach. There is another possible approach, based on the fact that a mixed fraction is a sum. Let's revisit Example 2.

You Try It!

Simplify:

$$7\frac{2}{5} + 3\frac{1}{8}$$

EXAMPLE 3. Simplify: $3\frac{1}{4} + 2\frac{1}{3}$.

Solution. Use the commutative and associative properties to change the order of addition, make equivalent fractions with a common denominator, then add.

$$\begin{aligned}
3\,\frac{1}{4} + 2\,\frac{1}{3} &= \left(3 + \frac{1}{4}\right) + \left(2 + \frac{1}{3}\right) && \text{Mixed fractions as sums.}\\
&= (3 + 2) + \left(\frac{1}{4} + \frac{1}{3}\right) && \text{Reorder and regroup.}\\
&= 5 + \left(\frac{1 \cdot 3}{4 \cdot 3} + \frac{1 \cdot 4}{3 \cdot 4}\right) && \text{Add whole numbers: } 3 + 2 = 5.\\
& && \text{Equivalent fractions; LCD} = 12.\\
&= 5 + \left(\frac{3}{12} + \frac{4}{12}\right) && \text{Simplify numerators and denominators.}\\
&= 5 + \frac{7}{12} && \text{Add numerators over common denominator.}
\end{aligned}$$

This result can be written in mixed fraction form. Thus,

$$3\,\frac{1}{4} + 2\,\frac{1}{3} = 5\,\frac{7}{12}.$$

Answer: $10\frac{21}{40}$

Note that this solution is identical to the result found in Example 2.

Example 3 leads us to the following result.

Adding Mixed Fractions. To add two mixed fractions, add the whole number parts, then add the fractional parts.

Working in Vertical Format

When adding mixed fractions, many prefer to work in a vertical format. For example, here is how we would arrange the solution from Example 2 and Example 3 in vertical format. We create equivalent fractions, then add the whole number parts and fractional parts.

$$\begin{array}{ccccc} 3\,\dfrac{1}{4} & = & 3\,\dfrac{1\cdot 3}{4\cdot 3} & = & 3\,\dfrac{3}{12} \\ +2\,\dfrac{1}{3} & = & +2\,\dfrac{1\cdot 4}{3\cdot 4} & = & +2\,\dfrac{4}{12} \\ \hline & & & & 5\,\dfrac{7}{12} \end{array}$$

Note that the answer is identical to the answer found in Example 2 and Example 3. That is,

$$3\,\frac{1}{4} + 2\,\frac{1}{3} = 5\,\frac{7}{12}.$$

You Try It!

Jim is working on a project that requires two boards, the first cut to a length of $6\frac{1}{2}$ feet, the second cut to a length of $5\frac{7}{8}$ feet. How many total feet of board is required?

EXAMPLE 4. Sarah is making window curtains for two rooms in her house. The kitchen will require $5\frac{2}{3}$ yards of material and the dining room will require $6\frac{5}{8}$ yards of material. How much total material is required?

Solution. To find the total material required for the two rooms, we must add $5\frac{2}{3}$ and $6\frac{5}{8}$. Create equivalent fractions with a common denominator, then add whole number parts and fractional parts.

$$\begin{array}{ccccc} 5\,\dfrac{2}{3} & = & 5\,\dfrac{2\cdot 8}{3\cdot 8} & = & 5\,\dfrac{16}{24} \\ +6\,\dfrac{5}{8} & = & +6\,\dfrac{5\cdot 3}{8\cdot 3} & = & +6\,\dfrac{15}{24} \\ \hline & & & & 11\,\dfrac{31}{24} \end{array}$$

An answer that is part mixed fraction, part improper fraction, is **not** allowed. To finish, we need to change the improper fractional part to a mixed fraction, then add. 31 divided by 24 is 1, with a remainder of 7. That is, $31/24 = 1\,\frac{7}{24}$. Now we can add whole number parts and fractional parts.

$$\begin{aligned} 11\,\frac{31}{24} &= 11 + 1\,\frac{7}{24} \\ &= 12\,\frac{7}{24}. \end{aligned}$$

Thus, the total material required is $12\frac{7}{24}$ yards.

Answer: $12\frac{3}{8}$ feet

□

Subtracting Mixed Fractions

Let's look at some examples that subtract two mixed fractions.

You Try It!

Simplify:

$$5\frac{2}{3} - 3\frac{1}{5}$$

EXAMPLE 5. Simplify: $4\frac{5}{8} - 2\frac{1}{16}$.

Solution. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then subtract.

$$\begin{aligned} 4\,\frac{5}{8} - 2\,\frac{1}{16} &= \frac{37}{8} - \frac{33}{16} && \text{Change to improper fractions.} \\ &= \frac{37 \cdot 2}{8 \cdot 2} - \frac{33}{16} && \text{Equivalent fractions with LCD} = 16. \\ &= \frac{74}{16} - \frac{33}{16} && \text{Simplify numerators and denominators.} \\ &= \frac{41}{16} && \text{Add numerators over common denominator.} \end{aligned}$$

Although this answer is perfectly acceptable, let's change the answer to a mixed fraction: 41 divided by 16 is 2, with a remainder of 9. Thus,

$$4\,\frac{5}{8} - 2\,\frac{1}{16} = 2\,\frac{9}{16}.$$

Answer: $2\frac{7}{15}$

□

You Try It!

Simplify:

$$4\frac{7}{9} - 2\frac{3}{18}$$

EXAMPLE 6. Simplify: $5\frac{3}{4} - 2\frac{1}{3}$.

Solution. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then subtract.

$$\begin{aligned} 5\,\frac{3}{4} - 2\,\frac{1}{3} &= \frac{23}{4} - \frac{7}{3} && \text{Change to improper fractions.} \\ &= \frac{23 \cdot 3}{4 \cdot 3} - \frac{7 \cdot 4}{3 \cdot 4} && \text{Equivalent fractions with LCD} = 12. \\ &= \frac{69}{12} - \frac{28}{12} && \text{Simplify numerators and denominators.} \\ &= \frac{41}{12} && \text{Add numerators over common denominator.} \end{aligned}$$

Although this answer is perfectly acceptable, let's change the answer to a mixed fraction: 41 divided by 12 is 3, with a remainder of 5. Thus,

$$5\,\frac{3}{4} - 2\,\frac{1}{3} = 3\,\frac{5}{12}.$$

Answer: $2\frac{11}{18}$

□

Mixed Fraction Approach. There is another possible approach, based on the fact that a mixed fraction is a sum. Let's revisit Example 6.

You Try It!

EXAMPLE 7. Simplify: $5\frac{3}{4} - 2\frac{1}{3}$.

Simplify:

$$8\frac{5}{6} - 4\frac{3}{8}$$

Solution. A mixed fraction is a sum.

$$5\,\frac{3}{4} - 2\,\frac{1}{3} = \left(5 + \frac{3}{4}\right) - \left(2 + \frac{1}{3}\right)$$

Distribute the negative sign.

$$= 5 + \frac{3}{4} - 2 - \frac{1}{3}$$

We could change the subtraction to adding the opposite, change the order of addition, then change the adding of opposites back to subtraction. However, it is much easier if we look at this last line as a request to add four numbers, two of which are positive and two of which are negative. Changing the order does not affect the answer.

$$= (5 - 2) + \left(\frac{3}{4} - \frac{1}{3}\right)$$

Note that we did not change the signs of any of the four numbers. We just changed the order. Subtract the whole number parts. Make equivalent fractions with a common denominator, then subtract the fractional parts.

$$= 3 + \left(\frac{3 \cdot 3}{4 \cdot 3} - \frac{1 \cdot 4}{3 \cdot 4}\right) \quad \text{Create equivalent fractions.}$$

$$= 3 + \left(\frac{9}{12} - \frac{4}{12}\right) \quad \text{Simplify numerators and denominators.}$$

$$= 3 + \frac{5}{12} \quad \text{Subtract fractional parts.}$$

Thus,

$$5\,\frac{3}{4} - 2\,\frac{1}{3} = 3\,\frac{5}{12}.$$

Note that this is exactly the same answer as that found in Example 6.

Answer: $4\frac{11}{24}$

□

In Example 6, we see that we handle subtraction of mixed fractions in exactly the same manner that we handle addition of mixed fractions.

Subtracting Mixed Fractions. To subtract two mixed fractions, subtract their whole number parts, then subtract their fractional parts.

Working in Vertical Format

When subtracting mixed fractions, many prefer to work in a vertical format. For example, here is how we would arrange the solution from Example 6 and Example 7 in vertical format. We create equivalent fractions, then subtract the whole number parts and fractional parts.

$$\begin{array}{rcrcr} 5\,\dfrac{3}{4} & = & 5\,\dfrac{3\cdot 3}{4\cdot 3} & = & 5\,\dfrac{9}{12} \\ -2\,\dfrac{1}{3} & = & -2\,\dfrac{1\cdot 4}{3\cdot 4} & = & -2\,\dfrac{4}{12} \\ \hline & & & & 3\,\dfrac{5}{12} \end{array}$$

Note that the answer is identical to the answer found in Example 6 and Example 7. That is,

$$5\,\frac{3}{4} - 2\,\frac{1}{3} = 3\,\frac{5}{12}.$$

Borrowing in Vertical Format

Consider the following example.

