Prealgebra Textbook

Second Edition

Chapter 5

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Chapter 5

Decimals

On January 29, 2001, the New York Stock exchange ended its 200-year tradition of quoting stock prices in fractions and switched to decimals.

It was said that pricing stocks the same way other consumer items were priced would make it easier for investors to understand and compare stock prices. Foreign exchanges had been trading in decimals for decades. Supporters of the change claimed that trading volume, the number of shares of stock traded, would increase and improve efficiency.

But switching to decimals would have another effect of *narrowing the spread*. The *spread* is the difference between the best price offered by buyers, called the *bid*, and the price requested by sellers called the *ask*. Stock brokers make commissions as a percentage of the spread which, using fractions, could be anywhere upwards from 12 cents per share.

When the New York Stock Exchange began back in 1792, the dollar was based on the Spanish *real*, (pronounced ray-al), also called *pieces of eight* as these silver coins were often cut into quarters or eighths to make change. This is what led to stock prices first denominated in eighths. Thus, the smallest spread that could occur would be 1/8 of a dollar, or 12.5 cents. That may seem like small change, but buying 1000 shares for $1 per share with a $0.125 spread is a $125.00 commision. Not bad for a quick trade!

Decimalization of stock pricing allowed for spreads as small as 1 cent. Since the number of shares traded on stock market exchanges have skyrocketed, with trillions of shares traded daily, stock broker commissions have not suffered. And the ease with which investors can quickly grasp the price of stock shares has contributed to the opening of markets for all classes of people.

In this chapter, we'll learn about how to compute and solve problems with decimals, and see how they relate to fractions.

5.1 Introduction to Decimals

Recall that whole numbers are constructed by using *digits*.

The Digits. The set

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

is called the set of *digits*.

As an example, the whole number 55,555 ("fifty-five thousand five hundred fifty-five") is constructed by using a single digit. However, the position of the digit 5 determines its value in the number 55,555. The first occurrence of the

5	5	5	5	5
ten thousands	thousands	hundreds	tens	ones
10,000	1,000	100	10	1

Table 5.1: Place value.

digit 5 happens in the ten thousands place, so its value is 5 ten thousands, or 50,000. The next occurrence of the digit 5 is in the thousands place, so its value is 5 thousands, or 5,000. Indeed, the whole number 55,555 in expanded form is

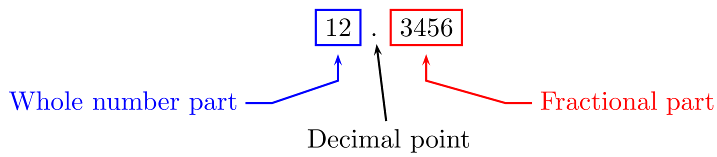
$$50000 + 5000 + 500 + 50 + 5,$$

which reflects the value of the digit 5 in each place.

Decimal Notation

In Table 5.1, each time you move one column to the left, the place value is 10 times larger than the place value of the preceding column. Vice-versa, each time you move one column to the right, the place value is 1/10 of the place value of the preceding column.

Now, consider the *decimal number* 12.3456, which consists of three parts: the whole number part, the decimal point, and the fractional part.



The whole number part of the decimal number is the part that lies strictly to the left of the decimal point, and the place value of each digit in the whole number part is given by the columns shown in Table 5.1.

The fractional part of the decimal number is the part that lies strictly to the right of the decimal point. As we saw in Table 5.1, each column has a value equal to 1/10 of the value of the column that lies to its immediate left. Thus, it should come as no surprise that:

- The first column to the right of the decimal point has place value 1/10 (tenths).
- The second column to the right of the decimal point has place value 1/100 (hundredths).
- The third column to the right of the decimal point has place value 1/1000 (thousandths).
- The fourth column to the right of the decimal point has place value 1/10000 (ten-thousandths).

These results are summarized for the decimal number 12.3456 in Table 5.2.

1	2	.	3	4	5	6
tens	ones	decimal point	tenths	hundredths	thousandths	ten-thousandths
10	1	.	1/10	1/100	1/1000	1/10000

Table 5.2: Place value.

Pronouncing Decimal Numbers

The decimal number 12.3456 is made up of 1 ten, 2 ones, 3 tenths, 4 hundredths, 5 thousandths, and 6 ten-thousandths (see Table 5.2), and can be written in *expanded form* as

$$12.3456 = 10 + 2 + \frac{3}{10} + \frac{4}{100} + \frac{5}{1000} + \frac{6}{10000}.$$

Note that the whole numbers can be combined and the fractions can be written with a common denominator and summed.

$$\begin{aligned} 12.3456 &= 12 + \frac{3 \cdot 1000}{10 \cdot 1000} + \frac{4 \cdot 100}{100 \cdot 100} + \frac{5 \cdot 10}{1000 \cdot 10} + \frac{6}{10000} \\ &= 12 + \frac{3000}{10000} + \frac{400}{10000} + \frac{50}{10000} + \frac{6}{10000} \\ &= 12 + \frac{3456}{10000} \end{aligned}$$

The result tells us how to pronounce the number 12.3456. It is pronounced "twelve and three thousand, four hundred fifty-six ten-thousandths."

You Try It!

Place the decimal number 3,502.23 in expanded form, then combine the whole number part and sum the fractional part over a common denominator

EXAMPLE 1. Place the decimal number 1,234.56 in expanded form, then combine the whole number part and sum the fractional part over a common denominator. Use the result to help pronounce the decimal number.

Solution. In expanded form,

$$1,234.56 = 1,000 + 200 + 30 + 4 + \frac{5}{10} + \frac{6}{100}$$

Sum the whole number parts. Express the fractional parts as equivalent fractions and combine over one common denominator.

$$\begin{aligned} &= 1,234 + \frac{5 \cdot 10}{10 \cdot 10} + \frac{6}{100} \\ &= 1,234 + \frac{50}{100} + \frac{6}{100} \\ &= 1,234 + \frac{56}{100} \end{aligned}$$

Hence, 1,234.56 is pronounced "one thousand, two hundred thirty-four and fifty-six hundredths."

Answer: $3,502 + \frac{23}{100}$

□

You Try It!

EXAMPLE 2. Place the decimal number 56.128 in expanded form, then combine the whole number part and sum the fractional part over a common denominator. Use the result to help pronounce the decimal number.

Place the decimal number 235.568 in expanded form, then combine the whole number part and sum the fractional part over a common denominator

Solution. In expanded form,

$$56.128 = 50 + 6 + \frac{1}{10} + \frac{2}{100} + \frac{8}{1000}$$

Sum the whole number parts. Express the fractional parts as equivalent fractions and combine over one common denominator.

$$\begin{aligned} &= 56 + \frac{1 \cdot 100}{10 \cdot 100} + \frac{2 \cdot 10}{100 \cdot 10} + \frac{8}{1000} \\ &= 56 + \frac{100}{1000} + \frac{20}{1000} + \frac{8}{1000} \\ &= 56 + \frac{128}{1000} \end{aligned}$$

Thus, 56.128 is pronounced "fifty-six and one hundred twenty-eight thousandths."

Answer: $235 + \frac{568}{1000}$

□

The discussion and example leads to the following result.

How to Read a Decimal Number

1. Pronounce the whole number part to the left of the decimal as you would any whole number.
2. Say the word "and" for the decimal point.
3. State the fractional part to the right of the decimal as you would any whole number, followed by the place value of the digit in the rightmost column.

You Try It!

EXAMPLE 3. Pronounce the decimal number 34.12.

Pronounce 28.73

Solution. The rightmost digit in the fractional part of 34.12 is in the hundredths column. Thus, 34.12 is pronounced "thirty-four and twelve hundredths."

Answer: "Twenty-eight and seventy-three hundredths"

□

Important Point. In pronouncing decimal numbers, the decimal point is read as "and." No other instance of the word "and" should appear in the pronunciation.

You Try It!

Pronounce 286.9.

Answer: "Four hundred thirty-four and two tenths"

EXAMPLE 4. Explain why "four hundred and thirty-four and two tenths" is an *incorrect* pronunciation of the decimal number 434.2.

Solution. The decimal point is read as "and." No other occurrence of the word "and" is allowed in the pronunciation. The correct pronunciation should be "four hundred thirty-four and two tenths."

□

You Try It!

Pronounce $7,002.207$.

Answer: "Seven thousand two and two hundred seven thousandths."

EXAMPLE 5. Pronounce the decimal number 5,678.123.

Solution. The rightmost digit in the fractional part of 5,678.123 is in the thousandths column. Hence, 5,678.123 is pronounced "5 thousand six hundred seventy-eight and one hundred twenty-three thousandths."

□

You Try It!

Pronounce 500.1205.

Asnwer: "Five hundred and one thousand two hundred five ten-thousandths."

EXAMPLE 6. Pronounce the decimal number 995.4325.

Solution. The rightmost digit in the fractional part of 995.4325 is in the ten-thousandths column. Hence, 995.4325 is pronounced "nine hundred ninety-five and four thousand three hundred twenty-five ten-thoudandths."

□

Decimals to Fractions

Because we now have the ability to pronounce decimal numbers, it is a simple exercise to change a decimal to a fraction.[1] For example, 134.12 is pronounced

[1]Changing fractions to decimals will be covered in Section 5.5.

"one hundred thirty-four and twelve hundredths," so it can easily be written as a mixed fraction.

$$134.12 = 134\,\frac{12}{100}$$

But this mixed fraction can be changed to an improper fraction.

$$\begin{aligned} 134\,\frac{12}{100} &= \frac{100 \cdot 134 + 12}{100} \\ &= \frac{13400 + 12}{100} \\ &= \frac{13412}{100} \end{aligned}$$

Note that the numerator is our original number without the decimal point. There are two decimal places in the original number and the denominator of the final improper fraction contains two zeros.

This discussion leads to the following result.

Changing Decimals to Improper Fractions. To change a decimal number to an improper fraction, proceed as follows:

1. Create a fraction.
2. Place the decimal number in the numerator **without the decimal point**.
3. Count the number of decimal places. Place an equal number of zeros in the denominator.

You Try It!

Change 17.205 to an improper fraction.

EXAMPLE 7. Change the following decimal numbers to improper fractions: (a) 1.2345, and (b) 27.198.

Solution. In each case, place the number in the numerator without the decimal point. In the denominator, add a number of zeros equal to the number of decimal places.

a) The decimal number 1.2345 has four decimal places. Hence,

$$1.2345 = \frac{12345}{10000}$$

b) The decimal number 27.198 has three decimal places. Hence,

$$27.198 = \frac{27198}{1000}$$

Answer: $\frac{17205}{100}$

□

You Try It!

Change 0.375 to a fraction, reduced to lowest terms.

EXAMPLE 8. Change each of the following decimals to fractions reduced to lowest terms: (a) 0.35, and (b) 0.125.

Solution. Place each number in the numerator without the decimal point. Place a number of zeros in the denominator equal to the number of decimal places. Reduce to lowest terms.

a) First, place 35 over 100.

$$0.35 = \frac{35}{100}$$

We can divide both numerator and denominator by the greatest common divisor.

$$\begin{aligned} &= \frac{35 \div 5}{100 \div 5} && \text{Divide numerator and denominator by 5.} \\ &= \frac{7}{20} && \text{Simplify numerator and denominator.} \end{aligned}$$

b) First, place 125 over 1000.

$$0.125 = \frac{125}{1000}$$

Prime factor and cancel common factors.

$$\begin{aligned} &= \frac{5 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5} && \text{Prime factor numerator and denominator.} \\ &= \frac{\cancel{5} \cdot \cancel{5} \cdot \cancel{5}}{2 \cdot 2 \cdot 2 \cdot \cancel{5} \cdot \cancel{5} \cdot \cancel{5}} && \text{Cancel common factors.} \\ &= \frac{1}{8} && \text{Simplify.} \end{aligned}$$

Answer: 3/8

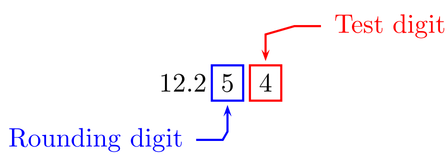
□

Rounding

The rules for rounding decimal numbers are almost identical to the rules for rounding whole numbers. First, a bit of terminology.

Rounding Digit and Test Digit. The digit in the place to which we wish to round is called the *rounding digit* and the digit that follows on its immediate right is called the *test* digit.

If we want to round the decimal number 12.254 to the nearest hundredth, then the rounding digit is 5 and the test digit is 4.



If we used the rules for rounding whole numbers, because the test digit 4 is less than 5, we would replace all digits to the right of the rounding digit with zeros to obtain the following approximation.

$$12.254 \approx 12.250$$

However, because

$$12.250 = 12\,\frac{250}{1000} = 12\,\frac{25}{100},$$

the trailing zero at the end of the fractional part is irrelevant. Hence, we *truncate* every digit after the rounding digit and use the following approximation.

$$12.254 \approx 12.25$$

Important Observation. Deleting trailing zeros from the end of the fractional part of a decimal number does not change its value.

The above discussion motivates the following algorithm for rounding decimal numbers.

Rounding Decimal Numbers. Locate the *rounding digit* and the *test digit.*

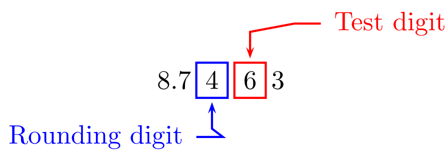
- If the test digit is greater than or equal to 5, add 1 to the rounding digit and truncate all digits to the right of the rounding digit.
- If the test digit is less than 5, simply truncate all digits to the right of the rounding digit.

You Try It!

Round 9.2768 to the nearest hundredth.

EXAMPLE 9. Round 8.7463 to the nearest hundredth.

Solution. Locate the rounding digit in the hundredths place and the test digit to its immediate right.



Because the test digit is greater than 5, add 1 to the rounding digit and truncate all digits to the right of the rounding digit. Hence, to the nearest hundredth:

$$8.7463 \approx 8.75$$

Answer: 9.28

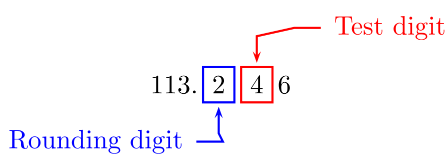
□

You Try It!

Round 58.748 to the nearest tenth.

EXAMPLE 10. Round 113.246 to the nearest tenth.

Solution. Locate the rounding digit in the tenths place and the test digit to its immediate right.