You Try It!

Simplify:

$$7\frac{1}{14} - 2\frac{5}{21}$$

EXAMPLE 8. Simplify: $8\frac{1}{4} - 5\frac{5}{6}$.

Solution. Create equivalent fractions with a common denominator.

$$\begin{array}{rcrcr} 8\,\dfrac{1}{4} & = & 8\,\dfrac{1\cdot 3}{4\cdot 3} & = & 8\,\dfrac{3}{12} \\ -5\,\dfrac{5}{6} & = & -5\,\dfrac{5\cdot 2}{6\cdot 2} & = & -5\,\dfrac{10}{12} \\ \hline \end{array}$$

You can see the difficulty. On the far right, we cannot subtract 10/12 from 3/12. The fix is to borrow 1 from 8 in the form of 12/12 and add it to the 3/12.

$$\begin{array}{rcrcr} 8\,\dfrac{3}{12} & = & 7+\dfrac{12}{12}+\dfrac{3}{12} & = & 7\,\dfrac{15}{12} \\ -5\,\dfrac{10}{12} & = & -5\,\dfrac{10}{12} & = & -5\,\dfrac{10}{12} \\ \hline & & & & 2\,\dfrac{5}{12} \end{array}$$

Now we can subtract. Hence, $8\frac{1}{4} - 5\frac{5}{6} = 2\frac{5}{12}$.

Answer: $4\frac{5}{6}$

You Try It!

EXAMPLE 9. Jim has a metal rod of length 10 inches. He cuts a length from the metal rod measuring $2\frac{7}{8}$ inches. What is the length of the remaining piece?

Sarah has a length of curtain material that measures 12 feet. She cuts a length of $6\frac{2}{3}$ feet from her curtain material. What is the length of the remaining piece?

Solution. To find the length of the remaining piece, we must subtract $2\frac{7}{8}$ from 10. There is no fractional part on the first number. To remedy this absence, we borrow 1 from 10 in the form of 8/8. Then we can subtract.

$$\begin{array}{rcrcr} 10 & = & 9+\dfrac{8}{8} & = & 9\,\dfrac{8}{8} \\ -2\,\dfrac{7}{8} & = & -2\,\dfrac{7}{8} & = & -2\,\dfrac{7}{8} \\ \hline & & & & 7\,\dfrac{1}{8} \end{array}$$

Hence, the length of the remaining piece of the metal rod is $7\frac{1}{8}$ inches.

Answer: $5\frac{1}{3}$ feet

Exercises

In Exercises 1-24, add or subtract the mixed fractions, as indicated, by first converting each mixed fraction to an improper fraction. Express your answer as a mixed fraction.

1. $9\,\frac{1}{4} + 9\,\frac{1}{2}$

2. $2\,\frac{1}{3} + 9\,\frac{1}{2}$

3. $6\,\frac{1}{2} - 1\,\frac{1}{3}$

4. $5\,\frac{1}{3} - 1\,\frac{3}{4}$

5. $9\,\frac{1}{2} + 7\,\frac{1}{4}$

6. $1\,\frac{1}{3} + 9\,\frac{3}{4}$

7. $5\,\frac{2}{3} + 4\,\frac{1}{2}$

8. $1\,\frac{9}{16} + 2\,\frac{3}{4}$

9. $3\,\frac{1}{3} - 1\,\frac{1}{4}$

10. $2\,\frac{1}{2} - 1\,\frac{1}{4}$

11. $8\,\frac{1}{2} - 1\,\frac{1}{3}$

12. $5\,\frac{1}{2} - 1\,\frac{2}{3}$

13. $4\,\frac{1}{2} - 1\,\frac{1}{8}$

14. $2\,\frac{1}{2} - 1\,\frac{1}{3}$

15. $4\,\frac{7}{8} + 1\,\frac{3}{4}$

16. $1\,\frac{1}{8} + 5\,\frac{1}{2}$

17. $2\,\frac{1}{3} - 1\,\frac{1}{4}$

18. $5\,\frac{1}{3} - 1\,\frac{1}{4}$

19. $9\,\frac{1}{2} - 1\,\frac{3}{4}$

20. $5\,\frac{1}{2} - 1\,\frac{3}{16}$

21. $4\,\frac{2}{3} + 1\,\frac{1}{4}$

22. $1\,\frac{1}{4} + 1\,\frac{1}{3}$

23. $9\,\frac{1}{2} + 3\,\frac{1}{8}$

24. $1\,\frac{1}{4} + 1\,\frac{2}{3}$

In Exercises 25-48, add or subtract the mixed fractions, as indicated, by using vertical format. Express your answer as a mixed fraction.

25. $3\,\frac{1}{2} + 3\,\frac{3}{4}$

26. $1\,\frac{1}{2} + 2\,\frac{2}{3}$

27. $1\,\frac{3}{8} + 1\,\frac{1}{4}$

28. $2\,\frac{1}{4} + 1\,\frac{2}{3}$

29. $1\,\frac{7}{8} + 1\,\frac{1}{2}$

30. $1\,\frac{3}{4} + 4\,\frac{1}{2}$

31. $8\,\frac{1}{2} - 5\,\frac{2}{3}$

32. $8\,\frac{1}{2} - 1\,\frac{2}{3}$

33. $7\,\frac{1}{2} - 1\,\frac{3}{16}$

34. $5\,\frac{1}{2} - 1\,\frac{1}{3}$

35. $9\,\frac{1}{2} - 1\,\frac{1}{3}$

36. $2\,\frac{1}{2} - 1\,\frac{3}{16}$

37. $5\,\frac{1}{3} - 2\,\frac{1}{2}$

38. $4\,\frac{1}{4} - 1\,\frac{1}{2}$

39. $9\,\frac{1}{2} - 2\,\frac{2}{3}$

40. $7\,\frac{1}{2} - 4\,\frac{2}{3}$

41. $1\,\frac{1}{16} + 1\,\frac{3}{4}$

42. $1\,\frac{1}{4} + 1\,\frac{1}{3}$

43. $8\,\frac{1}{2} + 3\,\frac{2}{3}$

44. $1\,\frac{2}{3} + 2\,\frac{1}{2}$

45. $6\,\frac{1}{2} - 1\,\frac{3}{16}$

46. $4\,\frac{1}{2} - 1\,\frac{1}{3}$

47. $2\,\frac{2}{3} + 1\,\frac{1}{4}$

48. $1\,\frac{1}{2} + 1\,\frac{1}{16}$

❧ ❧ ❧ Answers ☙ ☙ ☙

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1. $18\,\frac{3}{4}$

3. $5\,\frac{1}{6}$

5. $16\,\frac{3}{4}$

7. $10\,\frac{1}{6}$

9. $2\,\frac{1}{12}$

11. $7\,\frac{1}{6}$

13. $3\,\frac{3}{8}$

15. $6\,\frac{5}{8}$

17. $1\,\frac{1}{12}$

19. $7\,\frac{3}{4}$

21. $5\,\frac{11}{12}$

23. $12\,\frac{5}{8}$

25. $7\frac{1}{4}$

27. $2\frac{5}{8}$

29. $3\frac{3}{8}$

31. $2\frac{5}{6}$

33. $6\frac{5}{16}$

35. $8\frac{1}{6}$

37. $2\frac{5}{6}$

39. $6\frac{5}{6}$

41. $2\frac{13}{16}$

43. $12\frac{1}{6}$

45. $5\frac{5}{16}$

47. $3\frac{11}{12}$

4.7 Order of Operations with Fractions

Let's begin by taking powers of fractions. Recall that

$$a^m = \underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ times}}$$

You Try It!

EXAMPLE 1. Simplify: $(-3/4)^2$.

Solution. By definition,

$$\begin{aligned}
\left(-\frac{3}{4}\right)^2 &= \left(-\frac{3}{4}\right)\left(-\frac{3}{4}\right) && \text{Fact: } a^2 = a \cdot a. \\
&= \frac{3 \cdot 3}{4 \cdot 4} && \text{Multiply numerators and denominators. Product of even number of negative factors is positive.} \\
&= \frac{9}{16} && \text{Simplify.}
\end{aligned}$$

Simplify:

$$\left(-\frac{2}{5}\right)^2$$

Answer: $4/25$

□

You Try It!

EXAMPLE 2. Simplify: $(-2/3)^3$.

Solution. By definition,

$$\begin{aligned}
\left(-\frac{2}{3}\right)^3 &= \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right) && \text{Fact: } a^3 = a \cdot a \cdot a. \\
&= -\frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3} && \text{Multiply numerators and denominators. Product of odd number of negative factors is negative.} \\
&= -\frac{8}{27} && \text{Simplify.}
\end{aligned}$$

Simplify:

$$\left(-\frac{1}{6}\right)^3$$

Answer: $-1/216$

□

The last two examples reiterate a principle learned earlier.

Odd and Even.

- The product of an **even** number of negative factors is positive.
- The product of an **odd** number of negative factors is negative.

Order of Operations

For convenience, we repeat here the rules guiding order of operations.

Rules Guiding Order of Operations. When evaluating expressions, proceed in the following order.

1. Evaluate expressions contained in grouping symbols first. If grouping symbols are nested, evaluate the expression in the innermost pair of grouping symbols first.
2. Evaluate all exponents that appear in the expression.
3. Perform all multiplications and divisions in the order that they appear in the expression, moving left to right.
4. Perform all additions and subtractions in the order that they appear in the expression, moving left to right.

You Try It!

Simplify:

$$-\frac{2}{3}+\frac{3}{4}\left(-\frac{1}{2}\right)$$

EXAMPLE 3. Simplify: $-\frac{1}{2}+\frac{1}{4}\left(-\frac{1}{3}\right)$.

Solution. Multiply first, then add.

$$\begin{aligned}
-\frac{1}{2}+\frac{1}{4}\left(-\frac{1}{3}\right) &= -\frac{1}{2}+\left(-\frac{1}{12}\right) && \text{Multiply: } \frac{1}{4}\left(-\frac{1}{3}\right)=-\frac{1}{12}. \\
&= -\frac{1\cdot 6}{2\cdot 6}+\left(-\frac{1}{12}\right) && \text{Equivalent fractions, LCD} = 12. \\
&= -\frac{6}{12}+\left(-\frac{1}{12}\right) && \text{Simplify numerator and denominator.} \\
&= -\frac{7}{12} && \text{Add over common denominator.}
\end{aligned}$$

Answer: $-25/24$

□

You Try It!

EXAMPLE 4. Simplify: $2\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right)$.

Simplify:

$$3\left(-\frac{1}{3}\right)^2 - 2\left(-\frac{1}{3}\right)$$

Solution. Exponents first, then multiply, then add.

$$\begin{aligned}
2\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) &= 2\left(\frac{1}{4}\right) + 4\left(-\frac{1}{2}\right) && \text{Exponent first: } \left(-\frac{1}{2}\right)^2 = \frac{1}{4}. \\
&= \frac{1}{2} + \left(-\frac{2}{1}\right) && \text{Multiply: } 2\left(\frac{1}{4}\right) = \frac{1}{2} \\
& && \text{and } 4\left(-\frac{1}{2}\right) = -\frac{2}{1}. \\
&= \frac{1}{2} + \left(-\frac{2 \cdot 2}{1 \cdot 2}\right) && \text{Equivalent fractions, LCD} = 2. \\
&= \frac{1}{2} + \left(-\frac{4}{2}\right) && \text{Simplify numerator and denominator.} \\
&= -\frac{3}{2} && \text{Add over common denominator.}
\end{aligned}$$

Answer: 1

□

You Try It!

EXAMPLE 5. Given $a = -3/4$, $b = 1/2$, $c = 1/3$, and $d = -1/4$, evaluate the expression $ab - cd$.

Given $a = -1/2$, $b = 1/3$, and $c = -1/5$, evaluate $a + bc$.

Solution. Recall that it is good practice to prepare parentheses before substituting.

$$ad - bc = (\quad)(\quad) - (\quad)(\quad)$$

Substitute the given values into the algebraic expression, then simplify using order of operations.

$$\begin{aligned}
ab - cd &= \left(-\frac{3}{4}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{3}\right)\left(-\frac{1}{4}\right) && \text{Substitute: } -3/4 \text{ for } a,\ 1/2 \text{ for } b, \\
& && 1/3 \text{ for } c, \text{ and } -1/4 \text{ for } d. \\
&= -\frac{3}{8} - \left(-\frac{1}{12}\right) && \text{Multiply first: } \left(-\frac{3}{4}\right)\left(\frac{1}{2}\right) = -\frac{3}{8} \\
& && \text{and } \left(\frac{1}{3}\right)\left(-\frac{1}{4}\right) = -\frac{1}{12}. \\
&= -\frac{3}{8} + \frac{1}{12} && \text{Subtract by adding opposite.} \\
&= -\frac{3 \cdot 3}{8 \cdot 3} + \frac{1 \cdot 2}{12 \cdot 2} && \text{Equivalent fractions; LCD} = 24. \\
&= -\frac{9}{24} + \frac{2}{24} && \text{Simplify numerators and denominators.} \\
&= -\frac{7}{24} && \text{Add over common denominator.}
\end{aligned}$$

Answer: $-17/30$

□

You Try It!

Give $a = -1/2$ and $b = -1/3$, evaluate $ab \div (a + b)$.

EXAMPLE 6. Given $a = -1/4$ and $b = 1/2$, evaluate $(a^2 - b^2) \div (a + b)$.

Solution. Recall that it is good practice to prepare parentheses before substituting.

$$(a^2 - b^2) \div (a + b) = \Big((\quad)^2 - (\quad)^2\Big) \div \Big((\quad) + (\quad)\Big)$$

Substitute the given values into the algebraic expression, then evaluate exponents first.

$$\begin{aligned}
(a^2 - b^2) \div (a + b) &= \left(\left(-\frac{1}{4}\right)^2 - \left(\frac{1}{2}\right)^2\right) \div \left(\left(-\frac{1}{4}\right) + \left(\frac{1}{2}\right)\right) \\
&= \left(\frac{1}{16} - \frac{1}{4}\right) \div \left(-\frac{1}{4} + \frac{1}{2}\right)
\end{aligned}$$

We must evaluate parentheses first. Inside each set of parentheses, create equivalent fractions and perform subtractions and additions next.

$$\begin{aligned}
&= \left(\frac{1}{16} - \frac{1 \cdot 4}{4 \cdot 4}\right) \div \left(-\frac{1}{4} + \frac{1 \cdot 2}{2 \cdot 2}\right) \\
&= \left(\frac{1}{16} - \frac{4}{16}\right) \div \left(-\frac{1}{4} + \frac{2}{4}\right) \\
&= -\frac{3}{16} \div \frac{1}{4}
\end{aligned}$$

Invert and multiply.

$$\begin{aligned} &= -\frac{3}{16} \cdot \frac{4}{1} \\ &= -\frac{12}{16} \end{aligned}$$

Reduce.

$$\begin{aligned} &= -\frac{12 \div 4}{16 \div 4} \\ &= -\frac{3}{4} \end{aligned}$$

Note: In the last step, you could also reduce by prime factoring numerator and denominator and canceling common factors.

Answer: $-1/5$

Complex Fractions

Complex Fractions. When the numerator and denominator of a fraction contain fractions themselves, such an expression is called a *complex fraction.*

You can use the standard order of operations to simplify a complex fraction. Recall the advice when a fraction is present.

Fractional Expressions. If a fractional expression is present, simplify the numerator and denominator separately, then divide.

You Try It!

EXAMPLE 7. Simplify:

$$\frac{-\dfrac{1}{2} + \dfrac{1}{3}}{\dfrac{3}{4} - \dfrac{3}{2}}$$

Simplify:

$$\frac{\dfrac{1}{4} - \dfrac{1}{3}}{\dfrac{1}{4} + \dfrac{1}{3}}$$

Solution. We have addition in the numerator, subtraction in the denominator. In each case, we need equivalent fractions with a common denominator.

$$\begin{aligned}
\frac{-\frac{1}{2}+\frac{1}{3}}{\frac{3}{4}-\frac{3}{2}} &= \frac{-\frac{1\cdot 3}{2\cdot 3}+\frac{1\cdot 2}{3\cdot 2}}{\frac{3}{4}-\frac{3\cdot 2}{2\cdot 2}} && \text{Create equivalent fractions.}\\
&= \frac{-\frac{3}{6}+\frac{2}{6}}{\frac{3}{4}-\frac{6}{4}} && \text{Simplify numerator and denominator.}\\
&= \frac{-\frac{1}{6}}{-\frac{3}{4}} && \text{Numerator: } -\frac{3}{6}+\frac{2}{6}=-\frac{1}{6}.\\
& && \text{Denominator: } \frac{3}{4}-\frac{6}{4}=-\frac{3}{4}.
\end{aligned}$$

The last expression asks us to divide. Invert and multiply.

$$\begin{aligned}
&= -\frac{1}{6} \div \left(-\frac{3}{4}\right) && \text{A complex fraction means divide.}\\
&= -\frac{1}{6} \cdot \left(-\frac{4}{3}\right) && \text{Invert and multiply.}
\end{aligned}$$

Like signs (two negatives) give a positive product. Multiply numerators and denominators, then reduce.

$$\begin{aligned}
&= \frac{4}{18} && \text{Like signs yields positive answer.}\\
& && \text{Multiply numerators and denominators.}\\
&= \frac{4 \div 2}{18 \div 2} && \text{Divide both numerator and denominator by 2.}\\
&= \frac{2}{9} && \text{Simplify.}
\end{aligned}$$

Alternatively, one could prime factor and cancel to reduce to lowest terms; that is,

$$\begin{aligned}
\frac{4}{18} &= \frac{2\cdot 2}{2\cdot 3\cdot 3} && \text{Prime factor.}\\
&= \frac{\cancel{2}\cdot 2}{\cancel{2}\cdot 3\cdot 3} && \text{Cancel common factors.}\\
&= \frac{2}{9} && \text{Simplify.}
\end{aligned}$$

Answer: $-1/7$

□

Clearing Fractions. An alternate technique for simplifying complex fractions is available.