Because the test digit is less than 5, truncate all digits to the right of the rounding digit. Hence, to the nearest tenth:

$$113.246 \approx 113.2$$

Answer: 58.7

□

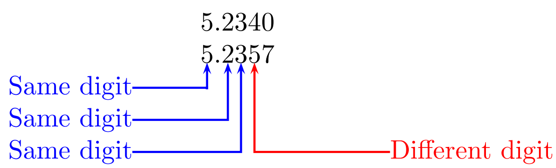
Comparing Decimals

We can compare two positive decimals by comparing digits in each place as we move from left to right, place by place. For example, suppose we wish to compare the decimal numbers 5.234 and 5.2357. First, add enough trailing zeros to the decimal number with the fewer decimal places so that the numbers have the same number of decimal places. In this case, note that

$$5.234 = 5\,\frac{234}{1000} = 5\,\frac{2340}{10000} = 5.2340.$$

Important Observation. Adding trailing zeros to the end of the fractional part of a decimal number does not change its value.

Next, align the numbers as follows.



As you scan the columns, moving left to right, the first place that has different digits occurs in the thousandths place, where the digit 5 is the second number is greater than the digit 4 in the first number in the same place. Because 5 is greater than 4, the second number is larger than the first. That is:

$$5.234 < 5.2357$$

This discussion suggests the following algorithm.

Comparing Positive Decimal Numbers.

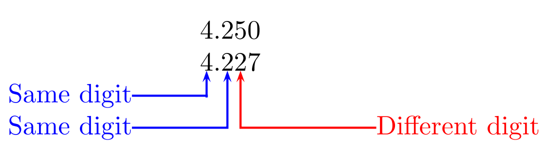
1. Add enough trailing zeros so that both numbers have the same number of decimal places.
2. Compare the digits in each place, moving left to right.
3. As soon as you find two digits in the same place that are different, the decimal number with the greatest digit in this place is the **larger** number.

You Try It!

Compare 8.34 and 8.348.

EXAMPLE 11. Compare 4.25 and 4.227.

Solution. Add a trailing zero to the first decimal number and align the numbers as follows.



The first difference is in the hundredths place, where the digit 5 in the first number is greater than the digit 2 in the same place of the second number. Hence, the first number is **larger** than the second; that is:

$$4.25 > 4.227$$

Answer: $8.34 < 8.348$

When comparing negative numbers, the number with the larger magnitude is the smaller number. Hence, we have to adjust our algorithm for comparing negative decimal numbers.

Comparing Negative Decimal Numbers.

1. Add enough trailing zeros so that both numbers have the same number of decimal places.
2. Compare the digits in each place, moving left to right.
3. As soon as you find two digits in the same place that are different, the decimal number with the greatest digit in this place is the **smaller** number.

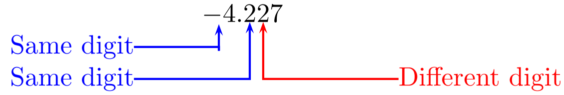
You Try It!

Compare -7.86 and -7.85.

EXAMPLE 12. Compare -4.25 and -4.227.

Solution. Add a trailing zero to the first decimal number and align the numbers as follows.

$$\begin{array}{r} -4.250 \\ -4.227 \end{array}$$



The first difference is in the hundredths place, where the digit 5 in the first number is greater than the digit 2 in the same place of the second number. Hence, the first number is **smaller** than the second; that is:

$$-4.25 < -4.227$$

Answer: $-7.86 < -7.85$

❧ ❧ ❧ Exercises ☙ ☙ ☙

1. Which digit is in the tenths column of the number 4,552.0908?

2. Which digit is in the thousandths column of the number 7,881.6127?

3. Which digit is in the tenths column of the number 4,408.2148?

4. Which digit is in the tenths column of the number 9,279.0075?

5. Which digit is in the ten-thousandths column of the number 2,709.5097?

6. Which digit is in the hundredths column of the number 1,743.1634?

7. Which digit is in the hundredths column of the number 3,501.4456?

8. Which digit is in the ten-thousandths column of the number 9,214.3625?

9. Which digit is in the hundredths column of the number 5,705.2193?

10. Which digit is in the hundredths column of the number 7,135.2755?

11. Which digit is in the tenths column of the number 8,129.3075?

12. Which digit is in the thousandths column of the number 6,971.4289?

In Exercises 13-20, write the given decimal number in expanded form.

13. 46.139

14. 68.392

15. 643.19

16. 815.64

17. 14.829

18. 45.913

19. 658.71

20. 619.38

In Exercises 21-28, follow the procedure shown in Examples 1 and 2 to write the decimal number in expanded form, then combine the whole number part and sum the fractional part over a common denominator.

21. 32.187

22. 35.491

23. 36.754

24. 89.357

25. 596.71

26. 754.23

27. 527.49

28. 496.15

In Exercises 29-40, pronounce the given decimal number. Write your answer out in words.

29. 0.9837

30. 0.6879

31. 0.2653

32. 0.8934

33. 925.47

34. 974.35

35. 83.427

36. 32.759

37. 63.729

38. 85.327

39. 826.57

40. 384.72

In Exercises 41-52, convert the given decimal to a mixed fraction. Do *not* simplify your answer.

41. 98.1

42. 625.591

43. 781.7

44. 219.999

45. 915.239

46. 676.037

47. 560.453

48. 710.9

49. 414.939

50. 120.58

51. 446.73

52. 653.877

In Exercises 53-60, convert the given decimal to an improper fraction. Do *not* simplify your answer.

53. 8.7

54. 3.1

55. 5.47

56. 5.27

57. 2.133

58. 2.893

59. 3.9

60. 1.271

In Exercises 61-68, convert the given decimal to a fraction. Reduce your answer to lowest terms.

61. 0.35

62. 0.38

63. 0.06

64. 0.84

65. 0.98

66. 0.88

67. 0.72

68. 0.78

---

69. Round 79.369 to the nearest hundredth.

70. Round 54.797 to the nearest hundredth.

71. Round 71.2427 to the nearest thousandth.

72. Round 59.2125 to the nearest thousandth.

73. Round 29.379 to the nearest tenth.

74. Round 42.841 to the nearest tenth.

75. Round 89.3033 to the nearest thousandth.

76. Round 9.0052 to the nearest thousandth.

77. Round 20.655 to the nearest tenth.

78. Round 53.967 to the nearest tenth.

79. Round 19.854 to the nearest hundredth.

80. Round 49.397 to the nearest hundredth.

In Exercises 81-92, determine which of the two given statements is true.

81.
$$0.30387617 < 0.3036562$$
or
$$0.30387617 > 0.3036562$$

82.
$$8.5934 < 8.554$$
or
$$8.5934 > 8.554$$

83.
$$-0.034 < -0.040493$$
or
$$-0.034 > -0.040493$$

84.
$$-0.081284 < -0.08118$$
or
$$-0.081284 > -0.08118$$

85.
$$-8.3527 < -8.36553$$
or
$$-8.3527 > -8.36553$$

86.
$$-0.00786 < -0.0051385$$
or
$$-0.00786 > -0.0051385$$

87.
$$18.62192 < 18.6293549$$
or
$$18.62192 > 18.6293549$$

88.
$$514.873553 < 514.86374$$
or
$$514.873553 > 514.86374$$

89.
$$36.8298 < 36.8266595$$
or
$$36.8298 > 36.8266595$$

90.
$$0.000681 < 0.00043174$$
or
$$0.000681 > 0.00043174$$

91.
$$-15.188392 < -15.187157$$
or
$$-15.188392 > -15.187157$$

92.
$$-0.049785 < -0.012916$$
or
$$-0.049785 > -0.012916$$

93. Write the decimal number in words.

i) A recently discovered 7.03-carat blue diamond auctioned at Sotheby's.

ii) The newly launched European Planck telescope will stay in orbit 1.75 years measuring radiation from the Big Bang.

iii) The sun composes 0.9985 of the mass in our solar system.

iv) Clay particles are small - only 0.0001 inch.

94. **Light speed.** The *index of refraction* for a given material is a value representing the number of times slower a light wave travels in that particular material than it travels in the vacuum of space.

i) Reorder the materials by their index of refraction from lowest to highest.

ii) How many times slower is a lightwave in a diamond compared with a vacuum?

Material	Index of Refraction
Diamond	2.417
Vacuum	1.0000
Plexiglas	1.51
Air	1.0003
Water	1.333
Zircon	1.923
Crown Glass	1.52
Ice	1.31

95. **Shorter day?** Scientists at NASA's Jet Propulsion Laboratory calculated that the earthquake in Chile may have shortened the length of a day on Earth by 1.26 millionths of a second.

i) Write that number completely as a decimal.

ii) Actual observations of the length of the day are accurate to five millionths of a second. Write this fraction as a decimal.

iii) Comparing the two decimals above and determine which is smaller. Do you think scientists can observe and measure the calculated slowing of the earth?

Answers

1. 0

3. 2

5. 7

7. 4

9. 1

11. 3

13. $40 + 6 + \frac{1}{10} + \frac{3}{100} + \frac{9}{1000}$

15. $600 + 40 + 3 + \frac{1}{10} + \frac{9}{100}$

17. $10 + 4 + \frac{8}{10} + \frac{2}{100} + \frac{9}{1000}$

19. $600 + 50 + 8 + \frac{7}{10} + \frac{1}{100}$

21. $32 + \frac{187}{1000}$

23. $36 + \frac{754}{1000}$

25. $596 + \frac{71}{100}$

27. $527 + \frac{49}{100}$

29. nine thousand eight hundred thirty-seven ten-thousandths

31. two thousand six hundred fifty-three ten-thousandths

33. nine hundred twenty-five and forty-seven hundredths

35. eighty-three and four hundred twenty-seven thousandths

37. sixty-three and seven hundred twenty-nine thousandths

39. eight hundred twenty-six and fifty-seven hundredths

41. $98\frac{1}{10}$

43. $781\frac{7}{10}$

45. $915\frac{239}{1000}$

47. $560\frac{453}{1000}$

49. $414\frac{939}{1000}$

51. $446\frac{73}{100}$

53. $\frac{87}{10}$

55. $\frac{547}{100}$

57. $\frac{2133}{1000}$

59. $\frac{39}{10}$

61. $\frac{7}{20}$

63. $\frac{3}{50}$

65. $\frac{49}{50}$

67. $\frac{18}{25}$

69. 79.37

71. 71.243

73. 29.4

75. 89.303

77. 20.7

79. 19.85

81. $0.30387617 > 0.3036562$

83. $-0.034 > -0.040493$

85. $-8.3527 > -8.36553$

87. $18.62192 < 18.6293549$

89. $36.8298 > 36.8266595$

91. $-15.188392 < -15.187157$

93.

i) seven and three hundredths

ii) one and seventy-five hundredths

iii) nine thousand nine hundred eighty-five ten-thousandths

iv) one ten-thousandth of an inch

95.

i) 0.00000126

ii) 0.000005

iii) $0.00000126 < 0.000005$; scientists would be unable to measure the calculated change in the length of a day.

5.2 Adding and Subtracting Decimals

Adding Decimals

Addition of decimal numbers is quite similar to addition of whole numbers. For example, suppose that we are asked to add 2.34 and 5.25. We could change these decimal numbers to mixed fractions and add.

$$\begin{aligned} 2.34 + 5.25 &= 2\frac{34}{100} + 5\frac{25}{100} \\ &= 7\frac{59}{100} \end{aligned}$$

However, we can also line the decimal numbers on their decimal points and add vertically, as follows.

$$\begin{array}{r} 2.34 \\ +\,5.25 \\ \hline 7.59 \end{array}$$

Note that this alignment procedure produces the same result, "seven and fifty nine hundredths." This motivates the following procedure for adding decimal numbers.

Adding Decimals. To add decimal numbers, proceed as follows:

1. Place the numbers to be added in vertical format, aligning the decimal points.
2. Add the numbers as if they were whole numbers.
3. Place the decimal point in the answer in the *same column* as the decimal points above it.

You Try It!

EXAMPLE 1. Add 3.125 and 4.814.

Add: $2.864 + 3.029$

Solution. Place the numbers in vertical format, aligning on their decimal points. Add, then place the decimal point in the answer in the same column as the decimal points that appear above the answer.

$$\begin{array}{r} 3.125 \\ +\,4.814 \\ \hline 7.939 \end{array}$$

Thus, $3.125 + 4.814 = 7.939$.

Answer: 5.893

□

You Try It!

Alice has \$8.63 in her purse and Joanna has \$2.29. If they combine sum their money, what is the total?

EXAMPLE 2. Jane has \$4.35 in her purse. Jim has \$5.62 in his wallet. If they sum their money, what is the total?

Solution. Arrange the numbers in vertical format, aligning decimal points, then add.

$$\begin{array}{r} \$4.35 \\ +\,\$5.62 \\ \hline \$9.97 \end{array}$$

Answer: \$10.91

Together they have \$9.97, nine dollars and ninety seven cents.

Before looking at another example, let's recall an important observation.

Important Observation. Adding zeros to the end of the fractional part of a decimal number does not change its value. Similarly, deleting trailing zeros from the end of a decimal number does not change its value.

For example, we could add two zeros to the end of the fractional part of 7.25 to obtain 7.2500. The numbers 7.25 and 7.2500 are identical as the following argument shows:

$$\begin{aligned} 7.2500 &= 7\frac{2500}{10000} \\ &= 7\frac{25}{100} \\ &= 7.25 \end{aligned}$$

You Try It!

Add: $9.7 + 15.86$

EXAMPLE 3. Add 7.5 and 12.23.

Solution. Arrange the numbers in vertical format, aligning their decimal points in a column. Note that we add a trailing zero to improve columnar alignment.

$$\begin{array}{r} 7.50 \\ +\,12.23 \\ \hline 19.73 \end{array}$$

Answer: 25.56

Hence, $7.5 + 12.23 = 19.73$.

You Try It!

EXAMPLE 4. Find the sum: $12.2 + 8.352 + 22.44$.

Add: $12.9 + 4.286 + 33.97$

Solution. Arrange the numbers in vertical format, aligning their decimal points in a column. Note that we add trailing zeros to improve the columnar alignment.

$$\begin{array}{r} 12.200 \\ 8.352 \\ +\,22.440 \\ \hline 42.992 \end{array}$$

Hence, $12.2 + 8.352 + 22.44 = 42.992$.

Answer: 51.156

Subtracting Decimals

Subtraction of decimal numbers proceeds in much the same way as addition of decimal numbers.

Subtracting Decimals. To subtract decimal numbers, proceed as follows:

1. Place the numbers to be subtracted in vertical format, aligning the decimal points.
2. Subtract the numbers as if they were whole numbers.
3. Place the decimal point in the answer in the *same column* as the decimal points above it.

You Try It!

EXAMPLE 5. Subtract 12.23 from 33.57.

Subtract: $58.76 - 38.95$

Solution. Arrange the numbers in vertical format, aligning their decimal points in a column, then subtract. Note that we subtract 12.23 **from** 33.57.

$$\begin{array}{r} 33.57 \\ -\,12.23 \\ \hline 21.34 \end{array}$$

Hence, $33.57 - 12.23 = 21.34$.

Asnwer: 19.81

As with addition, we add trailing zeros to the fractional part of the decimal numbers to help columnar alignment.

You Try It!

Subtract: $15.2 - 8.756$

EXAMPLE 6. Find the difference: $13.3 - 8.572$.

Solution. Arrange the numbers in vertical format, aligning their decimal points in a column. Note that we add trailing zeros to the fractional part of 13.3 to improve columnar alignment.

$$\begin{array}{r} 13.300 \\ -\,8.572 \\ \hline 4.728 \end{array}$$

Hence, $13.3 - 8.572 = 4.728$.

Answer: 6.444

Adding and Subtracting Signed Decimal Numbers

We use the same rules for addition of signed decimal numbers as we did for the addition of integers.

Adding Two Decimals with Like Signs. To add two decimals with like signs, proceed as follows:

1. Add the magnitudes of the decimal numbers.
2. Prefix the common sign.

You Try It!

Simplify: $-5.7 + (-83.85)$

EXAMPLE 7. Simplify: $-3.2 + (-18.95)$.

Solution. To add like signs, first add the magnitudes.

$$\begin{array}{r} 3.20 \\ +\,18.95 \\ \hline 22.15 \end{array}$$

Prefix the common sign. Hence, $-3.2 + (-18.95) = -22.15$

Answer: -89.55

We use the same rule as we did for integers when adding decimals with unlike signs.

Adding Two Decimals with Unlike Signs. To add two decimals with unlike signs, proceed as follows:

1. Subtract the smaller magnitude from the larger magnitude.
2. Prefix the sign of the decimal number with the larger magnitude.

You Try It!

EXAMPLE 8. Simplify: $-3 + 2.24$.

Simplify: $-8 + 5.74$

Solution. To add unlike signs, first subtract the smaller magnitude from the larger magnitude.

$$\begin{array}{r} 3.00 \\ -\ 2.24 \\ \hline 0.76 \end{array}$$

Prefix the sign of the decimal number with the larger magnitude. Hence, $-3 + 2.24 = -0.76$.

Answer: -2.26

Subtraction still means *add the opposite*.

You Try It!

EXAMPLE 9. Simplify: $-8.567 - (-12.3)$.

Simplify: $-2.384 - (-15.2)$

Solution. Subtraction must first be changed to addition by adding the opposite.

$$-8.567 - (-12.3) = -8.567 + 12.3$$

We have unlike signs. First, subtract the smaller magnitude from the larger magnitude.

$$\begin{array}{r} 12.300 \\ -\ 8.567 \\ \hline 3.733 \end{array}$$

Prefix the sign of the decimal number with the larger magnitude. Hence:

$$\begin{aligned} -8.567 - (-12.3) &= -8.567 + 12.3 \\ &= 3.733 \end{aligned}$$

Answer: 12.816

Order of operations demands that we simplify expressions contained in parentheses first.

You Try It!

EXAMPLE 10. Simplify: $-11.2 - (-8.45 + 2.7)$.

Simplify: $-12.8 - (-7.44 + 3.7)$

Solution. We need to add inside the parentheses first. Because we have unlike signs, subtract the smaller magnitude from the larger magnitude.

$$\begin{array}{r} 8.45 \\ -\ 2.70 \\ \hline 5.75 \end{array}$$

Prefix the sign of the number with the larger magnitude. Therefore,

$$-11.2 - (-8.45 + 2.7) = -11.2 - (-5.75)$$

Subtraction means add the opposite.

$$-11.2 - (-5.75) = -11.2 + 5.75$$

Again, we have unlike signs. Subtract the smaller magnitude from the larger magnitude.

$$\begin{array}{r} 11.20 \\ -\ 5.75 \\ \hline 5.45 \end{array}$$

Prefix the sign of the number with the large magnitude.

$$-11.2 + 5.75 = -5.45$$

Answer: -9.06

□

Writing Mathematics. The solution to the previous example should be written as follows:

$$\begin{aligned} -11.2 - (-8.45 + 2.7) &= -11.2 - (-5.75) \\ &= -11.2 + 5.75 \\ &= -5.45 \end{aligned}$$

Any scratch work, such as the computations in vertical format in the previous example, should be done in the margin or on a scratch pad.

You Try It!

EXAMPLE 11. Simplify: $-12.3 - |-4.6 - (-2.84)|$.

Solution. We simplify the expression inside the absolute value bars first, take the absolute value of the result, then subtract.

$$\begin{aligned} &-12.3 - |-4.6 - (-2.84)| \\ &\quad = -12.3 - |-4.6 + 2.84| && \text{Add the opposite.} \\ &\quad = -12.3 - |-1.76| && \text{Add: } -4.6 + 2.84 = -1.76. \\ &\quad = -12.3 - 1.76 && |-1.76| = 1.76. \\ &\quad = -12.3 + (-1.76) && \text{Add the opposite.} \\ &\quad = -14.06 && \text{Add: } -12.3 + (-1.76) = -14.06. \end{aligned}$$

Simplify:

$$-8.6 - |-5.5 - (-8.32)|$$

Answer: -11.42

□

❧ ❧ ❧ Exercises ☙ ☙ ☙

In Exercises 1-12, add the decimals.

1. $31.9+84.7$

2. $9.39+7.7$

3. $4+97.18$

4. $2.645+2.444$

5. $4+87.502$

6. $23.69+97.8$

7. $95.57+7.88$

8. $18.7+7$

9. $52.671+5.97$

10. $9.696+28.2$

11. $4.76+2.1$

12. $1.5+46.4$

In Exercises 13-24, subtract the decimals.

13. $9-2.261$

14. $98.14-7.27$

15. $80.9-6$

16. $9.126-6$

17. $55.672-3.3$

18. $4.717-1.637$

19. $60.575-6$

20. $8.91-2.68$

21. $39.8-4.5$

22. $8.210-3.7$

23. $8.1-2.12$

24. $7.675-1.1$

In Exercises 25-64, add or subtract the decimals, as indicated.

25. $-19.13-7$

26. $-8-79.8$

27. $6.08-76.8$

28. $5.76-36.8$

29. $-34.7+(-56.214)$

30. $-7.5+(-7.11)$

31. $8.4+(-6.757)$

32. $-1.94+72.85$

33. $-50.4+7.6$

34. $1.4+(-86.9)$

35. $-43.3+2.2$

36. $0.08+(-2.33)$

37. $0.19-0.7$

38. $9-18.01$

39. $-7-1.504$

40. $-4.28-2.6$

41. $-4.47+(-2)$

42. $-9+(-43.67)$

43. $71.72-(-6)$

44. $6-(-8.4)$

45. $-9.829 - (-17.33)$

46. $-95.23 - (-71.7)$

47. $2.001 - 4.202$

48. $4 - 11.421$

49. $2.6 - 2.99$

50. $3.57 - 84.21$

51. $-4.560 - 2.335$

52. $-4.95 - 96.89$

53. $-54.3 - 3.97$

54. $-2 - 29.285$

55. $-6.32 + (-48.663)$

56. $-8.8 + (-34.27)$

57. $-8 - (-3.686)$

58. $-2.263 - (-72.3)$

59. $9.365 + (-5)$

60. $-0.12 + 6.973$

61. $2.762 - (-7.3)$

62. $65.079 - (-52.6)$

63. $-96.1 + (-9.65)$

64. $-1.067 + (-4.4)$

In Exercises 65-80, simplify the given expression.

65. $-12.05 - |17.83 - (-17.16)|$

66. $15.88 - | - 5.22 - (-19.94)|$

67. $-6.4 + |9.38 - (-9.39)|$

68. $-16.74 + |16.64 - 2.6|$

69. $-19.1 - (1.51 - (-17.35))$

70. $17.98 - (10.07 - (-10.1))$

71. $11.55 + (6.3 - (-1.9))$

72. $-8.14 + (16.6 - (-15.41))$

73. $-1.7 - (1.9 - (-16.25))$

74. $-4.06 - (4.4 - (-10.04))$

75. $1.2 + |8.74 - 16.5|$

76. $18.4 + |16.5 - 7.6|$

77. $-12.4 - |3.81 - 16.4|$

78. $13.65 - |11.55 - (-4.44)|$

79. $-11.15 + (11.6 - (-16.68))$

80. $8.5 + (3.9 - 6.98)$

---

81. **Big Banks.** Market capitalization of nation's four largest banks (*as of April 23, 2009*)

JPMorgan Chase & Co	$124.8 billion
Wells Fargo & Co.	$85.3 billion
Goldman Sachs Group Inc.	$61.8 billion
Bank of America	$56.4 billion

What is the total value of the nation's four largest banks? *Associated Press Times-Standard 4/22/09*

82. **Telescope Mirror.** The newly launched Herschel Telescope has a mirror 11.5 feet in diameter while Hubble's mirror is 7.9 feet in diameter. How much larger is Herschel's mirror in diameter than Hubble's?

83. **Average Temperature.** The average temperatures in Sacramento, California in July are a high daytime temperature of 93.8 degrees Fahrenheit and a low nighttime temperature of 60.9 degrees Fahrenheit. What is the change in temperature from day to night? *Hint: See Section 2.3 for the formula for comparing temperatures.*

84. **Average Temperature.** The average temperatures in Redding, California in July are a high daytime temperature of 98.2 degrees Fahrenheit and a low nighttime temperature of 64.9 degrees Fahrenheit. What is the change in temperature from day to night? *Hint: See Section 2.3 for the formula for comparing temperatures.*

85. **Net Worth.** Net worth is defined as *assets* minus *liabilities*. *Assets* are everything of value that can be converted to cash while *liabilities* are the total of debts. General Growth Properties, the owners of the Bayshore Mall, have $29.6 billion in assets and $27 billion in liabilities, and have gone bankrupt. What was General Growth Properties net worth before bankruptcy? *Times-Standard 4/17/2009*

86. **Grape crush.** The California Department of Food and Agriculture's preliminary grape crush report shows that the state produced 3.69 million tons of wine grapes in 2009. That's just shy of the record 2005 crush of 3.76 million tons. By how many tons short of the record was the crush of 2009? *Associated Press-Times-Standard Calif. winegrapes harvest jumped 23% in '09.*

87. **Turnover.** The Labor Department's Job Openings and Labor Turnover Survey claims that employers hired about 4.08 million people in January 2010 while 4.12 million people were fired or otherwise left their jobs. How many more people lost jobs than were hired? Convert your answer to a whole number. *Associated Press-Times-Standard 03/10/10 Job openings up sharply in January to* 2.7*M.*

❧ ❧ ❧ Answers ☙ ☙ ☙

1. 116.6

3. 101.18

5. 91.502

7. 103.45

9. 58.641

11. 6.86

13. 6.739

15. 74.9

17. 52.372

19. 54.575

21. 35.3

23. 5.98

25. −26.13

27. −70.72

29. −90.914

31. 1.643

33. −42.8

35. -41.1

37. -0.51

39. -8.504

41. -6.47

43. 77.72

45. 7.501

47. -2.201

49. -0.39

51. -6.895

53. -58.27

55. -54.983

57. -4.314

59. 4.365

61. 10.062

63. -105.75

65. -47.04

67. 12.37

69. -37.96

71. 19.75

73. -19.85

75. 8.96

77. -24.99

79. 17.13

81. \$328.3 billion

83. -32.9 degrees Fahrenheit

85. \$2.6 billion

87. $40,000$

5.3 Multiplying Decimals

Multiplying decimal numbers involves two steps: (1) multiplying the numbers as whole numbers, ignoring the decimal point, and (2) placing the decimal point in the correct position in the product or answer.

For example, consider (0.7)(0.08), which asks us to find the product of "seven tenths" and "eight hundredths." We could change these decimal numbers to fractions, then multiply.

$$\begin{aligned}(0.7)(0.08) &= \frac{7}{10} \cdot \frac{8}{100} \\ &= \frac{56}{1000} \\ &= 0.056\end{aligned}$$

The product is 56/1000, or "fifty six thousandths," which as a decimal is written 0.056.

Important Observations. There are two very important observations to be made about the example (0.7)(0.08).

1. In fractional form

$$\frac{7}{10} \cdot \frac{8}{100} = \frac{56}{1000},$$

   note that the numerator of the product is found by taking the product of the whole numbers 7 and 8. That is, you ignore the decimal points in 0.7 and 0.08 and multiply 7 and 8 as if they were whole numbers.

2. The first factor, 0.7, has one digit to the right of the decimal point. Its fractional equivalent, 7/10, has one zero in its denominator. The second factor, 0.08, has two digits to the right of the decimal point. Its fractional equivalent, 8/100, has two zeros in its denominator. Therefore, the product 56/1000 is forced to have three zeros in its denominator and its decimal equivalent, 0.056, must therefore have three digits to the right of the decimal point.

Let's look at another example.

You Try It!

Multiply: (1.86)(9.5)

EXAMPLE 1. Simplify: (2.34)(1.2).

Solution. Change the decimal numbers "two and thirty four hundredths" and "one and two tenths" to fractions, then multiply.

$$\begin{aligned}
(2.34)(1.2) &= 2\frac{34}{100} \cdot 1\frac{2}{10} && \text{Change decimals to fractions.} \\
&= \frac{234}{100} \cdot \frac{12}{10} && \text{Change mixed to improper fractions.} \\
&= \frac{2808}{1000} && \text{Multiply numerators and denominators.} \\
&= 2\frac{808}{1000} && \text{Change to mixed fraction.} \\
&= 2.808 && \text{Change back to decimal form.}
\end{aligned}$$

Answer: 17.67

□

Important Observations. We make the same two observations as in the previous example.

1. If we treat the decimal numbers as whole numbers without decimal points, then $(234)(12) = 2808$, which is the numerator of the fraction $2808/1000$ in the solution shown in Example 1. These are also the same digits shown in the answer 2.808.

2. There are two digits to the right of the decimal point in the first factor 2.34 and one digit to the right of the decimal point in the second factor 1.2. This is a total of three digits to the right of the decimal points in the factors, which is precisely the same number of digits that appear to the right of the decimal point in the answer 2.808.

The observations made at the end of the previous two examples lead us to the following method.

Multiplying Decimal Numbers. To multiply two decimal numbers, perform the following steps:

1. Ignore the decimal points in the factors and multiply the two factors as if they were whole numbers.

2. Count the number of digits to the right of the decimal point in each factor. Sum these two numbers.

3. Place the decimal point in the product so that the number of digits to the right of the decimal point equals the sum found in step 2.

You Try It!

Multiply: (5.98)(3.7)

EXAMPLE 2. Use the steps outlined in *Multiplying Decimal Numbers* to find the product in Example 1.

Solution. We follow the steps outlined in *Multiplying Decimal Numbers.*

1. The first step is to multiply the factors 2.34 and 1.2 as whole numbers, ignoring the decimal points.

$$\begin{array}{r} 234 \\ \times\, 12 \\ \hline 468 \\ 234 \\ \hline 2808 \end{array}$$

2. The second step is to find the sum of the number of digits to the right of the decimal points in each factor. Note that 2.34 has two digits to the right of the decimal point, while 1.2 has one digit to the right of the decimal point. Thus, we have a total of three digits to the right of the decimal points in the factors.