Clearing Fractions from Complex Fractions. You can clear fractions from a complex fraction using the following algorithm:

1. Determine an LCD_1 for the numerator.
2. Determine an LCD_2 for the denominator.
3. Determine an LCD for both LCD_1 and LCD_2.
4. Multiply both numerator and denominator by this "combined" LCD.

Let's apply this technique to the complex fraction of Example 7.

You Try It!

EXAMPLE 8. Simplify:

$$\frac{-\frac{1}{2}+\frac{1}{3}}{\frac{3}{4}-\frac{3}{2}}$$

Simplify:

$$\frac{-\frac{2}{3}+\frac{1}{5}}{\frac{4}{5}-\frac{1}{2}}$$

Solution. As we saw in the solution in Example 7, common denominators of 6 and 4 were used for the numerator and denominator, respectively. Thus, a common denominator for both numerator and denominator would be 12. We begin the alternate solution technique by multiplying both numerator and denominator by 12.

$$\begin{aligned}
\frac{-\frac{1}{2}+\frac{1}{3}}{\frac{3}{4}-\frac{3}{2}} &= \frac{12\left(-\frac{1}{2}+\frac{1}{3}\right)}{12\left(\frac{3}{4}-\frac{3}{2}\right)} && \text{Multiply numerator and denominator by 12.} \\
&= \frac{12\left(-\frac{1}{2}\right)+12\left(\frac{1}{3}\right)}{12\left(\frac{3}{4}\right)-12\left(\frac{3}{2}\right)} && \text{Distribute the 12.} \\
&= \frac{-6+4}{9-18} && \text{Multiply: } 12(-1/2)=-6,\ 12(1/2)=4. \\
& && 12(3/4)=9, \text{ and } 12(3/2)=18. \\
&= \frac{-2}{-9} && \text{Simplify.} \\
&= \frac{2}{9} && \text{Like signs yields positive.}
\end{aligned}$$

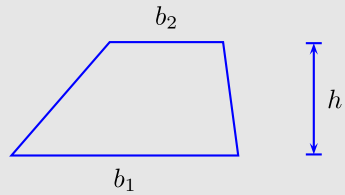
Answer: $-14/9$

□

Application — Trapezoid

A trapezoid is a special type of quadrilateral (four-sided polygon).

Trapezoid. A quadrilateral with one pair of parallel opposite sides is called a *trapezoid.*



The pair of parallel sides are called the *bases* of the trapezoid. Their lengths are marked by the variables b_1 and b_2 in the figure above. The distance between the parallel bases is called the *height* or *altitude* of the trapezoid. The height is marked by the variable h in the figure above.

Mathematicians use *subscripts* to create new variables. Thus, b_1 ("b sub 1") and b_2 ("b sub 2") are two distinct variables, used in this case to represent the length of the bases of the trapezoid.

By drawing in a diagonal, we can divide the trapezoid into two triangles (see Figure 4.14).

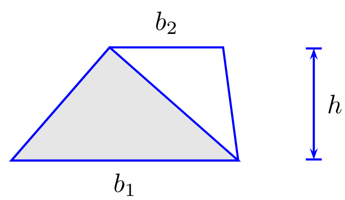


Figure 4.14: Dividing the trapezoid into two triangles.

We can find the area of the trapezoid by summing the areas of the two triangles.

- The shaded triangle in Figure 4.14 has base b_1 and height h. Hence, the area of the shaded triangle is $(1/2)b_1h$.
- The unshaded triangle in Figure 4.14 has base b_2 and height h. Hence, the area of the unshaded triangle is $(1/2)b_2h$.

Summing the areas, the area of the trapezoid is

$$\text{Area of Trapezoid} = \frac{1}{2}b_1h + \frac{1}{2}b_2h.$$

We can use the distributive property to factor out a $(1/2)h$.

Area of a Trapezoid. A trapezoid with bases b_1 and b_2 and height h has area

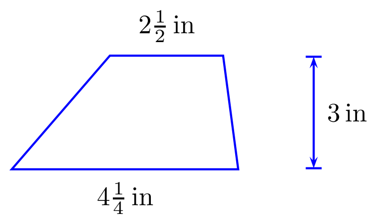
$$A = \frac{1}{2}h\left(b_1 + b_2\right).$$

That is, to find the area, sum the bases, multiply by the height, and take one-half of the result.

You Try It!

A trapezoid has bases measuring 6 and 15 feet, respectively. The height of the trapezoid is 5 feet. Find the area of the trapezoid.

EXAMPLE 9. Find the area of the trapezoid pictured below.



Solution. The formula for the area of a trapezoid is

$$A = \frac{1}{2}h\left(b_1 + b_2\right)$$

Substituting the given bases and height, we get

$$A = \frac{1}{2}(3)\left(4\frac{1}{4} + 2\frac{1}{2}\right).$$

Simplify the expression inside the parentheses first. Change mixed fractions to improper fractions, make equivalent fractions with a common denominator, then add.

$$\begin{aligned} A &= \frac{1}{2}(3)\left(\frac{17}{4} + \frac{5}{2}\right) \\ &= \frac{1}{2}(3)\left(\frac{17}{4} + \frac{5 \cdot 2}{2 \cdot 2}\right) \\ &= \frac{1}{2}(3)\left(\frac{17}{4} + \frac{10}{4}\right) \\ &= \frac{1}{2}\left(\frac{3}{1}\right)\left(\frac{27}{4}\right) \end{aligned}$$

Multiply numerators and denominators.

$$= \frac{81}{8}$$

This improper fraction is a perfectly good answer, but let's change this result to a mixed fraction (81 divided by 8 is 10 with a remainder of 1). Thus, the area of the trapezoid is

$$A = 10\,\frac{1}{8}\,\text{square inches.}$$

Answer: $52\frac{1}{2}$ square feet

❧ ❧ ❧ Exercises ☙ ☙ ☙

In Exercises 1-8, simplify the expression.

1. $\left(-\dfrac{7}{3}\right)^3$

2. $\left(\dfrac{1}{2}\right)^3$

3. $\left(\dfrac{5}{3}\right)^4$

4. $\left(-\dfrac{3}{5}\right)^4$

5. $\left(\dfrac{1}{2}\right)^5$

6. $\left(\dfrac{3}{4}\right)^5$

7. $\left(\dfrac{4}{3}\right)^2$

8. $\left(-\dfrac{8}{5}\right)^2$

---

9. If $a = 7/6$, evaluate a^3.

10. If $e = 1/6$, evaluate e^3.

11. If $e = -2/3$, evaluate $-e^2$.

12. If $c = -1/5$, evaluate $-c^2$.

13. If $b = -5/9$, evaluate b^2.

14. If $c = 5/7$, evaluate c^2.

15. If $b = -1/2$, evaluate $-b^3$.

16. If $a = -2/9$, evaluate $-a^3$.

In Exercises 17-36, simplify the expression.

17. $\left(-\dfrac{1}{2}\right)\left(\dfrac{1}{6}\right) - \left(\dfrac{7}{8}\right)\left(-\dfrac{7}{9}\right)$

18. $\left(-\dfrac{3}{4}\right)\left(\dfrac{1}{2}\right) - \left(\dfrac{3}{5}\right)\left(\dfrac{1}{4}\right)$

19. $\left(-\dfrac{9}{8}\right)^2 - \left(-\dfrac{3}{2}\right)\left(\dfrac{7}{3}\right)$

20. $\left(\dfrac{3}{2}\right)^2 - \left(\dfrac{7}{8}\right)\left(-\dfrac{1}{2}\right)$

21. $\left(-\dfrac{1}{2}\right)\left(-\dfrac{7}{4}\right) - \left(-\dfrac{1}{2}\right)^2$

22. $\left(\dfrac{1}{5}\right)\left(-\dfrac{9}{4}\right) - \left(\dfrac{7}{4}\right)^2$

23. $-\dfrac{7}{6} - \dfrac{1}{7} \cdot \dfrac{7}{9}$

24. $-\dfrac{4}{9} - \dfrac{8}{5} \cdot \dfrac{8}{9}$

25. $\dfrac{3}{4} + \dfrac{9}{7}\left(-\dfrac{7}{6}\right)$

26. $\dfrac{3}{2} + \dfrac{1}{4}\left(-\dfrac{9}{8}\right)$

27. $\left(-\dfrac{1}{3}\right)^2 + \left(\dfrac{7}{8}\right)\left(-\dfrac{1}{3}\right)$

28. $\left(-\dfrac{2}{9}\right)^2 + \left(\dfrac{2}{3}\right)\left(\dfrac{1}{2}\right)$

29. $\dfrac{5}{9} + \dfrac{5}{9} \cdot \dfrac{7}{9}$

30. $-\dfrac{1}{2} + \dfrac{9}{8} \cdot \dfrac{1}{3}$

31. $\left(-\frac{5}{6}\right)\left(\frac{3}{8}\right)+\left(-\frac{7}{9}\right)\left(-\frac{3}{4}\right)$

32. $\left(\frac{7}{4}\right)\left(\frac{6}{5}\right)+\left(-\frac{2}{5}\right)\left(\frac{8}{3}\right)$

33. $\frac{4}{3}-\frac{2}{9}\left(-\frac{3}{4}\right)$

34. $-\frac{1}{3}-\frac{1}{5}\left(-\frac{4}{3}\right)$

35. $\left(-\frac{5}{9}\right)\left(\frac{1}{2}\right)+\left(-\frac{1}{6}\right)^2$

36. $\left(\frac{1}{4}\right)\left(\frac{1}{6}\right)+\left(-\frac{5}{6}\right)^2$

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37. Given $a=-5/4$, $b=1/2$, and $c=3/8$, evaluate $a+bc$.