3. The third and final step is to place the decimal point in the product or answer so that there are a total of three digits to the right of the decimal point. Thus,

$$(2.34)(1.2) = 2.808.$$

   Note that this is precisely the same solution found in Example 1.

What follows is a convenient way to arrange your work in vertical format.

$$\begin{array}{r} 2.34 \\ \times\, 1.2 \\ \hline 468 \\ 2\,34 \\ \hline 2.808 \end{array}$$

Answer: 22.126

□

You Try It!

Multiply: (9.582)(8.6)

EXAMPLE 3. Simplify: (8.235)(2.3).

Solution. We use the convenient vertical format introduced at the end of Example 2.

$$\begin{array}{r} 8.235 \\ \times\, 2.3 \\ \hline 2\;4705 \\ 16\;470 \\ \hline 18.9405 \end{array}$$

The factor 8.235 has three digits to the right of the decimal point; the factor 2.3 has one digit to the right of the decimal point. Therefore, there must be a total of four digits to the right of the decimal point in the product or answer.

Answer: 82.4052

Multiplying Signed Decimal Numbers

The rules governing multiplication of signed decimal numbers are identical to the rules governing multiplication of integers.

Like Signs. The product of two decimal numbers with like signs is positive. That is:

$$(+)(+) = + \qquad \text{and} \qquad (-)(-) = +$$

Unike Signs. The product of two decimal numbers with unlike signs is negative. That is:

$$(+)(-) = - \qquad \text{and} \qquad (-)(+) = -$$

You Try It!

EXAMPLE 4. Simplify: $(-2.22)(-1.23)$.

Multiply: $(-3.86)(-5.77)$

Solution. Ignore the signs to do the multiplication on the left, then consider the signs in the final answer on the right.

As each factor has two digits to the right of the decimal point, there should be a total of 4 decimals to the right of the decimal point in the product.

$$\begin{array}{r} 2.22 \\ \times\, 1.23 \\ \hline 666 \\ 444 \\ 1\;11 \\ \hline 1.6206 \end{array}$$

Like signs give a positive product. Hence:

$$(-2.22)(-1.23) = 1.6206$$

Answer: 22.2722

You Try It!

Multiply: $(9.23)(-0.018)$

EXAMPLE 5. Simplify: $(5.68)(-0.012)$.

Solution. Ignore the signs to do the multiplication on the left, then consider the signs in the final answer on the right.

The first factor has two digits to the right of the decimal point, the second factor has three. Therefore, there must be a total of five digits to the right of the decimal point in the product or answer. This necessitates prepending an extra zero in front of our product.

$$\begin{array}{r} 5.68 \\ \times\, 0.012 \\ \hline 1136 \\ 568 \\ \hline 0.06816 \end{array}$$

Unlike signs give a negative product. Hence:

$$(5.68)(-0.012) = -0.06816$$

Answer: -0.16614

□

Order of Operations

The same *Rules Guiding Order of Operations* also apply to decimal numbers.

Rules Guiding Order of Operations. When evaluating expressions, proceed in the following order.

1. Evaluate expressions contained in grouping symbols first. If grouping symbols are nested, evaluate the expression in the innermost pair of grouping symbols first.
2. Evaluate all exponents that appear in the expression.
3. Perform all multiplications and divisions in the order that they appear in the expression, moving left to right.
4. Perform all additions and subtractions in the order that they appear in the expression, moving left to right.

You Try It!

If $a = 3.8$ and $b = -4.6$, evaluate the expression:

$$2.5a^2 - b^2$$

EXAMPLE 6. If $a = 3.1$ and $b = -2.4$, evaluate $a^2 - 3.2b^2$.

Solution. Prepare the expression for substitution using parentheses.

$$a^2 - 3.2b^2 = (\quad)^2 - 3.2(\quad)^2$$

Substitute 3.1 for a and -2.4 for b and simplify.

$$\begin{aligned} a^2 - 3.2b^2 &= (3.1)^2 - 3.2(-2.4)^2 && \text{Substitute: 3.1 for } a, -2.4 \text{ for } b. \\ &= 9.61 - 3.2(5.76) && \text{Exponents first: } (3.1)^2 = 9.61,\ (-2.4)^2 = 5.76 \\ &= 9.61 - 18.432 && \text{Multiply: } 3.2(5.76) = 18.432 \\ &= -8.822 && \text{Subtract: } 9.61 - 18.432 = -8.822 \end{aligned}$$

Answer: 14.94

□

Powers of Ten

Consider:

$$\begin{aligned} 10^1 &= 10 \\ 10^2 &= 10 \cdot 10 = 100 \\ 10^3 &= 10 \cdot 10 \cdot 10 = 1,000 \\ 10^4 &= 10 \cdot 10 \cdot 10 \cdot 10 = 10,000 \end{aligned}$$

Note the answer for 10^4, a one followed by four zeros! Do you see the pattern?

Powers of Ten. In the expression 10^n, the exponent matches the number of zeros in the answer. Hence, 10^n will be a 1 followed by n zeros.

You Try It!

EXAMPLE 7. Simplify: 10^9.

Simplify: 10^6

Solution. 10^9 should be a 1 followed by 9 zeros. That is,

$$10^9 = 1,000,000,000,$$

or "one billion."

Answer: 1,000,000

□

Multiplying Decimal Numbers by Powers of Ten

Let's multiply 1.234567 by 10^3, or equivalently, by 1,000. Ignore the decimal point and multiply the numbers as whole numbers.

$$\begin{array}{r} 1.234567 \\ \times 1000 \\ \hline 1234.567000 \end{array}$$

The sum total of digits to the right of the decimal point in each factor is 6. Therefore, we place the decimal point in the product so that there are six digits to the right of the decimal point.

However, the trailing zeros may be removed without changing the value of the product. That is, 1.234567 times 1000 is 1234.567. Note that the decimal point in the product is three places further to the right than in the original factor. This observation leads to the following result.

Multiplying a Decimal Number by a Power of Ten. Multiplying a decimal number by 10^n will move the decimal point n places to the right.

You Try It!

Simplify: $1.234567 \cdot 10^2$

Answer: 123.4567

EXAMPLE 8. Simplify: $1.234567 \cdot 10^4$

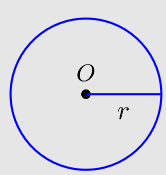
Solution. Multiplying by 10^4 (or equivalently, by 10,000) moves the decimal 4 places to the right. Thus, $1.234567 \cdot 10,000 = 12345.67$.

□

The Circle

Let's begin with a definition.

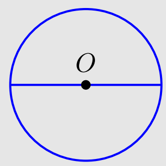
The Circle. A *circle* is the collection of all points equidistant from a given point O, called the *center* of the circle.



The segment joining any point on the circle to the center of the circle is called a *radius* of the circle. In the figure above, the variable r represents the length of the radius of the circle.

We need another term, the *diameter* of a circle.

The Diameter of a Circle. If two points on a circle are connected with a line segment, then that segment is called a *chord* of the circle. If the chord passes through the center of the circle, then the chord is called the *diameter* of the circle.



In the figure above, the variable d represents the length of the diameter of the circle. Note that the diameter is twice the length of the radius; in symbols,

$$d = 2r.$$

The Circumference of a Circle

When we work with polygons, the *perimeter* of the polygon is found by summing the lengths of its edges. The circle uses a different name for its perimeter.

The Circumference of a Circle. The length of the circle is called its *circumference*. We usually use the variable C to denote the circumference of a circle.

That is, if one were to walk along the circle, the total distance traveled in one revolution is the circumference of the circle.

The ancient mathematicians of Egypt and Greece noted a striking relation between the circumference of a circle and its diameter. They discovered that whenever you divide a circle's circumference by its diameter, you get a constant. That is, if you take a very large circle and divide its circumference by its diameter, you get exactly the same number if you take a very small circle and divide its circumference by its diameter. This common constant was named π ("pi").

Relating the Circumference and Diameter. Whenever a circle's circumference is divided by its diameter, the answer is the constant π. That is, if C is the circumference of the circle and d is the circle's diameter, then

$$\frac{C}{d} = \pi.$$

In modern times, we usually multiply both sides of this equation by d to obtain the formula for the circumference of a circle.

$$C = \pi d$$

Because the diameter of a circle is twice the length of its radius, we can substitute $d = 2r$ in the last equation to get an alternate form of the circumference equation.

$$C = \pi(2r) = 2\pi r$$

The number π has a rich and storied history. Ancient geometers from Egypt, Babylonia, India, and Greece knew that π was slightly larger than 3. The earliest known approximations date from around 1900 BC (Wikipedia); they are 25/8 (Babylonia) and 256/81 (Egypt). The Indian text *Shatapatha Brahmana* gives π as $339/108 \approx 3.139$. Archimedes (287-212 BC) was the first to estimate π rigorously, approximating the circumference of a circle with inscribed and circumscribed polygons. He was able to prove that $223/71 < \pi < 22/7$. Taking the average of these values yields $\pi \approx 3.1419$. Modern mathematicians have proved that π is an *irrational* number, an infinite decimal that never repeats any pattern. Mathematicians, with the help of computers, routinely produce approximations of π with billions of digits after the decimal point.

Digits of Pi. Here is π, correct to the first fifty digits.

$$\pi = 3.14159265358979323846264338327950288419716939937510\ldots$$

The number of digits of π used depends on the application. Working at very small scales, one might keep many digits of π, but if you are building a circular garden fence in your backyard, then fewer digits of π are needed.

You Try It!

Find the radius of a circle having radius 14 inches. Use $\pi \approx 3.14$

EXAMPLE 9. Find the circumference of a circle given its radius is 12 feet.

Solution. The circumference of the circle is given by the formula $C = \pi d$, or, because $d = 2r$,

$$C = 2\pi r.$$

Substitute 12 for r.

$$C = 2\pi r = 2\pi(12) = 24\pi$$

Therefore, the circumference of the circle is *exactly* $C = 24\pi$ feet.

We can approximate the circumference by entering an approximation for π. Let's use $\pi \approx 3.14$. *Note: The symbol ≈ is read "approximately equal to."*

$$C = 24\pi \approx 24(3.14) \approx 75.36 \text{ feet}$$

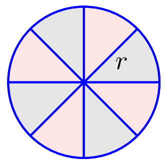
It is important to understand that the solution $C = 24\pi$ feet is the **exact** circumference, while $C \approx 75.36$ feet is only an approximation.

Answer: 87.92 inches

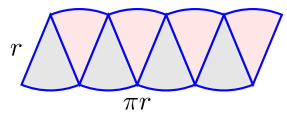
□

The Area of a Circle

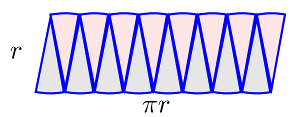
Here's a reasonable argument used to help develop a formula for the area of a circle. Start with a circle of radius r and divide it into 8 equal wedges, as shown in the figure that follows.



Rearrange the pieces as shown in the following figure.



Note that the rearranged pieces almost form a rectangle with length approximately half the circumference of the circle, πr, and width approximately r. The area would be approximately $A \approx (\text{length})(\text{width}) \approx (\pi r)(r) \approx \pi r^2$. This approximation would be even better if we doubled the number of wedges of the circle.



If we doubled the number of wedges again, the resulting figure would even more closely resemble a rectangle with length πr and width r. This leads to the following conclusion.

The Area of a Circle. The area of a circle of radius r is given by the formula

$$A = \pi r^2.$$

You Try It!

EXAMPLE 10. Find the area of a circle having a diameter of 12.5 meters. Use 3.14 for π and round the answer for the area to the nearest tenth of a square meter.

Find the area of a circle having radius 12.2 centimeters. Use $\pi \approx 3.14$

Solution. The diameter is twice the radius.

$$d = 2r$$

Substitute 12.5 for d and solve for r.

$$
\begin{aligned}
12.5 &= 2r && \text{Substitute 12.5 for } d. \\
\frac{12.5}{2} &= \frac{2r}{2} && \text{Divide both sides by 2.} \\
6.25 &= r && \text{Simiplify.}
\end{aligned}
$$

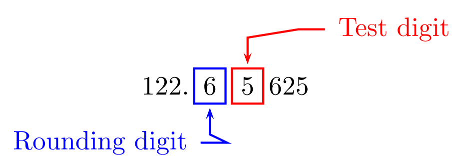
Hence, the radius is 6.25 meters. To find the area, use the formula

$$A = \pi r^2$$

and substutite: 3.14 for π and 6.25 for r.

$$
\begin{aligned}
A &= (3.14)(6.25)^2 && \text{Substitute: 3.14 for } \pi\text{, 6.25 for } r. \\
&= (3.14)(39.0625) && \text{Square first: } (6.25)^2 = 39.0625. \\
&= 122.65625 && \text{Multiply: } (3.14)(39.0625) = 122.65625.
\end{aligned}
$$

Hence, the approximate area of the circle is $A = 122.65625$ square meters. To round to the nearest tenth of a square meter, identify the rounding digit and the test digit.



Because the test digit is greater than or equal to 5, add 1 to the rounding digit and truncate. Thus, correct to the nearest tenth of a square meter, the area of the circle is approximately

$$A \approx 122.7\,\text{m}^2.$$

Answer: $467.3576\,\text{cm}^2$

□

❧ ❧ ❧ Exercises ☙ ☙ ☙

In Exercises 1-28, multiply the decimals.

1. (6.7)(0.03)

2. (2.4)(0.2)

3. (28.9)(5.9)

4. (33.5)(2.1)

5. (4.1)(4.6)

6. (2.6)(8.2)

7. (75.3)(0.4)

8. (21.4)(0.6)

9. (6.98)(0.9)

10. (2.11)(0.04)

11. (57.9)(3.29)

12. (3.58)(16.3)

13. (47.3)(0.9)

14. (30.7)(0.4)

15. (9.9)(6.7)

16. (7.2)(6.1)

17. (19.5)(7.9)

18. (43.4)(8.9)

19. (6.9)(0.3)

20. (7.7)(0.7)

21. (35.3)(3.81)

22. (5.44)(9.58)

23. (2.32)(0.03)

24. (4.48)(0.08)

25. (3.02)(6.7)

26. (1.26)(9.4)

27. (4.98)(6.2)

28. (3.53)(2.9)

In Exercises 29-56, multiply the decimals.

29. $(-9.41)(0.07)$

30. $(4.45)(-0.4)$

31. $(-7.4)(-0.9)$

32. $(-6.9)(0.05)$

33. $(-8.2)(3.7)$

34. $(-7.5)(-6.6)$

35. $(9.72)(-9.1)$

36. $(6.22)(-9.4)$

37. $(-6.4)(2.6)$

38. $(2.3)(-4.4)$

39. $(-39.3)(-0.8)$

40. $(57.7)(-0.04)$

41. $(63.1)(-0.02)$

42. $(-51.1)(-0.8)$

43. $(-90.8)(3.1)$

44. $(-74.7)(2.9)$

45. $(47.5)(-82.1)$

46. $(-14.8)(-12.7)$

47. $(-31.1)(-4.8)$

48. $(-28.7)(-6.8)$

49. $(-2.5)(-0.07)$

50. $(-1.3)(-0.05)$

51. $(1.02)(-0.2)$

52. $(-7.48)(-0.1)$

53. $(7.81)(-5.5)$

54. $(-1.94)(4.2)$

55. $(-2.09)(37.9)$

56. $(20.6)(-15.2)$

In Exercises 57-68, multiply the decimal by the given power of 10.

57. $24.264 \cdot 10$

58. $65.722 \cdot 100$

59. $53.867 \cdot 10^4$

60. $23.216 \cdot 10^4$

61. $5.096 \cdot 10^3$

62. $60.890 \cdot 10^3$

63. $37.968 \cdot 10^3$

64. $43.552 \cdot 10^3$

65. $61.303 \cdot 100$

66. $83.837 \cdot 1000$

67. $74.896 \cdot 1000$

68. $30.728 \cdot 100$

In Exercises 69-80, simplify the given expression.

69. $(0.36)(7.4) - (-2.8)^2$

70. $(-8.88)(-9.2) - (-2.3)^2$

71. $9.4 - (-7.7)(1.2)^2$

72. $0.7 - (-8.7)(-9.4)^2$

73. $5.94 - (-1.2)(-1.8)^2$

74. $-2.6 - (-9.8)(9.9)^2$

75. $6.3 - 4.2(9.3)^2$

76. $9.9 - (-4.1)(8.5)^2$

77. $(6.3)(1.88) - (-2.2)^2$

78. $(-4.98)(-1.7) - 3.5^2$

79. $(-8.1)(9.4) - 1.8^2$

80. $(-3.63)(5.2) - 0.8^2$

---

81. Given $a = -6.24$, $b = 0.4$, and $c = 7.2$, evaluate the expression $a - bc^2$.

82. Given $a = 4.1$, $b = -1.8$, and $c = -9.5$, evaluate the expression $a - bc^2$.

83. Given $a = -2.4$, $b = -2.1$, and $c = -4.6$, evaluate the expression $ab - c^2$.

84. Given $a = 3.3$, $b = 7.3$, and $c = 3.4$, evaluate the expression $ab - c^2$.

85. Given $a = -3.21$, $b = 3.5$, and $c = 8.3$, evaluate the expression $a - bc^2$.

86. Given $a = 7.45$, $b = -6.1$, and $c = -3.5$, evaluate the expression $a - bc^2$.

87. Given $a = -4.5$, $b = -6.9$, and $c = 4.6$, evaluate the expression $ab - c^2$.

88. Given $a = -8.3$, $b = 8.2$, and $c = 5.4$, evaluate the expression $ab - c^2$.

---

89. A circle has a diameter of 8.56 inches. Using $\pi \approx 3.14$, find the circumference of the circle, correct to the nearest tenth of an inch.

90. A circle has a diameter of 14.23 inches. Using $\pi \approx 3.14$, find the circumference of the circle, correct to the nearest tenth of an inch.

91. A circle has a diameter of 12.04 inches. Using $\pi \approx 3.14$, find the circumference of the circle, correct to the nearest tenth of an inch.

92. A circle has a diameter of 14.11 inches. Using $\pi \approx 3.14$, find the circumference of the circle, correct to the nearest tenth of an inch.

93. A circle has a diameter of 10.75 inches. Using $\pi \approx 3.14$, find the area of the circle, correct to the nearest hundredth of a square inch.

94. A circle has a diameter of 15.49 inches. Using $\pi \approx 3.14$, find the area of the circle, correct to the nearest hundredth of a square inch.

95. A circle has a diameter of 13.96 inches. Using $\pi \approx 3.14$, find the area of the circle, correct to the nearest hundredth of a square inch.

96. A circle has a diameter of 15.95 inches. Using $\pi \approx 3.14$, find the area of the circle, correct to the nearest hundredth of a square inch.

---

97. Sue has decided to build a circular fish pond near her patio. She wants it to be 15 feet in diameter and 1.5 feet deep. What is the volume of water it will hold? Use $\pi \approx 3.14$. *Hint: The volume of a cylinder is given by the formula* $V = \pi r^2 h$, *which is the area of the circular base times the height of the cylinder.*

98. John has a decision to make regarding his employment. He currently has a job at Taco Loco in Fortuna. After taxes, he makes about $9.20 per hour and works about 168 hours a month. He currently pays $400 per month for rent. He has an opportunity to move to Santa Rosa and take a job at Mi Ultimo Refugio which would pay $10.30 per hour after taxes for 168 hours a month, but his rent would cost $570 per month.

a) After paying for housing in Fortuna, how much does he have left over each month for other expenditures?

b) After paying for housing in Santa Rosa, how much would he have left over each month for other expenditures?

c) For which job would he have more money left after paying rent and how much would it be?

99. John decided to move to Santa Rosa and take the job at Mi Ultimo Refugio (see Exercise 98). He was able to increase his income because he could work 4 Sundays a month at time-and-a half. So now he worked 32 hours a month at time-and-a-half and 136 hours at the regular rate of $10.30 (all after taxes were removed). *Note: He previously had worked 168 hours per month at $10.30 per hour.*

a) What was his new monthly income?

b) How much did his monthly income increase?

100. **Electric Bill.** On a recent bill, PGE charged \$0.11531 per Kwh for the first 333 Kwh of electrical power used. If a household used 312 Kwh of power, what was their electrical bill?

101. **Cabernet.** In Napa Valley, one acre of good land can produce about 3.5 tons of quality grapes. At an average price of \$3,414 per ton for premium cabernet, how much money could you generate on one acre of cabernet farming? *Associated Press-Times-Standard 03/11/10 Grape moth threatens Napa Valley growing method.*

102. **Fertilizer.** Using the 2008 Ohio Farm Custom Rates, the average cost for spreading dry bulk fertilizer is about \$4.50 per acre. What is the cost to fertilize 50 acres?

103. **Agribusiness.** Huge corporate agribarns house 1000 pigs each.

a) If each pig weighs approximately 100 pounds, how many pounds of pig is in each warehouse?

b) At an average \$1.29 per pound, what is the total cash value for a corporate agribarn? *Associated Press-Times-Standard 12/29/09 Pressure rises to stop antibiotics in agriculture.*

104. **Shipwrecks.** A dozen centuries-old shipwrecks were found in the Baltic Sea by a gas company building an underwater pipeline between Russia and Germany. The 12 wrecks were found in a 30-mile-long and 1.2-mile-wide corridor at a depth of 430 feet. Model the corridor with a rectangle and find the approximate area of the region where the ships were found. *Associated Press-Times-Standard 03/10/10 Centuries-old shipwrecks found in Baltic Sea.*

105. **Radio dish.** The diameter of the "workhorse fleet" of radio telescopes, like the one in Goldstone, California, is 230 feet. What is the circumference of the radio telescope dish to the nearest tenth? *Associated Press-Times-Standard 03/09/2010 NASA will repair deep space antenna in California desert.*

❧ ❧ ❧ Answers ☙ ☙ ☙

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1. 0.201

3. 170.51

5. 18.86

7. 30.12

9. 6.282

11. 190.491

13. 42.57

15. 66.33

17. 154.05

19. 2.07

21. 134.493

23. 0.0696

25. 20.234

27. 30.876

29. −0.6587

31. 6.66

33. -30.34

35. -88.452

37. -16.64

39. 31.44

41. -1.262

43. -281.48

45. -3899.75

47. 149.28

49. 0.175

51. -0.204

53. -42.955

55. -79.211

57. 242.64

59. 538670

61. 5096

63. 37968

65. 6130.3

67. 74896

69. -5.176

71. 20.488

73. 9.828

75. -356.958

77. 7.004

79. -79.38

81. -26.976

83. -16.12

85. -244.325

87. 9.89

89. 26.9 in.

91. 37.8 in.

93. 90.72 square inches

95. 152.98 square inches

97. 264.9375 cubic feet

99.
a) \$1895.20
b) \$164.80

101. \$11,949

103.
a) 100,000 pounds
b) \$129,000

105. 722.2 feet

5.4 Dividing Decimals

In this and following sections we make use of the terms *divisor*, *dividend*, *quotient*, and *remainder*.

Divisor, Dividend, Quotient, and Remainder. This schematic reminds readers of the position of these terms in the division process.

$$\begin{array}{r}\text{quotient}\\ \text{divisor}\overline{)\ \text{dividend}}\\ \dots\\ \hline \text{remainder}\end{array}$$

Now that these terms are defined, we begin the discussion of division of decimal numbers.

Suppose that we wish to divide 637 by 100. We could do this in fraction form, change the result to a mixed fraction, then the mixed fraction to decimal form.

$$\frac{637}{100} = 6\,\frac{37}{100} = 6.37$$

We can also arrange the division much as we would the division of two whole numbers.

$$\begin{array}{r}6.37\\ 100\overline{)637.00}\\ \underline{600}\\ 37\ 0\\ \underline{30\ 0}\\ 7\ 00\\ \underline{7\ 00}\\ 0\end{array}$$

Note that adding two zeros after the decimal point in the dividend does not change the value of 637. Further, note that we proceed as if we are dividing two whole numbers, placing the decimal point in the quotient directly above the decimal point in the dividend.

These observations lead to the following algorithm.

Dividing a Decimal by a Whole Number. To divide a decimal number by a whole number, proceed as follows:

1. Set up the long division as you would the division of two whole numbers.
2. Perform the division as if the numbers were both whole numbers, adding zeros to the right of the decimal point in the dividend as necessary to complete the division.
3. Place the decimal point in the quotient immediately above the decimal point in the dividend.

You Try It!

EXAMPLE 1. Divide 23 by 20.

Solution. Arrange as if using long division to divide whole numbers, adding enough zeros to the right of the decimal point in the dividend to complete the division.

$$\begin{array}{r} 1.15 \\ 20\overline{)23.00} \\ \underline{20} \\ 3\,0 \\ \underline{2\,0} \\ 1\,00 \\ \underline{1\,00} \\ 0 \end{array}$$

Hence, 23 divided by 20 is 1.15.

□

Adding Zeros to the Right of the Decimal Point. Usually, one does not immediately see how many zeros to the right of the decimal point in the dividend are needed. These zeros are usually added at each step of the division, until the division is completed or the user is willing to terminate the process and accept only an estimate of the quotient.

You Try It!

EXAMPLE 2. Divide: $155.2 \div 25$.

Divide: $42.55 \div 23$

Solution. Arrange as if using long division to divide whole numbers, and begin.

$$\begin{array}{r} 6.2 \\ 25\overline{)155.2} \\ \underline{150} \\ 5\,2 \\ \underline{5\,0} \\ 2 \end{array}$$

We still have a nonzero remainder. Adding another zero does no good.

$$\begin{array}{r} 6.20 \\ 25\overline{)155.20} \\ \underline{150} \\ 5\,2 \\ \underline{5\,0} \\ 20 \end{array}$$

However, if we add one more additional zero, the division completes with a zero remainder.

$$\begin{array}{r} 6.208 \\ 25\overline{)155.200} \\ \underline{150} \\ 5\;2 \\ \underline{5\;0} \\ 200 \\ \underline{200} \\ 0 \end{array}$$

Answer: 1.85

Thus, 155.2 divided by 25 is 6.208.

Decimal Divisors

When the divisor contains a decimal point, we have a little work to do before we begin the division process. Suppose that we wish to divide 1.25 by 2.5. In fraction form, we could start with

$$\frac{1.25}{2.5},$$

then clear the decimals from the denominator by multiplying both numerator and denominator by 10. *Note: Recall that multiplying by 10 moves the decimal point one place to the right.*

$$\begin{aligned} \frac{1.25}{2.5} &= \frac{1.25 \cdot 10}{2.5 \cdot 10} \\ &= \frac{12.5}{25} \end{aligned}$$

Thus, dividing 1.25 by 2.5 is equivalent to dividing 12.5 by 25. This we know how to do.

$$\begin{array}{r} 0.5 \\ 25\overline{)12.5} \\ \underline{12\;5} \\ 0 \end{array}$$

Thus, 1.25 divided by 2.5 is 0.5.

Writing Mathematics. Never write .5. Always add the leading zero in the ones place and write 0.5.

Instead of working in fraction form, we can take care of positioning the decimal point in the long division framework. Start with:

$$2.5\overline{)1.25}$$

Move the decimal point in the divisor to the end of the divisor, then move the decimal point in the dividend an equal number of places.

$$2.5\,\overline{)1.2\,5}$$

Thus, the division becomes

$$25\overline{)12.5}$$

and we proceed as above to find the quotient.

This discussion motivates the following algorithm.

> **Dividing by a Decimal Divisor.** If the divisor contains a decimal, proceed as follows:
>
> 1. Move the decimal to the end of the divisor.
>
> 2. Move the decimal in the dividend an equal number of places.

We can then complete the division using the rules for dividing a decimal by a whole number.

You Try It!

Divide: $0.45\overline{)36.99}$

EXAMPLE 3. Divide: $0.36\overline{)4.392}$

Solution. Move the decimal in the divisor to the end of the divisor. Move the decimal in the dividend an equal number of places (two places) to the right.

$$0.36\,\overline{)4.39\,2}$$

Now we can follow the algorithm for dividing a decimal number by a whole number.

$$\begin{array}{r}
12.2\\
36\overline{)439.2}\\
\underline{36}\\
79\\
\underline{72}\\
7\,2\\
\underline{7\,2}\\
0
\end{array}$$

Answer: 82.2

Thus, 4.392 divided by 0.36 is 12.2.

Dividing Signed Decimal Numbers

The rules governing division of signed decimal numbers are identical to the rules governing division of integers.

Like Signs. The quotient of two decimal numbers with like signs is positive. That is:

$$\frac{(+)}{(+)} = + \qquad \text{and} \qquad \frac{(-)}{(-)} = +$$

Unlike Signs. The quotient of two decimal numbers with unlike signs is negative. That is:

$$\frac{(+)}{(-)} = - \qquad \text{and} \qquad \frac{(-)}{(+)} = -$$

You Try It!

Divide: $-0.0113 \div 0.05$

EXAMPLE 4. Divide: $-0.03 \div 0.024$.

Solution. First, divide the magnitudes. Move the decimal in the divisor to the end of the divisor. Move the decimal in the dividend an equal number of places (three places) to the right. Note that this requires an extra trailing zero in the dividend.

$$0.024\overline{)0.030}$$

Our problem then becomes:

$$24\overline{)30}$$

We can now follow the algorithm for dividing a decimal number by a whole number. Note that we have to add two trailing zeros in the dividend to complete the division with a zero remainder.

$$\begin{array}{r}
1.25 \\
24\overline{)30.00} \\
\underline{24} \\
6\,0 \\
\underline{4\,8} \\
1\,20 \\
\underline{1\,20} \\
0
\end{array}$$

Finally, because the quotient of unlike signs is negative, −0.03 divided by 0.024 is −1.25. That is,

$$\frac{-0.03}{0.024} = -1.25.$$

Answer: −0.226

Rounding

Sometimes an exact decimal representation of a fraction is not needed and is an approximation is more than adequate.