38. Given $a=-3/5$, $b=1/5$, and $c=1/3$, evaluate $a+bc$.

39. Given $x=-1/8$, $y=5/2$, and $z=-1/2$, evaluate the expression $x+yz$.

40. Given $x=-5/9$, $y=1/4$, and $z=-2/3$, evaluate the expression $x+yz$.

41. Given $a=3/4$, $b=5/7$, and $c=1/2$, evaluate the expression $a-bc$.

42. Given $a=5/9$, $b=2/3$, and $c=2/9$, evaluate the expression $a-bc$.

43. Given $x=-3/2$, $y=1/4$, and $z=-5/7$, evaluate x^2-yz.

44. Given $x=-3/2$, $y=-1/2$, and $z=5/3$, evaluate x^2-yz.

45. Given $a=6/7$, $b=2/3$, $c=-8/9$, and $d=-6/7$, evaluate $ab+cd$.

46. Given $a=4/9$, $b=-3/2$, $c=7/3$, and $d=-8/9$, evaluate $ab+cd$.

47. Given $w=-1/8$, $x=-2/7$, $y=-1/2$, and $z=8/7$, evaluate $wx-yz$.

48. Given $w=2/7$, $x=-9/4$, $y=-3/4$, and $z=-9/2$, evaluate $wx-yz$.

49. Given $x=3/8$, $y=3/5$, and $z=-3/2$, evaluate $xy+z^2$.

50. Given $x=-1/2$, $y=7/5$, and $z=-3/2$, evaluate $xy+z^2$.

51. Given $u=9/7$, $v=2/3$, and $w=-3/7$, evaluate $uv-w^2$.

52. Given $u=8/7$, $v=-4/3$, and $w=2/3$, evaluate $uv-w^2$.

53. Given $a=7/8$, $b=-1/4$, and $c=-3/2$, evaluate a^2+bc.

54. Given $a=-5/8$, $b=3/2$, and $c=-3/2$, evaluate a^2+bc.

55. Given $u=1/3$, $v=5/2$, and $w=-2/9$, evaluate the expression $u-vw$.

56. Given $u=-1/2$, $v=1/4$, and $w=-1/4$, evaluate the expression $u-vw$.

In Exercises 57-68, simplify the complex rational expression.

57. $$\frac{\frac{8}{3}+\frac{7}{6}}{-\frac{9}{2}-\frac{1}{4}}$$

58. $$\frac{\frac{7}{8}+\frac{1}{9}}{\frac{8}{9}-\frac{1}{6}}$$

59. $$\frac{\frac{3}{4}+\frac{4}{3}}{\frac{1}{9}+\frac{5}{3}}$$

60. $$\frac{-\frac{9}{8}-\frac{6}{5}}{\frac{7}{4}+\frac{1}{2}}$$

61. $$\frac{\frac{7}{5}+\frac{5}{2}}{-\frac{1}{4}+\frac{1}{2}}$$

62. $$\frac{\frac{5}{6}+\frac{2}{3}}{\frac{3}{5}+\frac{2}{3}}$$

63. $$\frac{-\frac{3}{2}-\frac{2}{3}}{-\frac{7}{4}-\frac{2}{3}}$$

64. $$\frac{\frac{8}{9}+\frac{3}{4}}{-\frac{2}{3}-\frac{1}{6}}$$

65. $$\frac{-\frac{1}{2}-\frac{4}{7}}{-\frac{5}{7}+\frac{1}{6}}$$

66. $$\frac{-\frac{3}{2}-\frac{5}{8}}{\frac{3}{4}-\frac{1}{2}}$$

67. $$\frac{-\frac{3}{7}-\frac{1}{3}}{\frac{1}{3}-\frac{6}{7}}$$

68. $$\frac{-\frac{5}{8}-\frac{6}{5}}{-\frac{5}{4}-\frac{3}{8}}$$

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69. A trapezoid has bases measuring $3\frac{3}{8}$ and $5\frac{1}{2}$ feet, respectively. The height of the trapezoid is 7 feet. Find the area of the trapezoid.

70. A trapezoid has bases measuring $2\frac{1}{2}$ and $6\frac{7}{8}$ feet, respectively. The height of the trapezoid is 3 feet. Find the area of the trapezoid.

71. A trapezoid has bases measuring $2\frac{1}{4}$ and $7\frac{3}{8}$ feet, respectively. The height of the trapezoid is 7 feet. Find the area of the trapezoid.

72. A trapezoid has bases measuring $3\frac{1}{8}$ and $6\frac{1}{2}$ feet, respectively. The height of the trapezoid is 3 feet. Find the area of the trapezoid.

73. A trapezoid has bases measuring $2\frac{3}{4}$ and $6\frac{5}{8}$ feet, respectively. The height of the trapezoid is 3 feet. Find the area of the trapezoid.

74. A trapezoid has bases measuring $2\frac{1}{4}$ and $7\frac{1}{8}$ feet, respectively. The height of the trapezoid is 5 feet. Find the area of the trapezoid.

Answers

1. $-\frac{343}{27}$

3. $\frac{625}{81}$

5. $\frac{1}{32}$

7. $\frac{16}{9}$

9. $\frac{343}{216}$

11. $-\frac{4}{9}$

13. $\frac{25}{81}$

15. $\frac{1}{8}$

17. $\frac{43}{72}$

19. $\frac{305}{64}$

21. $\frac{5}{8}$

23. $-\frac{23}{18}$

25. $-\frac{3}{4}$

27. $-\frac{13}{72}$

29. $\frac{80}{81}$

31. $\frac{13}{48}$

33. $\frac{3}{2}$

35. $-\frac{1}{4}$

37. $-\frac{17}{16}$

39. $-\frac{11}{8}$

41. $\frac{11}{28}$

43. $\frac{17}{7}$

45. $\frac{4}{3}$

47. $\frac{17}{28}$

49. $\frac{99}{40}$

51. $\frac{33}{49}$

53. $\frac{73}{64}$

55. $\frac{8}{9}$

57. $-\frac{46}{57}$

59. $\frac{75}{64}$

61. $\frac{78}{5}$

63. $\frac{26}{29}$

65. $\frac{45}{23}$

67. $\frac{16}{11}$

69. $31\frac{1}{16}$

71. $33\frac{11}{16}$

73. $14\frac{1}{16}$

4.8 Solving Equations with Fractions

Undoing Subtraction

We can still add the same amount to both sides of an equation without changing the solution.

You Try It!

Solver for x:

$$x - \frac{2}{3} = \frac{1}{5}$$

EXAMPLE 1. Solve for x: $x - \dfrac{5}{6} = \dfrac{1}{3}$.

Solution. To "undo" subtracting 5/6, add 5/6 to both sides of the equation and simplify.

$$\begin{aligned} x - \frac{5}{6} &= \frac{1}{3} && \text{Original equation.} \\ x - \frac{5}{6} + \frac{5}{6} &= \frac{1}{3} + \frac{5}{6} && \text{Add } \frac{5}{6} \text{ to both sides.} \\ x &= \frac{1 \cdot 2}{3 \cdot 2} + \frac{5}{6} && \text{Equivalent fractions, LCD} = 6. \\ x &= \frac{2}{6} + \frac{5}{6} && \text{Simplify.} \\ x &= \frac{7}{6} && \text{Add.} \end{aligned}$$

It is perfectly acceptable to leave your answer as an improper fraction. If you desire, or if you are instructed to do so, you can change your answer to a mixed fraction (7 divided by 6 is 1 with a remainder of 1). That is $x = 1\frac{1}{6}$.

Checking the Solution. Substitute 7/6 for x in the original equation and simplify.

$$\begin{aligned} x - \frac{5}{6} &= \frac{1}{3} && \text{Original equation.} \\ \frac{7}{6} - \frac{5}{6} &= \frac{1}{3} && \text{Substitute } 7/6 \text{ for } x. \\ \frac{2}{6} &= \frac{1}{3} && \text{Subtract.} \\ \frac{1}{3} &= \frac{1}{3} && \text{Reduce.} \end{aligned}$$

Because the last statement is true, we conclude that 7/6 is a solution of the equation $x - 5/6 = 1/3$.

Answer: 13/15

□

Undoing Addition

You can still subtract the same amount from both sides of an equation without changing the solution.

You Try It!