You Try It!

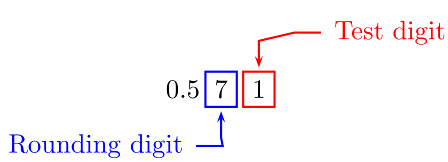
EXAMPLE 5. Convert 4/7 to a decimal. Round your answer to the nearest hundredth.

Convert 5/7 to a decimal. Round your answer to the nearest hundredth.

Solution. We need to carry the division one place beyond the hundredths place.

$$\begin{array}{r} 0.571 \\ 7\overline{)4.000} \\ \underline{3\,5} \\ 50 \\ \underline{49} \\ 10 \\ \underline{7} \\ 3 \end{array}$$

To round to the nearest hundredth, first identify the rounding and test digits.



Because the "test digit" is less 5, leave the rounding digit alone and truncate. Therefore, correct to the nearest hundredth, $4/7 \approx 0.57$.

Answer: 0.71

Dividing by Powers of Ten

Recall:

$$
\begin{aligned}
10^1 &= 10 \\
10^2 &= 10 \cdot 10 = 100 \\
10^3 &= 10 \cdot 10 \cdot 10 = 1000
\end{aligned}
$$

Powers of Ten. In the expression 10^n, the exponent matches the number of zeros in the answer. Hence, 10^n will be a 1 followed by n zeros.

Thus, $10^4 = 10,000$, $10^5 = 100,000$, etc. The exponent tells us how many zeros will follow the 1.

Let's divide 123456.7 by 1000.

$$
\begin{array}{r}
123.4567 \\
1000\overline{)123456.7000} \\
\underline{1000} \\
2345 \\
\underline{2000} \\
3456 \\
\underline{3000} \\
456\,7 \\
\underline{400\,0} \\
56\,70 \\
\underline{50\,00} \\
6\,700 \\
\underline{6\,000} \\
7000 \\
\underline{7000} \\
0
\end{array}
$$

Note the result: 123456.7 divided by 1000 is 123.4567. Dividing by 1000 moves the decimal point 3 places to the left!

$$123456.7 \div 1000 = 123\,456.7 = 123.4567$$

This discussion leads to the following result.

Dividing a Decimal by a Power of Ten. Dividing a decimal number by 10^n will move the decimal point n places to the left.

You Try It!

EXAMPLE 6. Simplify: $123456.7 \div 10^4$

Simplify: $123456.7 \div 10^2$

Solution. Dividing by 10^4 (or equivalently, 10,000) moves the decimal point four places to the left. Thus, $123456.7 \div 10^4 = 12.34567$.

Answer: 1234.567

□

Order of Operations

We remind readers of the *Rules Guiding Order of Operations*.

Rules Guiding Order of Operations. When evaluating expressions, proceed in the following order.

1. Evaluate expressions contained in grouping symbols first. If grouping symbols are nested, evaluate the expression in the innermost pair of grouping symbols first.
2. Evaluate all exponents that appear in the expression.
3. Perform all multiplications and divisions in the order that they appear in the expression, moving left to right.
4. Perform all additions and subtractions in the order that they appear in the expression, moving left to right.

In addition, when fractions are present:

Fractional Expressions. If a fractional expression is present, simplify the numerator and denominator separately, then divide.

You Try It!

EXAMPLE 7. Evaluate $ab/(c+d)$, given that $a = 2.1$, $b = -3.4$, $c = -1.3$, and $d = 1.1$.

If $a = -2.1$, $b = 1.7$, $c = 4$, and $d = 0.05$, evaluate:

$$\frac{a+b}{cd}$$

Solution. Recall that it is good practice to prepare an expression for substitution by using parentheses.

$$ab/(c+d) = (\quad)(\quad)/((\quad)+(\quad))$$

Substitute the given values for a, b, c, and d, then use the *Rules Guiding Order of Operations* to simplify the resulting expression.

$$\begin{aligned} ab/(c+d) &= (2.1)(-3.4)/((-1.3)+(1.1)) && \text{2.1, −3.4, −1.3, 1.1 for } a, b, c, d. \\ &= (2.1)(-3.4)/(-0.2) && \text{Parens: } (-1.3)+(1.1) = -0.2. \\ &= -7.14/(-0.2) && \text{Multiply: } (2.1)(-3.4) = -7.14. \\ &= 35.7 && \text{Divide: } -7.14/(-0.2) = 35.7. \end{aligned}$$

Answer: -2

You Try It!

If $a = 0.5$ and $b = -0.125$, evaluate:

$$\frac{2a-b}{a+2b}$$

EXAMPLE 8. Given $a = 0.1$ and $b = -0.3$, evaluate the expression

$$\frac{a+2b}{2a+b}.$$

Solution. Substitute the given values, then use the *Rules Guiding Order of Operations* to simplify the resulting expression.

$$\frac{a+2b}{2a+b} = \frac{(0.1)+2(-0.3)}{2(0.1)+(-0.3)} \qquad 0.1 \text{ for } a,\ -0.3 \text{ for } b.$$

Simplify the numerator, simplify the denominator, then divide.

$$\begin{aligned} &= \frac{0.1+(-0.6)}{0.2+(-0.3)} && \text{Numerator: } 2(-0.3) = -0.6. \\ & && \text{Denominator: } 2(0.1) = 0.2. \\ &= \frac{-0.5}{-0.1} && \text{Numerator: } 0.1+(-0.6) = -0.5. \\ & && \text{Denominator: } 0.2+(-0.3) = -0.1. \\ &= 5 && \text{Divide: } -0.5/(-0.1) = 5. \end{aligned}$$

Answer: 4.5

❧ ❧ ❧ Exercises ☙ ☙ ☙

In Exercises 1-16, divide the numbers.

1. $\frac{39}{52}$

2. $\frac{16}{25}$

3. $\frac{755.3}{83}$

4. $\frac{410.4}{76}$

5. $\frac{333}{74}$

6. $\frac{117}{65}$

7. $\frac{32.12}{73}$

8. $\frac{12.32}{44}$

9. $\frac{37.63}{71}$

10. $\frac{20.46}{31}$

11. $\frac{138}{92}$

12. $\frac{110}{25}$

13. $\frac{17}{25}$

14. $\frac{18}{75}$

15. $\frac{229.5}{51}$

16. $\frac{525.6}{72}$

In Exercises 17-40, divide the decimals.

17. $\frac{0.3478}{0.47}$

18. $\frac{0.4559}{0.97}$

19. $\frac{1.694}{2.2}$

20. $\frac{1.008}{1.8}$

21. $\frac{43.61}{4.9}$

22. $\frac{22.78}{3.4}$

23. $\frac{1.107}{0.41}$

24. $\frac{2.465}{0.29}$

25. $\frac{2.958}{0.51}$

26. $\frac{5.141}{0.53}$

27. $\frac{71.76}{7.8}$

28. $\frac{14.08}{8.8}$

29. $\frac{0.8649}{0.93}$

30. $\frac{0.3901}{0.83}$

31. $\frac{0.6958}{0.71}$

32. $\frac{0.1829}{0.31}$

33. $\frac{1.248}{0.52}$

34. $\frac{6.375}{0.85}$

35. $\frac{62.56}{9.2}$

36. $\frac{28.08}{7.8}$

37. $\frac{6.278}{8.6}$

38. $\frac{3.185}{4.9}$

39. $\frac{2.698}{7.1}$

40. $\frac{4.959}{8.7}$

In Exercises 41-64, divide the decimals.

41. $\frac{-11.04}{1.6}$

42. $\frac{-31.27}{5.3}$

43. $\frac{-3.024}{5.6}$

44. $\frac{-3.498}{5.3}$

45. $\frac{-0.1056}{0.22}$

46. $\frac{-0.2952}{-0.72}$

47. $\frac{0.3204}{-0.89}$

48. $\frac{0.3306}{-0.38}$

49. $\frac{-1.419}{0.43}$

50. $\frac{-1.625}{-0.25}$

51. $\frac{-16.72}{-2.2}$

52. $\frac{-66.24}{9.2}$

53. $\frac{-2.088}{-0.87}$

54. $\frac{-2.025}{-0.75}$

55. $\frac{-1.634}{-8.6}$

56. $\frac{-3.094}{3.4}$

57. $\frac{-0.119}{0.85}$

58. $\frac{0.5766}{-0.62}$

59. $\frac{-3.591}{-6.3}$

60. $\frac{-3.016}{5.8}$

61. $\frac{36.96}{-4.4}$

62. $\frac{-78.26}{-8.6}$

63. $\frac{-2.156}{-0.98}$

64. $\frac{-6.072}{0.66}$

In Exercises 65-76, divide the decimal by the given power of 10.

65. $\frac{524.35}{100}$

66. $\frac{849.39}{100}$

67. $\dfrac{563.94}{10^3}$

68. $\dfrac{884.15}{10^3}$

69. $\dfrac{116.81}{10^2}$

70. $\dfrac{578.01}{10^3}$

71. $\dfrac{694.55}{10}$

72. $\dfrac{578.68}{100}$

73. $\dfrac{341.16}{10^3}$

74. $\dfrac{46.63}{10^4}$

75. $\dfrac{113.02}{1000}$

76. $\dfrac{520.77}{1000}$

---

77. Compute the quotient 52/83, and round your answer to the nearest tenth.

78. Compute the quotient 43/82, and round your answer to the nearest tenth.

79. Compute the quotient 51/59, and round your answer to the nearest tenth.

80. Compute the quotient 17/69, and round your answer to the nearest tenth.

81. Compute the quotient 5/74, and round your answer to the nearest hundredth.

82. Compute the quotient 3/41, and round your answer to the nearest hundredth.

83. Compute the quotient 5/94, and round your answer to the nearest hundredth.

84. Compute the quotient 3/75, and round your answer to the nearest hundredth.

85. Compute the quotient 7/72, and round your answer to the nearest hundredth.

86. Compute the quotient 4/57, and round your answer to the nearest hundredth.

87. Compute the quotient 16/86, and round your answer to the nearest tenth.

88. Compute the quotient 21/38, and round your answer to the nearest tenth.

In Exercises 89-100, simplify the given expression.

89. $\dfrac{7.5 \cdot 7.1 - 19.5}{0.54}$

90. $\dfrac{1.5(-8.8) - (-18.6)}{1.8}$

91. $\dfrac{17.76 - (-11.7)}{0.5^2}$

92. $\dfrac{-14.8 - 2.1}{2.6^2}$

93. $\dfrac{-18.22 - 6.7}{14.75 - 7.75}$

94. $\dfrac{1.4 - 13.25}{-6.84 - (-2.1)}$

95. $\dfrac{-12.9 - (-10.98)}{0.5^2}$

96. $\dfrac{5.1 - (-16.5)}{(-1.5)^2}$

97. $\dfrac{-9.5 \cdot 1.6 - 3.7}{-3.6}$

98. $\dfrac{6.5(-1.6) - 3.35}{-2.75}$

99. $\dfrac{-14.98 - 9.6}{17.99 - 19.99}$

100. $\dfrac{-5.6 - 7.5}{-5.05 - 1.5}$

101. Given $a = -2.21$, $c = 3.3$, and $d = 0.5$, evaluate and simplify the following expression.

$$\frac{a-c}{d^2}$$

102. Given $a = 2.8$, $c = -14.68$, and $d = 0.5$, evaluate and simplify the following expression.

$$\frac{a-c}{d^2}$$

103. Given $a = -5.8$, $b = 10.37$, $c = 4.8$, and $d = 5.64$, evaluate and simplify the following expression:

$$\frac{a-b}{c-d}$$

104. Given $a = -10.79$, $b = 3.94$, $c = -3.2$, and $d = -8.11$, evaluate and simplify the following expression:

$$\frac{a-b}{c-d}$$

105. Given $a = -1.5$, $b = 4.7$, $c = 18.8$, and $d = -11.75$, evaluate and simplify the following expression.

$$\frac{ab-c}{d}$$

106. Given $a = 9.3$, $b = 6.6$, $c = 14.27$, and $d = 0.2$, evaluate and simplify the following expression.

$$\frac{ab-c}{d}$$

107. **Biodiesel plants.** There are about 180 biodiesel plants operating in about 40 states. Of the states that have them, what is the average number of biodiesel plants per state? *Associated Press-Times-Standard 01/02/10 Fledgling biofuel industry ends year on a dour note.*

108. **Bat fungus.** A fungus called ”white-nose syndrome” has killed an estimated 500,000 bats throughout the country. This means about 2,400,000 pounds of bugs aren't eaten over the year, says Forest Service biologist Becky Ewing. How many pounds of insects does an average bat eat annually? *Associated Press-Times-Standard 5/2/09*

109. **Patent backlog.** In the U.S. Patent and Trademark Office, 6000 examiners have a backlog of 770,000 new, unexamined applications for patents. How many applications is that for each examiner to catch up on? Round your answer to the nearest tenth. *Associated Press-Times-Standard 5/5/09*

110. **Doing well.** The large health insurer Wellpoint, Inc., owner of Anthem Blue Cross, earned $536 million in the last three months of 2009. What was the average earnings per month for the insurer over that period? Round to the nearest million. *Associated Press-Times-Standard 02/09/10 HHS secretary asks Anthem Blue Cross to justify rate hike.*

111. **Cyber attacks.** The Pentagon has spent $100 million over a six-month period responding to and repairing damage from cyber-attacks and other computer network problems. What's the average amount of money spent per month over that time? Round your answer to the nearest hundredth of a million. *Associated Press-Times-Standard 4/19/09*

112. Daily milk. The average California cow can produce 2,305 gallons of milk annually. How much milk can a cow produce each day? Round your answer to the nearest hundredth of a gallon. *http://www.moomilk.com/faq.htm*

113. Media mail. To promote her business, Theresa mails several packages via Media Mail. One package weighing 2 lbs. costs $2.77, another package weighing 3 lbs. costs $3.16, and the third package weighing 5 lbs. costs $3.94 to mail. What was the average cost per pound to mail the packages? Round your result to the nearest penny. *http://www.usps.com/prices/media-mail-prices.htm*

Answers

1. 0.75

3. 9.1

5. 4.5

7. 0.44

9. 0.53

11. 1.5

13. 0.68

15. 4.5

17. 0.74

19. 0.77

21. 8.9

23. 2.7

25. 5.8

27. 9.2

29. 0.93

31. 0.98

33. 2.4

35. 6.8

37. 0.73

39. 0.38

41. -6.9

43. -0.54

45. -0.48

47. -0.36

49. -3.3

51. 7.6

53. 2.4

55. 0.19

57. -0.14

59. 0.57

61. -8.4

63. 2.2

65. 5.2435

67. 0.56394

69. 1.1681

71. 69.455

73. 0.34116

75. 0.11302

77. 0.6

79. 0.9

81. 0.07

83. 0.05

85. 0.10

87. 0.2

89. 62.5

91. 117.84

93. −3.56

95. −7.68

97. 5.25

99. 12.29

101. −22.04

103. 19.25

105. 2.2

107. 4.5 biodiesel plants

109. 128.3

111. \$16.67 million

113. \$0.99 per pound

5.5 Fractions and Decimals

When converting a fraction to a decimal, only one of two things can happen. Either the process will terminate or the decimal representation will begin to repeat a pattern of digits. In each case, the procedure for changing a fraction to a decimal is the same.