EXAMPLE 2. Solve for x: $x + \dfrac{2}{3} = -\dfrac{3}{5}$.

Solve for x:

$$x + \frac{3}{4} = -\frac{1}{2}$$

Solution. To "undo" adding 2/3, subtract 2/3 from both sides of the equation and simplify.

$$\begin{aligned} x + \frac{2}{3} &= -\frac{3}{5} && \text{Original equation.} \\ x + \frac{2}{3} - \frac{2}{3} &= -\frac{3}{5} - \frac{2}{3} && \text{Subtract } \frac{2}{3} \text{ from both sides.} \\ x &= -\frac{3 \cdot 3}{5 \cdot 3} - \frac{2 \cdot 5}{3 \cdot 5} && \text{Equivalent fractions, LCD} = 15. \\ x &= -\frac{9}{15} - \frac{10}{15} && \text{Simplify.} \\ x &= -\frac{19}{15} && \text{Subtract.} \end{aligned}$$

Readers are encouraged to check this solution in the original equation.

Answer: $-5/4$

□

Undoing Multiplication

We "undo" multiplication by dividing. For example, to solve the equation $2x = 6$, we would divide both sides of the equation by 2. In similar fashion, we could divide both sides of the equation

$$\frac{3}{5}x = \frac{4}{10}$$

by 3/5. However, it is more efficient to take advantage of reciprocals. For convenience, we remind readers of the *Multiplicative Inverse Property*.

Multiplicative Inverse Property. Let a/b be any fraction. The number b/a is called the *multiplicative inverse* or *reciprocal* of a/b. The product of reciprocals is 1.

$$\frac{a}{b} \cdot \frac{b}{a} = 1.$$

Let's put our knowledge of reciprocals to work.

You Try It!

EXAMPLE 3. Solve for x: $\dfrac{3}{5}x = \dfrac{4}{10}$.

Solve for y:

$$\frac{2}{3}y = \frac{4}{5}$$

Solution. To "undo" multiplying by 3/5, multiply both sides by the reciprocal 5/3 and simplify.

$$
\begin{aligned}
\frac{3}{5}x &= \frac{4}{10} && \text{Original equation.} \\
\frac{5}{3}\left(\frac{3}{5}x\right) &= \frac{5}{3}\left(\frac{4}{10}\right) && \text{Multiply both sides by } 5/3. \\
\left(\frac{5}{3}\cdot\frac{3}{5}\right)x &= \frac{20}{30} && \text{On the left, use the associative property to regroup. On the right, multiply.} \\
1x &= \frac{2}{3} && \text{On the left, } \frac{5}{3}\cdot\frac{3}{5} = 1. \text{ On the right, reduce: } \frac{20}{30} = \frac{2}{3}. \\
x &= \frac{2}{3} && \text{On the left, } 1x = x.
\end{aligned}
$$

Checking the Solution. Substitute 2/3 for x in the original equation and simplify.

$$
\begin{aligned}
\frac{3}{5}x &= \frac{4}{10} && \text{Original equation.} \\
\frac{3}{5}\left(\frac{2}{3}\right) &= \frac{4}{10} && \text{Substitute } 2/3 \text{ for } x. \\
\frac{6}{15} &= \frac{4}{10} && \text{Multiply numerators; multiply denominators.} \\
\frac{2}{5} &= \frac{2}{5} && \text{Reduce both sides to lowest terms.}
\end{aligned}
$$

Because this last statement is true, we conclude that 2/3 is a solution of the equation $(3/5)x = 4/10$.

Answer: 6/5

□

You Try It!

Solve for z:

$$-\frac{2}{7}z = \frac{4}{21}$$

EXAMPLE 4. Solve for x: $-\dfrac{8}{9}x = \dfrac{5}{18}$.

Solution. To "undo" multiplying by $-8/9$, multiply both sides by the reciprocal $-9/8$ and simplify.

$$-\frac{8}{9}x = \frac{5}{18} \qquad \text{Original equation.}$$

$$-\frac{9}{8}\left(-\frac{8}{9}x\right) = -\frac{9}{8}\left(\frac{5}{18}\right) \qquad \text{Multiply both sides by } -9/8.$$

$$\left[-\frac{9}{8}\cdot\left(-\frac{8}{9}\right)\right]x = -\frac{3\cdot 3}{2\cdot 2\cdot 2}\cdot\frac{5}{2\cdot 3\cdot 3} \qquad \text{On the left, use the associative property to regroup. On the right, prime factor.}$$

$$1x = -\frac{\cancel{3}\cdot\cancel{3}}{2\cdot 2\cdot 2}\cdot\frac{5}{2\cdot\cancel{3}\cdot\cancel{3}} \qquad \text{On the left, } -\frac{9}{8}\cdot\left(-\frac{8}{9}\right)=1. \text{ On the right, cancel common factors.}$$

$$x = -\frac{5}{16} \qquad \text{On the left, } 1x = x. \text{ Multiply on right.}$$

Readers are encouraged to check this solution in the original equation.

Answer: $-2/3$

Clearing Fractions from the Equation

Although the technique demonstrated in the previous examples is a solid mathematical technique, working with fractions in an equation is not always the most efficient use of your time.

Clearing Fractions from the Equation. To clear all fractions from an equation, multiply both sides of the equation by the least common denominator of the fractions that appear in the equation.

Let's put this idea to work.

You Try It!

EXAMPLE 5. In Example 1, we were asked to solve the following equation for x:

$$x - \frac{5}{6} = \frac{1}{3}.$$

Take a moment to review the solution technique in Example 1. We will now solve this equation by first clearing all fractions from the equation.

Solve for t:

$$t - \frac{2}{7} = -\frac{1}{4}$$

Solution. Multiply both sides of the equation by the least common denominator for the fractions appearing in the equation.

$$\begin{aligned} x - \frac{5}{6} &= \frac{1}{3} && \text{Original equation.} \\ 6\left(x - \frac{5}{6}\right) &= 6\left(\frac{1}{3}\right) && \text{Multiply both sides by 6.} \\ 6x - 6\left(\frac{5}{6}\right) &= 6\left(\frac{1}{3}\right) && \text{Distribute the 6.} \\ 6x - 5 &= 2 && \text{On each side, multiply first.} \\ & && 6\left(\frac{5}{6}\right) = 5 \text{ and } 6\left(\frac{1}{3}\right) = 2. \end{aligned}$$

Note that the equation is now entirely clear of fractions, making it a much simpler equation to solve.

$$\begin{aligned} 6x - 5 + 5 &= 2 + 5 && \text{Add 5 to both sides.} \\ 6x &= 7 && \text{Simplify both sides.} \\ \frac{6x}{6} &= \frac{7}{6} && \text{Divide both sides by 6.} \\ x &= \frac{7}{6} && \text{Simplify.} \end{aligned}$$

Answer: 1/28

Note that this is the same solution found in Example 1.

□

You Try It!

Solve for u:

$$-\frac{7}{9}u = \frac{14}{27}$$

EXAMPLE 6. In Example 4, we were asked to solve the following equation for x.

$$-\frac{8}{9}x = \frac{5}{18}$$

Take a moment to review the solution in Example 4. We will now solve this equation by first clearing all fractions from the equation.

Solution. Multiply both sides of the equation by the least common denominator for the fractions that appear in the equation.

$$\begin{aligned} -\frac{8}{9}x &= \frac{5}{18} && \text{Original equation.} \\ 18\left(-\frac{8}{9}x\right) &= 18\left(\frac{5}{18}\right) && \text{Multiply both sides by 18.} \\ -16x &= 5 && \text{On each side, cancel and multiply.} \\ & && 18\left(-\frac{8}{9}\right) = -16 \text{ and } 18\left(\frac{5}{18}\right) = 5. \end{aligned}$$

Note that the equation is now entirely free of fractions. Continuing,

$$\frac{-16x}{-16} = \frac{5}{-16} \qquad \text{Divide both sides by } -16.$$

$$x = -\frac{5}{16} \qquad \text{Simplify.}$$

Note that this is the same as the solution found in Example 4.

Answer: $-2/3$

□

You Try It!

EXAMPLE 7. Solve for x: $\dfrac{2}{3}x + \dfrac{3}{4} = \dfrac{1}{2}$.

Solve for r:

$$\frac{3}{4}r + \frac{2}{3} = \frac{1}{2}$$

Solution. Multiply both sides of the equation by the least common denominator for the fractions appearing in the equation.

$$\frac{2}{3}x + \frac{3}{4} = \frac{1}{2} \qquad \text{Original equation.}$$

$$12\left(\frac{2}{3}x + \frac{3}{4}\right) = 12\left(\frac{1}{2}\right) \qquad \text{Multiply both sides by 12.}$$

$$12\left(\frac{2}{3}x\right) + 12\left(\frac{3}{4}\right) = 12\left(\frac{1}{2}\right) \qquad \text{On the left, distribute 12.}$$

$$8x + 9 = 6 \qquad \text{Multiply: } 12\left(\frac{2}{3}x\right) = 8x,\ 12\left(\frac{3}{4}\right) = 9, \text{ and } 12\left(\frac{1}{2}\right) = 6.$$

Note that the equation is now entirely free of fractions. We need to isolate the terms containing x on one side of the equation.

$$8x + 9 - 9 = 6 - 9 \qquad \text{Subtract 9 from both sides.}$$

$$8x = -3 \qquad \text{Simplify both sides.}$$

$$\frac{8x}{8} = \frac{-3}{8} \qquad \text{Divide both sides by 8.}$$

$$x = -\frac{3}{8} \qquad \text{Simplify both sides.}$$

Readers are encouraged to check this solution in the original equation.

Answer: $-2/9$

□

You Try It!

EXAMPLE 8. Solve for x: $\dfrac{2}{3} - \dfrac{3x}{4} = \dfrac{x}{2} - \dfrac{1}{8}$.

Solve for s:

$$\frac{3}{2} - \frac{2s}{5} = \frac{s}{3} - \frac{1}{5}$$

Solution. Multiply both sides of the equation by the least common denominator for the fractions in the equation.

$$\frac{2}{3} - \frac{3x}{4} = \frac{x}{2} - \frac{1}{8}$$ Original equation.

$$24\left(\frac{2}{3} - \frac{3x}{4}\right) = 24\left(\frac{x}{2} - \frac{1}{8}\right)$$ Multiply both sides by 24.

$$24\left(\frac{2}{3}\right) - 24\left(\frac{3x}{4}\right) = 24\left(\frac{x}{2}\right) - 24\left(\frac{1}{8}\right)$$ On both sides, distribute 24.

$$16 - 18x = 12x - 3$$ Left: $24\left(\frac{2}{3}\right) = 16$, $24\left(\frac{3x}{4}\right) = 18x$.
Right: $24\left(\frac{x}{2}\right) = 12x$, $24\left(\frac{1}{8}\right) = 3$.

Note that the equation is now entirely free of fractions. We need to isolate the terms containing x on one side of the equation.

$$16 - 18x - 12x = 12x - 3 - 12x$$ Subtract $12x$ from both sides.

$$16 - 30x = -3$$ Left: $-18x - 12x = -30x$.
Right: $12x - 12x = 0$.

$$16 - 30x - 16 = -3 - 16$$ Subtract 16 from both sides.

$$-30x = -19$$ Left: $16 - 16 = 0$.
Right: $-3 - 16 = -19$.

$$\frac{-30x}{-30} = \frac{-19}{-30}$$ Divide both sides by -30.

$$x = \frac{19}{30}$$ Simplify both sides.

Answer: 51/22

Readers are encouraged to check this solution in the original equation.

□

Applications

Let's look at some applications that involve equations containing fractions. For convenience, we repeat the *Requirements for Word Problem Solutions.*

Requirements for Word Problem Solutions.

1. **Set up a Variable Dictionary.** You must let your readers know what each variable in your problem represents. This can be accomplished in a number of ways:

   - Statements such as "Let P represent the perimeter of the rectangle."

- Labeling unknown values with variables in a table.
- Labeling unknown quantities in a sketch or diagram.

2. Set up an Equation. Every solution to a word problem must include a carefully crafted equation that accurately describes the constraints in the problem statement.

3. Solve the Equation. You must always solve the equation set up in the previous step.

4. Answer the Question. This step is easily overlooked. For example, the problem might ask for Jane's age, but your equation's solution gives the age of Jane's sister Liz. Make sure you answer the original question asked in the problem. Your solution should be written in a sentence with appropriate units.

5. Look Back. It is important to note that this step does not imply that you should simply check your solution in your equation. After all, it's possible that your equation incorrectly models the problem's situation, so you could have a valid solution to an incorrect equation. The important question is: "Does your answer make sense based on the words in the original problem statement."

You Try It!

Attendance for the Celtics game was 9,510. If this is 3/4 of capacity, what is the capacity of the Celtics' arena?

EXAMPLE 9. In the third quarter of a basketball game, announcers informed the crowd that attendance for the game was 12,250. If this is two-thirds of the capacity, find the full seating capacity for the basketball arena.

Solution. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let F represent the full seating capacity. *Note: It is much better to use a variable that "sounds like" the quantity that it represents. In this case, letting F represent the full seating capacity is much more descriptive than using x to represent the full seating capacity.*

2. *Set up an Equation.* Two-thirds of the full seating capacity is 12,250.

Two-thirds	of	Full Seating Capacity	is	12,250
$\frac{2}{3}$	$\cdot$	F	$=$	12,250

Hence, the equation is

$$\frac{2}{3}F = 12250.$$

3. *Solve the Equation.* Multiply both sides by 3 to clear fractions, then solve.

$$\begin{aligned}
\frac{2}{3}F &= 12250 && \text{Original equation.}\\
3\left(\frac{2}{3}F\right) &= 3(12250) && \text{Multiply both sides by 3.}\\
2F &= 36750 && \text{Simplify both sides.}\\
\frac{2F}{2} &= \frac{36750}{2} && \text{Divide both sides by 2.}\\
F &= 18375 && \text{Simplify both sides.}
\end{aligned}$$

4. *Answer the Question.* The full seating capacity is 18,375.

5. *Look Back.* The words of the problem state that 2/3 of the seating capacity is 12,250. Let's take two-thirds of our answer and see what we get.

$$\begin{aligned}
\frac{2}{3} \cdot 18375 &= \frac{2}{3} \cdot \frac{18375}{1}\\
&= \frac{2}{3} \cdot \frac{3 \cdot 6125}{1}\\
&= \frac{2}{\cancel{3}} \cdot \frac{\cancel{3} \cdot 6125}{1}\\
&= 12250
\end{aligned}$$

This is the correct attendance, so our solution is correct.

Answer: 12,680

□

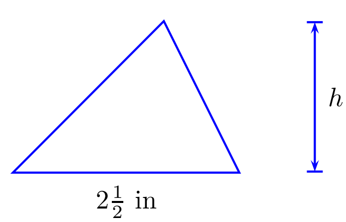
You Try It!

The area of a triangle is 161 square feet. If the base of the triangle measures $40\frac{1}{4}$ feet, find the height of the triangle.

EXAMPLE 10. The area of a triangle is 20 square inches. If the length of the base is $2\frac{1}{2}$ inches, find the height (altitude) of the triangle.

Solution. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Our variable dictionary will take the form of a well labeled diagram.



2. *Set up an Equation.* The area A of a triangle with base b and height h is

$$A = \frac{1}{2}bh.$$

Substitute $A = 20$ and $b = 2\frac{1}{2}$.

$$20 = \frac{1}{2}\left(2\frac{1}{2}\right)h.$$

3. *Solve the Equation.* Change the mixed fraction to an improper fraction, then simplify.

$$\begin{aligned}
20 &= \frac{1}{2}\left(2\frac{1}{2}\right)h && \text{Original equation.}\\
20 &= \frac{1}{2}\left(\frac{5}{2}\right)h && \text{Mixed to improper: } 2\frac{1}{2} = \frac{5}{2}.\\
20 &= \left(\frac{1}{2}\cdot\frac{5}{2}\right)h && \text{Associative property.}\\
20 &= \frac{5}{4}h && \text{Multiply: } \frac{1}{2}\cdot\frac{5}{2} = \frac{5}{4}.
\end{aligned}$$

Now, multiply both sides by 4/5 and solve.

$$\begin{aligned}
\frac{4}{5}(20) &= \frac{4}{5}\left(\frac{5}{4}h\right) && \text{Multiply both sides by 4/5.}\\
16 &= h && \text{Simplify: } \frac{4}{5}(20) = 16\\
&&& \text{and } \frac{4}{5}\cdot\frac{5}{4} = 1.
\end{aligned}$$

4. *Answer the Question.* The height of the triangle is 16 inches.

5. *Look Back.* If the height is 16 inches and the base is $2\frac{1}{2}$ inches, then the area is

$$\begin{aligned}
A &= \frac{1}{2}\left(2\,\frac{1}{2}\right)(16)\\
&= \frac{1}{2}\cdot\frac{5}{2}\cdot\frac{16}{1}\\
&= \frac{5\cdot 16}{2\cdot 2}\\
&= \frac{(5)\cdot(2\cdot 2\cdot 2\cdot 2)}{(2)\cdot(2)}\\
&= \frac{5\cdot\cancel{2}\cdot\cancel{2}\cdot 2\cdot 2}{\cancel{2}\cdot\cancel{2}}\\
&= 20
\end{aligned}$$

This is the correct area (20 square inches), so our solution is correct. Answer: 8 feet

□

❧ ❧ ❧ Exercises ☙ ☙ ☙

1. Is 1/4 a solution of the equation

$$x + \frac{5}{8} = \frac{5}{8}\ ?$$

2. Is 1/4 a solution of the equation

$$x + \frac{1}{3} = \frac{5}{12}\ ?$$

3. Is −8/15 a solution of the equation

$$\frac{1}{4}x = -\frac{1}{15}\ ?$$

4. Is −18/7 a solution of the equation

$$-\frac{3}{8}x = \frac{25}{28}\ ?$$

5. Is 1/2 a solution of the equation

$$x + \frac{4}{9} = \frac{17}{18}\ ?$$

6. Is 1/3 a solution of the equation

$$x + \frac{3}{4} = \frac{13}{12}\ ?$$

7. Is 3/8 a solution of the equation

$$x - \frac{5}{9} = -\frac{13}{72}\ ?$$

8. Is 1/2 a solution of the equation

$$x - \frac{3}{5} = -\frac{1}{10}\ ?$$

9. Is 2/7 a solution of the equation

$$x - \frac{4}{9} = -\frac{8}{63}\ ?$$

10. Is 1/9 a solution of the equation

$$x - \frac{4}{7} = -\frac{31}{63}\ ?$$

11. Is 8/5 a solution of the equation

$$\frac{11}{14}x = \frac{44}{35}\ ?$$

12. Is 16/9 a solution of the equation

$$\frac{13}{18}x = \frac{104}{81}\ ?$$

In Exercises 13-24, solve the equation and simplify your answer.