Changing a Fraction to a Decimal. To change a fraction to a decimal, divide the numerator by the denominator. *Hint: If you first reduce the fraction to lowest terms, the numbers will be smaller and the division will be a bit easier as a result.*

Terminating Decimals

Terminating Decimals. First reduce the fraction to lowest terms. If the denominator of the resulting fraction has a prime factorization consisting of strictly twos and/or fives, then the decimal representation will "terminate."

You Try It!

EXAMPLE 1. Change 15/48 to a decimal.

Change 10/16 to a decimal.

Solution. First, reduce the fraction to lowest terms.

$$\begin{aligned}\frac{15}{48} &= \frac{3\cdot 5}{3\cdot 16}\\ &= \frac{5}{16}\end{aligned}$$

Next, note that the denominator of 5/16 has prime factorization $16 = 2\cdot 2\cdot 2\cdot 2$. It consists only of twos. Hence, the decimal representation of 5/16 should terminate.

$$\begin{array}{r} 0.3125\\ 16\overline{)5.0000}\\ \underline{4\,8}\\ 20\\ \underline{16}\\ 40\\ \underline{32}\\ 80\\ \underline{80}\\ 0 \end{array}$$

The zero remainder terminates the process. Hence, $5/16 = 0.3125$.

Answer: 0.625

You Try It!

Change $7\frac{11}{20}$ to a decimal.

EXAMPLE 2. Change $3\,\frac{7}{20}$ to a decimal.

Solution. Note that $7/20$ is reduced to lowest terms and its denominator has prime factorization $20 = 2 \cdot 2 \cdot 5$. It consists only of twos and fives. Hence, the decimal representation of $7/20$ should terminate.

$$\begin{array}{r} 0.35 \\ 20\overline{)7.00} \\ \underline{6\ 0} \\ 1\ 00 \\ \underline{1\ 00} \\ 0 \end{array}$$

The zero remainder terminates the process. Hence, $7/20 = 0.35$. Therefore, $3\,\frac{7}{20} = 3.35$.

Answer: 7.55

□

Repeating Decimals

Repeating Decimals. First reduce the fraction to lowest terms. If the prime factorization of the resulting denominator does not consist strictly of twos and fives, then the division process will never have a remainder of zero. However, repeated patterns of digits must eventually reveal themselves.

You Try It!

Change 5/12 to a decimal.

EXAMPLE 3. Change 1/12 to a decimal.

Solution. Note that $1/12$ is reduced to lowest terms and the denominator has a prime factorization $12 = 2 \cdot 2 \cdot 3$ that does **not** consist strictly of twos and fives. Hence, the decimal representation of $1/12$ will not "terminate." We need to carry out the division until a remainder reappears for a second time. This will indicate repetition is beginning.

$$\begin{array}{r} .083 \\ 12\overline{)1.000} \\ \underline{96} \\ 40 \\ \underline{36} \\ 4 \end{array}$$

Note the second appearance of 4 as a remainder in the division above. This is an indication that repetition is beginning. However, to be sure, let's carry the division out for a couple more places.

```
      .08333
12)1.00000
    96
     40
     36
      40
      36
       40
       36
        4
```

Note how the remainder 4 repeats over and over. In the quotient, note how the digit 3 repeats over and over. It is pretty evident that if we were to carry out the division a few more places, we would get

$$\frac{1}{12} = 0.833333\ldots$$

The ellipsis is a symbolic way of saying that the threes will repeat forever. It is the mathematical equivalent of the word "etcetera."

Answer: $0.41666\ldots$

□

There is an alternative notation to the ellipsis, namely

$$\frac{1}{12} = 0.08\overline{3}.$$

The bar over the 3 (called a "repeating bar") indicates that the 3 will repeat indefinitely. That is,

$$0.08\overline{3} = 0.083333\ldots.$$

Using the Repeating Bar. To use the repeating bar notation, take whatever block of digits are under the repeating bar and duplicate that block of digits infinitely to the right.

Thus, for example:

- $5.3\overline{45} = 5.3454545\ldots.$
- $0.\overline{142857} = 0.142857142857142857\ldots.$

Important Observation. Although $0.8\overline{33}$ will also produce $0.8333333\ldots$, as a rule we should use as few digits as possible under the repeating bar. Thus, $0.8\overline{3}$ is preferred over $0.8\overline{33}$.

You Try It!

Change 5/33 to a decimal.

EXAMPLE 4. Change 23/111 to a decimal.

Solution. The denominator of 23/111 has prime factorization $111 = 3 \cdot 37$ and does **not** consist strictly of twos and fives. Hence, the decimal representation will not "terminate." We need to perform the division until we spot a repeated remainder.

$$\begin{array}{r} 0.207 \\ 111\overline{)23.000} \\ \underline{22\,2} \\ 800 \\ \underline{777} \\ 23 \end{array}$$

Note the return of 23 as a remainder. Thus, the digit pattern in the quotient should start anew, but let's add a few places more to our division to be sure.

$$\begin{array}{r} 0.207207 \\ 111\overline{)23.000000} \\ \underline{22\,2} \\ 800 \\ \underline{777} \\ 230 \\ \underline{222} \\ 800 \\ \underline{777} \\ 23 \end{array}$$

Aha! Again a remainder of 23. Repetition! At this point, we are confident that

$$\frac{23}{111} = 0.207207\ldots.$$

Using a "repeating bar," this result can be written

$$\frac{23}{111} = 0.\overline{207}.$$

Answer: $0.151515\ldots$

□

Expressions Containing Both Decimals and Fractions

At this point we can convert fractions to decimals, and vice-versa, we can convert decimals to fractions. Therefore, we should be able to evaluate expressions that contain a mix of fraction and decimal numbers.

You Try It!

Simplify: $-\dfrac{7}{8} - 6.5$

EXAMPLE 5. Simplify: $-\dfrac{3}{8} - 1.25$.

Solution. Let's change 1.25 to an improper fraction.

$$\begin{aligned} 1.25 &= \frac{125}{100} && \text{Two decimal places} \implies \text{two zeros.} \\ &= \frac{5}{4} && \text{Reduce to lowest terms.} \end{aligned}$$

In the original problem, replace 1.25 with 5/4, make equivalent fractions with a common denominator, then subtract.

$$\begin{aligned} -\frac{3}{8} - 1.25 &= -\frac{3}{8} - \frac{5}{4} && \text{Replace 1.25 with 5/4.} \\ &= -\frac{3}{8} - \frac{5 \cdot 2}{4 \cdot 2} && \text{Equivalent fractions, LCD = 8.} \\ &= -\frac{3}{8} - \frac{10}{8} && \text{Simplify numerator and denominator.} \\ &= -\frac{3}{8} + \left(-\frac{10}{8}\right) && \text{Add the opposite.} \\ &= -\frac{13}{8} && \text{Add.} \end{aligned}$$

Thus, $-3/8 - 1.25 = -13/8$.

Alternate Solution. Because $-3/8$ is reduced to lowest terms and $8 = 2 \cdot 2 \cdot 2$ consists only of twos, the decimal representation of $-3/8$ will terminate.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ \underline{2\,4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \end{array}$$

Hence, $-3/8 = -0.375$. Now, replace $-3/8$ in the original problem with -0.375, then simplify.

$$\begin{aligned} -\frac{3}{8} - 1.25 &= -0.375 - 1.25 && \text{Replace } -3/8 \text{ with } -0.375. \\ &= -0.375 + (-1.25) && \text{Add the opposite.} \\ &= -1.625 && \text{Add.} \end{aligned}$$

Thus, $-3/8 - 1.25 = -1.625$.

Are They the Same? The first method produced $-13/8$ as an answer; the second method produced -1.625. Are these the same results? One way to find out is to change -1.625 to an improper fraction.

$$\begin{aligned} -1.625 &= -\frac{1625}{1000} && \text{Three places} \implies \text{three zeros.} \\ &= -\frac{5\cdot5\cdot5\cdot5\cdot13}{2\cdot2\cdot2\cdot5\cdot5\cdot5} && \text{Prime factor.} \\ &= -\frac{13}{2\cdot2\cdot2} && \text{Cancel common factors.} \\ &= -\frac{13}{8} && \text{Simplify.} \end{aligned}$$

Answer: $-7\frac{3}{8}$ or -7.375

Thus, the two answers are the same.

□

Let's look at another example.

You Try It!

Simplify: $-\frac{4}{9} + 0.25$

EXAMPLE 6. Simplify: $-\frac{2}{3} + 0.35$.

Solution. Let's attack this expression by first changing 0.35 to a fraction.

$$\begin{aligned} -\frac{2}{3} + 0.35 &= -\frac{2}{3} + \frac{35}{100} && \text{Change 0.35 to a fraction.} \\ &= -\frac{2}{3} + \frac{7}{20} && \text{Reduce 35/100 to lowest terms.} \end{aligned}$$

Find an LCD, make equivalent fractions, then add.

$$\begin{aligned} &= -\frac{2\cdot20}{3\cdot20} + \frac{7\cdot3}{20\cdot3} && \text{Equivalent fractions with LCD = 60.} \\ &= -\frac{40}{60} + \frac{21}{60} && \text{Simplify numerators and denominators.} \\ &= -\frac{19}{60} && \text{Add.} \end{aligned}$$

Answer: $-7/36$

Thus, $-\frac{2}{3} + 0.35 = -\frac{19}{60}$.

□

In Example 6, we run into trouble if we try to change $-2/3$ to a decimal. The decimal representation for $-2/3$ is a repeating decimal (the denominator is not made up of only twos and fives). Indeed, $-2/3 = -0.\overline{6}$. To add $-0.\overline{6}$ and 0.35, we have to align the decimal points, then begin adding at the right end. But $-0.\overline{6}$ has no right end! This observation leads to the following piece of advice.

Important Observation. When presented with a problem containing both decimals and fractions, if the decimal representation of any fraction repeats, its best to first change all numbers to fractions, then simplify.

❧ ❧ ❧ Exercises ☙ ☙ ☙

In Exercises 1-20, convert the given fraction to a terminating decimal.

1. $\frac{59}{16}$

2. $\frac{19}{5}$

3. $\frac{35}{4}$

4. $\frac{21}{4}$

5. $\frac{1}{16}$

6. $\frac{14}{5}$

7. $\frac{6}{8}$

8. $\frac{7}{175}$

9. $\frac{3}{2}$

10. $\frac{15}{16}$

11. $\frac{119}{175}$

12. $\frac{4}{8}$

13. $\frac{9}{8}$

14. $\frac{5}{2}$

15. $\frac{78}{240}$

16. $\frac{150}{96}$

17. $\frac{25}{10}$

18. $\frac{2}{4}$

19. $\frac{9}{24}$

20. $\frac{216}{150}$

In Exercises 21-44, convert the given fraction to a repeating decimal. Use the "repeating bar" notation.

21. $\frac{256}{180}$

22. $\frac{268}{180}$

23. $\frac{364}{12}$

24. $\frac{292}{36}$

25. $\frac{81}{110}$

26. $\frac{82}{99}$

27. $\frac{76}{15}$

28. $\frac{23}{9}$

29. $\frac{50}{99}$

30. $\frac{53}{99}$

31. $\frac{61}{15}$

32. $\frac{37}{18}$

33. $\dfrac{98}{66}$

34. $\dfrac{305}{330}$

35. $\dfrac{190}{495}$

36. $\dfrac{102}{396}$

37. $\dfrac{13}{15}$

38. $\dfrac{65}{36}$

39. $\dfrac{532}{21}$

40. $\dfrac{44}{60}$

41. $\dfrac{26}{198}$

42. $\dfrac{686}{231}$

43. $\dfrac{47}{66}$

44. $\dfrac{41}{198}$

In Exercises 45-52, simplify the given expression by first converting the fraction into a terminating decimal.

45. $\dfrac{7}{4} - 7.4$

46. $\dfrac{3}{2} - 2.73$

47. $\dfrac{7}{5} + 5.31$

48. $-\dfrac{7}{4} + 3.3$

49. $\dfrac{9}{10} - 8.61$

50. $\dfrac{3}{4} + 3.7$

51. $\dfrac{6}{5} - 7.65$

52. $-\dfrac{3}{10} + 8.1$

In Exercises 53-60, simplify the given expression by first converting the decimal into a fraction.

53. $\dfrac{7}{6} - 2.9$

54. $-\dfrac{11}{6} + 1.12$

55. $-\dfrac{4}{3} - 0.32$

56. $\dfrac{11}{6} - 0.375$

57. $-\dfrac{2}{3} + 0.9$

58. $\dfrac{2}{3} - 0.1$

59. $\dfrac{4}{3} - 2.6$

60. $-\dfrac{5}{6} + 2.3$

In Exercises 61-64, simplify the given expression.

61. $\frac{5}{6} + 2.375$

62. $\frac{5}{3} + 0.55$

63. $\frac{11}{8} + 8.2$

64. $\frac{13}{8} + 8.4$

65. $-\frac{7}{10} + 1.2$

66. $-\frac{7}{5} - 3.34$

67. $-\frac{11}{6} + 0.375$

68. $\frac{5}{3} - 1.1$

❧ ❧ ❧ Answers ☙ ☙ ☙

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1. 3.6875

3. 8.75

5. 0.0625

7. 0.75

9. 1.5

11. 0.68

13. 1.125

15. 0.325

17. 2.5

19. 0.375

21. $1.4\overline{2}$

23. $30.\overline{3}$

25. $0.7\overline{36}$

27. $5.0\overline{6}$

29. $0.\overline{50}$

31. $4.0\overline{6}$

33. $1.\overline{48}$

35. $0.\overline{38}$

37. $0.8\overline{6}$

39. $25.\overline{3}$

41. $0.\overline{13}$

43. $0.7\overline{12}$

45. -5.65

47. 6.71

49. -7.71

51. -6.45

53. $-\frac{26}{15}$

55. $-\frac{124}{75}$

57. $\frac{7}{30}$

59. $-\frac{19}{15}$

61. $\frac{77}{24}$

63. 9.575

65. 0.5

67. $-\frac{35}{24}$

5.6 Equations With Decimals

We can add or subtract the same decimal number from both sides of an equation without affecting the solution.