13. $2x - 3 = 6x + 7$

14. $9x - 8 = -9x - 3$

15. $-7x + 4 = 3x$

16. $6x + 9 = -6x$

17. $-2x = 9x - 4$

18. $-6x = -9x + 8$

19. $-8x = 7x - 7$

20. $-6x = 5x + 4$

21. $-7x + 8 = 2x$

22. $-x - 7 = 3x$

23. $-9x + 4 = 4x - 6$

24. $-2x + 4 = x - 7$

In Exercises 25-48, solve the equation and simplify your answer.

25. $x+\frac{3}{2}=\frac{1}{2}$

26. $x-\frac{3}{4}=\frac{1}{4}$

27. $-\frac{9}{5}x=\frac{1}{2}$

28. $\frac{7}{3}x=-\frac{7}{2}$

29. $\frac{3}{8}x=\frac{8}{7}$

30. $-\frac{1}{9}x=-\frac{3}{5}$

31. $\frac{2}{5}x=-\frac{1}{6}$

32. $\frac{1}{6}x=\frac{2}{9}$

33. $-\frac{3}{2}x=\frac{8}{7}$

34. $-\frac{3}{2}x=-\frac{7}{5}$

35. $x+\frac{3}{4}=\frac{5}{9}$

36. $x-\frac{1}{9}=-\frac{3}{2}$

37. $x-\frac{4}{7}=\frac{7}{8}$

38. $x+\frac{4}{9}=-\frac{3}{4}$

39. $x+\frac{8}{9}=\frac{2}{3}$

40. $x-\frac{5}{6}=\frac{1}{4}$

41. $x+\frac{5}{2}=-\frac{9}{8}$

42. $x+\frac{1}{2}=\frac{5}{3}$

43. $-\frac{8}{5}x=\frac{7}{9}$

44. $-\frac{3}{2}x=-\frac{5}{9}$

45. $x-\frac{1}{4}=-\frac{1}{8}$

46. $x-\frac{9}{2}=-\frac{7}{2}$

47. $-\frac{1}{4}x=\frac{1}{2}$

48. $-\frac{8}{9}x=-\frac{8}{3}$

In Exercises 49-72, solve the equation and simplify your answer.

49. $-\frac{7}{3}x-\frac{2}{3}=\frac{3}{4}x+\frac{2}{3}$

50. $\frac{1}{2}x-\frac{1}{2}=\frac{3}{2}x+\frac{3}{4}$

51. $-\frac{7}{2}x-\frac{5}{4}=\frac{4}{5}$

52. $-\frac{7}{6}x+\frac{5}{6}=-\frac{8}{9}$

53. $-\frac{9}{7}x+\frac{9}{2}=-\frac{5}{2}$

54. $\frac{5}{9}x-\frac{7}{2}=\frac{1}{4}$

55. $\frac{1}{4}x-\frac{4}{3}=-\frac{2}{3}$

56. $\frac{8}{7}x+\frac{3}{7}=\frac{5}{3}$

57. $\frac{5}{3}x+\frac{3}{2}=-\frac{1}{4}$

58. $\frac{1}{2}x-\frac{8}{3}=-\frac{2}{5}$

59. $-\frac{1}{3}x+\frac{4}{5}=-\frac{9}{5}x-\frac{5}{6}$

60. $-\frac{2}{9}x-\frac{3}{5}=\frac{4}{5}x-\frac{3}{2}$

61. $-\frac{4}{9}x - \frac{8}{9} = \frac{1}{2}x - \frac{1}{2}$

62. $-\frac{5}{4}x - \frac{5}{3} = \frac{8}{7}x + \frac{7}{3}$

63. $\frac{1}{2}x - \frac{1}{8} = -\frac{1}{8}x + \frac{5}{7}$

64. $-\frac{3}{2}x + \frac{8}{3} = \frac{7}{9}x - \frac{1}{2}$

65. $-\frac{3}{7}x - \frac{1}{3} = -\frac{1}{9}$

66. $\frac{2}{3}x + \frac{2}{9} = -\frac{9}{5}$

67. $-\frac{3}{4}x + \frac{2}{7} = \frac{8}{7}x - \frac{1}{3}$

68. $\frac{1}{2}x + \frac{1}{3} = -\frac{5}{2}x - \frac{1}{4}$

69. $-\frac{3}{4}x - \frac{2}{3} = -\frac{2}{3}x - \frac{1}{2}$

70. $\frac{1}{3}x - \frac{5}{7} = \frac{3}{2}x + \frac{4}{3}$

71. $-\frac{5}{2}x + \frac{9}{5} = \frac{5}{8}$

72. $\frac{9}{4}x + \frac{4}{3} = -\frac{1}{6}$

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73. At a local soccer game, announcers informed the crowd that attendance for the game was 4,302. If this is 2/9 of the capacity, find the full seating capacity for the soccer stadium.

74. At a local basketball game, announcers informed the crowd that attendance for the game was 5,394. If this is 2/7 of the capacity, find the full seating capacity for the basketball stadium.

75. The area of a triangle is 51 square inches. If the length of the base is $8\,\frac{1}{2}$ inches, find the height (altitude) of the triangle.

76. The area of a triangle is 20 square inches. If the length of the base is $2\,\frac{1}{2}$ inches, find the height (altitude) of the triangle.

77. The area of a triangle is 18 square inches. If the length of the base is $4\,\frac{1}{2}$ inches, find the height (altitude) of the triangle.

78. The area of a triangle is 44 square inches. If the length of the base is $5\,\frac{1}{2}$ inches, find the height (altitude) of the triangle.

79. At a local hockey game, announcers informed the crowd that attendance for the game was 4,536. If this is 2/11 of the capacity, find the full seating capacity for the hockey stadium.

80. At a local soccer game, announcers informed the crowd that attendance for the game was 6,970. If this is 2/7 of the capacity, find the full seating capacity for the soccer stadium.

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81. **Pirates.** About one-third of the world's pirate attacks in 2008 occurred off the Somali coast. If there were 111 pirate attacks off the Somali coast, estimate the number of pirate attacks worldwide in 2008.

82. **Nuclear arsenal.** The U.S. and Russia agreed to cut nuclear arsenals of long-range nuclear weapons by about a third, down to $1,550$. How many long-range nuclear weapons are there now? *Associated Press-Times-Standard 04/04/10 Nuclear heartland anxious about missile cuts.*

83. **Seed vault.** The Svalbard Global Seed Vault has amassed half a million seed samples, and now houses at least one-third of the world's crop seeds. Estimate the total number of world's crop seeds. *Associated Press-Times-Standard 03/15/10 Norway doomsday seed vault hits half-million mark.*

84. **Freight train.** The three and one-half mile long Union Pacific train is about $2\frac{1}{2}$ times the length of a typical freight train. How long is a typical freight train? *Associated Press-Times-Standard 01/13/10 Unususally long train raises safety concerns.*

Answers

1. No

3. No

5. Yes

7. Yes

9. No

11. Yes

13. $-\dfrac{5}{2}$

15. $\dfrac{2}{5}$

17. $\dfrac{4}{11}$

19. $\dfrac{7}{15}$

21. $\dfrac{8}{9}$

23. $\dfrac{10}{13}$

25. -1

27. $-\dfrac{5}{18}$

29. $\dfrac{64}{21}$

31. $-\dfrac{5}{12}$

33. $-\dfrac{16}{21}$

35. $-\dfrac{7}{36}$

37. $\dfrac{81}{56}$

39. $-\dfrac{2}{9}$

41. $-\dfrac{29}{8}$

43. $-\dfrac{35}{72}$

45. $\dfrac{1}{8}$

47. -2

49. $-\dfrac{16}{37}$

51. $-\dfrac{41}{70}$

53. $\dfrac{49}{9}$

55. $\dfrac{8}{3}$

57. $-\dfrac{21}{20}$

59. $-\dfrac{49}{44}$

61. $-\dfrac{7}{17}$

63. $\dfrac{47}{35}$

65. $-\dfrac{14}{27}$

67. $\dfrac{52}{159}$

69. -2

71. $\dfrac{47}{100}$

73. 19,359

75. 12

77. 8

79. 24,948

81. There were about 333 pirate attacks worldwide.

83. $1,500,000$

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