You Try It!

EXAMPLE 1. Solve for x: $x - 1.35 = -2.6$.

Solution. To undo subtracting 1.35, add 1.35 to both sides of the equation.

$$\begin{aligned} x - 1.35 &= -2.6 && \text{Original equation.} \\ x - 1.35 + 1.35 &= -2.6 + 1.35 && \text{Add 1.35 to both sides.} \\ x &= -1.25 && \text{Simplify: } -2.6 + 1.35 = -1.25. \end{aligned}$$

Solve for x:

$$x + 1.25 = 0.6$$

Answer: -0.65

We can still multiply both sides of an equation by the same decimal number without affecting the solution.

You Try It!

EXAMPLE 2. Solve for x: $\dfrac{x}{-0.35} = 4.2$.

Solution. To undo dividing by -0.35, multiply both sides of the equation by -0.35.

$$\begin{aligned} \frac{x}{-0.35} &= 4.2 && \text{Original equation.} \\ -0.35\left(\frac{x}{-0.35}\right) &= -0.35\,(4.2) && \text{Multiply both sides by } -0.35. \\ x &= -1.470 && \text{Simplify: } -0.35(4.2) = -1.470. \end{aligned}$$

Solve for y:

$$\frac{y}{0.37} = -1.52$$

Answer: -0.5624

We can still divide both sides of an equation by the same decimal number without affecting the solution.

You Try It!

EXAMPLE 3. Solve for x: $-1.2x = -4.08$.

Solution. To undo multiplying by -1.2, divide both sides of the equation by -1.2.

$$\begin{aligned} -1.2x &= -4.08 && \text{Original equation.} \\ \frac{-1.2x}{-1.2} &= \frac{-4.08}{-1.2} && \text{Divide both sides by } -1.2. \\ x &= 3.4 && \text{Simplify: } -4.08/(-1.2) = 3.4. \end{aligned}$$

Solve for z:

$$-2.5z = 1.4$$

Answer: -0.56

Combining Operations

We sometimes need to combine operations.

You Try It!

Solve for u:

$$-0.02u - 3.2 = -1.75$$

EXAMPLE 4. Solve for x: $-3.8x - 1.7 = -17.28$.

Solution. To undo subtracting 1.7, add 1.7 to both sides of the equation.

$$\begin{aligned} -3.8x - 1.7 &= -17.28 && \text{Original equation.} \\ -3.8x - 1.7 + 1.7 &= -17.28 + 1.7 && \text{Add 1.7 to both sides} \\ -3.8x &= -15.58 && \text{Simplify: } -17.28 + 1.7 = -15.58. \end{aligned}$$

Next, to undo multiplying by -3.8, divide both sides of the equation by -3.8.

$$\begin{aligned} \frac{-3.8x}{-3.8} &= \frac{-15.58}{-3.8} && \text{Divide both sides by } -3.8. \\ x &= 4.1 && \text{Simplify: } -15.58/(-3.8) = 4.1. \end{aligned}$$

Answer: -72.5

□

Combining Like Terms

Combining like terms with decimal coefficients is done in the same manner as combining like terms with integer coefficients.

You Try It!

Simplify:

$$-1.185t + 3.2t$$

EXAMPLE 5. Simplify the expression: $-3.2x + 1.16x$.

Solution. To combine these like terms we must add the coefficients.

To add coefficients with unlike signs, first subtract the coefficient with the smaller magnitude from the coefficient with the larger magnitude.

$$\begin{array}{r} 3.20 \\ -\ 1.16 \\ \hline 2.04 \end{array}$$

Prefix the sign of the decimal number having the larger magnitude. Hence:

$$-3.2 + 1.16 = -2.04.$$

We can now combine like terms as follows:

$$-3.2x + 1.16x = -2.04x$$

Answer: $2.015t$

□

When solving equations, we sometimes need to combine like terms.

You Try It!

EXAMPLE 6. Solve the equation for x: $4.2 - 3.1x + 2x = -7.02$.

Solve for r:

$$-4.2 + 3.6r - 4.1r = 1.86$$

Solution. Combine like terms on the left-hand side of the equation.

$$\begin{aligned}
4.2 - 3.1x + 2x &= -7.02 && \text{Original equation.}\\
4.2 - 1.1x &= -7.02 && \text{Combine like terms: } -3.1x + 2x = -1.1x.\\
4.2 - 1.1x - 4.2 &= -7.02 - 4.2 && \text{Subtract 4.2 from both sides.}\\
-1.1x &= -11.02 && \text{Subtract: } -7.02 - 4.2 = -11.22.\\
\frac{-1.1x}{-1.1} &= \frac{-11.22}{-1.1} && \text{Divide both sides by } -1.1.\\
x &= 10.2 && \text{Divide: } -11.22/(-1.1) = 10.2.
\end{aligned}$$

Thus, the solution of the equation is 10.2.

Check. Like all equations, we can check our solution by substituting our answer in the original equation.

$$\begin{aligned}
4.2 - 3.1x + 2x &= -7.02 && \text{Original equation.}\\
4.2 - 3.1(10.2) + 2(10.2) &= -7.02 && \text{Substitute 10.2 for } x.\\
4.2 - 31.62 + 20.4 &= -7.02 && \text{Multiply: } 3.1(10.2) = 31.62,\ 2(10.2) = 20.4.\\
-27.42 + 20.4 &= -7.02 && \text{Order of Ops: Add, left to right.}\\
& && 4.2 - 31.62 = -27.42.\\
-7.02 &= -7.02 && \text{Add: } -27.42 + 20.4 = -7.02.
\end{aligned}$$

Because the last line is a true statement, the solution $x = 10.2$ checks.

Answer: -12.12

□

Using the Distributive Property

Sometimes we will need to employ the distributive property when solving equations.

Distributive Property. Let a, b, and c be any numbers. Then,

$$a(b + c) = ab + ac.$$

You Try It!

Solve for x:

$$-2.5x - 0.1(x - 2.3) = 8.03$$

EXAMPLE 7. Solve the equation for x: $-6.3x - 0.4(x - 1.2) = -0.86$.

Solution. We first distribute the -0.4 times each term in the parentheses, then combine like terms.

$$\begin{aligned} -6.3x - 0.4(x - 1.2) &= -0.86 && \text{Original equation.} \\ -6.3x - 0.4x + 0.48 &= -0.86 && \text{Distribute. Note that } -0.4(-1.2) = 0.48. \\ -6.7x + 0.48 &= -0.86 && \text{Combine like terms.} \end{aligned}$$

Next, subtract 0.48 from both sides, then divide both sides of the resulting equation by -6.7.

$$\begin{aligned} -6.7x + 0.48 - 0.48 &= -0.86 - 0.48 && \text{Subtract 0.48 from both sides.} \\ -6.7x &= -1.34 && \text{Simplify: } -0.86 - 0.48 = -1.34. \\ \frac{-6.7x}{-6.7} &= \frac{-1.34}{-6.7} && \text{Divide both sides by } -6.7. \\ x &= 0.2 && \text{Simplify: } -1.34/(-6.7) = 0.2. \end{aligned}$$

Answer: -3

□

Rounding Solutions

Sometimes an approximate solution is adequate.

You Try It!

Solve for x:

$$4.2x - 1.25 = 3.4 + 0.71x$$

EXAMPLE 8. Solve the equation $3.1x + 4.6 = 2.5 - 2.2x$ for x. Round the answer to the nearest tenth.

Solution. We need to isolate the terms containing x on one side of the equation. We begin by adding $2.2x$ to both sides of the equation.

$$\begin{aligned} 3.1x + 4.6 &= 2.5 - 2.2x && \text{Original equation.} \\ 3.1x + 4.6 + 2.2x &= 2.5 - 2.2x + 2.2x && \text{Add } 2.2x \text{ to both sides.} \\ 5.3x + 4.6 &= 2.5 && \text{Combine terms: } 3.1x + 2.2x = 5.3x. \end{aligned}$$

To undo adding 4.6, subtract 4.6 from both sides of the equation.

$$\begin{aligned} 5.3x + 4.6 - 4.6 &= 2.5 - 4.6 && \text{Subtract 4.6 from both sides.} \\ 5.3x &= -2.1 && \text{Simplify: } 2.5 - 4.6 = -2.1. \end{aligned}$$

To undo the effect of multiplying by 5.3, divide both sides of the equation by 5.3.

$$\frac{5.3x}{5.3} = \frac{-2.1}{5.3} \qquad \text{Divide both sides by 5.3.}$$

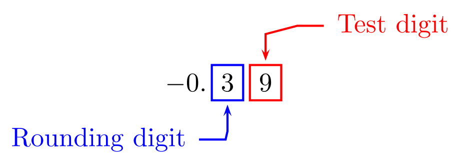
$$x \approx -0.4 \qquad \text{Round solution to nearest tenth.}$$

To round the answer to the nearest tenth, we must carry the division out one additional place.

$$\begin{array}{r} 0.39 \\ 53\overline{)21.00} \\ \underline{15\ 9} \\ 5\ 10 \\ \underline{4\ 77} \\ 33 \end{array}$$

Thus, $-2.1/5.3 \approx -0.39$.

Because the "test digit" is greater than or equal to 5, add 1 to the rounding digit and truncate.



Thus, $-0.39 \approx -0.4$.

Answer: 1.33

Applications

Let's look at some applications that involve equations containing decimals. For convenience, we repeat the *Requirements for Word Problem Solutions.*

Requirements for Word Problem Solutions.

1. **Set up a Variable Dictionary.** You must let your readers know what each variable in your problem represents. This can be accomplished in a number of ways:
   - Statements such as "Let P represent the perimeter of the rectangle."
   - Labeling unknown values with variables in a table.
   - Labeling unknown quantities in a sketch or diagram.
2. **Set up an Equation.** Every solution to a word problem must include a carefully crafted equation that accurately describes the constraints in the problem statement.
3. **Solve the Equation.** You must always solve the equation set up in the previous step.

4. **Answer the Question.** This step is easily overlooked. For example, the problem might ask for Jane's age, but your equation's solution gives the age of Jane's sister Liz. Make sure you answer the original question asked in the problem. Your solution should be written in a sentence with appropriate units.

5. **Look Back.** It is important to note that this step does not imply that you should simply check your solution in your equation. After all, it's possible that your equation incorrectly models the problem's situation, so you could have a valid solution to an incorrect equation. The important question is: "Does your answer make sense based on the words in the original problem statement."

Let's start with a rectangular garden problem.

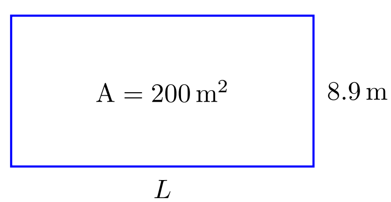
You Try It!

Eta's dog run is in the shape of a rectangle with area 500 square feet. If the length of the run is 28 feet, find the width of the run, correct to the nearest tenth of a foot.

EXAMPLE 9. Molly needs to create a rectangular garden plot covering 200 square meters (200 m^2). If the width of the plot is 8.9 meters, find the length of the plot correct to the nearest tenth of a meter.

Solution. We will follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We will use a sketch to define our variables.



   Note that L represents the length of the rectangle.

2. *Set Up an Equation.* The area A of a rectangle is given by the formula

$$A = LW,$$

   where L and W represent the length and width of the rectangle, respectively. Substitute 200 for A and 8.9 for W in the formula to obtain

$$200 = L(8.9),$$

   or equivalently,

$$200 = 8.9L.$$

3. *Solve the Equation.* Divide both sides of the last equation by 8.9, then round your answer to the nearest tenth.

$$\frac{200}{8.9} = \frac{8.9L}{8.9} \qquad \text{Divide both sides by 8.9.}$$

$$22.5 \approx L \qquad \text{Round to nearest tenth.}$$

To round the answer to the nearest tenth, we must carry the division out one additional place.

```
        22.47
89)2000.00
   178
    220
    178
     42 0
     35 6
      6 40
      6 23
         0
```

Because the "test digit" is greater than or equal to 5, add 1 to the rounding digit and truncate.

Test digit

22. [4] [7]

Rounding digit

Thus, $200/8.9 \approx 22.5$.

4. *Answer the Question.* To the nearest tenth of a meter, the length of the rectangular plot is $L \approx 22.5$ meters.

5. *Look Back.* We have $L \approx 22.5$ meters and $W = 8.9$ meters. Multiply length and width to find the area.

$$\text{Area} \approx (22.5\,\text{m})(8.9\,\text{m}) \approx 200.25\,\text{m}^2.$$

Note that this is very nearly the exact area of 200 square meters. The discrepancy is due to the fact that we found the length rounded to the nearest tenth of a meter.

Answer: 17.9 feet

□

You Try It!

EXAMPLE 10. Children's tickets to the circus go on sale for \$6.75. The Boys and Girls club of Eureka has \$1,000 set aside to purchase these tickets. Approximately how many tickets can the Girls and Boys club purchase?

Adult tickets to the circus cost \$12.25 apiece. If the club has \$1,200 set aside for adult ticket purchase, how many adult tickets can they purchase?

Solution. We will follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let N represent the number of tickets purchased by the Boys and Girls club of Eureka.

2. *Set Up an Equation.* Note that

Price per Ticket	times	Number of Tickets	is	Full Purchase Price
6.75	·	N	=	1,000

Hence, our equation is

$$6.75N = 1000.$$

3. *Solve the Equation.* Divide both sides of the equation by 6.75.

$$\frac{6.75N}{6.75} = \frac{1000}{6.75} \quad \text{Divide both sides by 6.75.}$$

$$N \approx 148 \quad \text{Truncate to nearest unit.}$$

Push the decimal point to the right-end of the divisor and the decimal point in the dividend an equal number of places.

```
6.75)1000.00
```

We'll stop the division at the units position.

```
        148
   675)100000
        675
        3250
        2700
         5500
         5400
          100
```

4. *Answer the Question.* The Boys and Girls club can purchase 148 tickets.

5. *Look Back.* Let's calculate the cost of 148 tickets at $6.75 apiece.

```
      1 48
   ×  6.75
      7 40
    103 6
    888
    999.00
```

Thus, at $6.75 apiece, 148 tickets will cost $999. Because the Boys and Girls club of Eureka has $1,000 to work with, note that the club doesn't have enough money left for another ticket.

Answer: 97

□

You Try It!

Dylan has a circular dog pen with circumference 100 feet. Find the radius of the pen, correct to the nearest tenth of a foot. Use $\pi \approx 3.14$

EXAMPLE 11. Marta has 20 feet of decorative fencing which she will use for the border of a small circular garden. Find the diameter of the circular garden, correct to the nearest hundredth of a foot. Use $\pi \approx 3.14$.

Solution. The formula governing the relation between the circumference and diameter of a circle is

$$C = \pi d.$$

The 20 feet of decorative fencing will be the circumference of the circular garden. Substitute 20 for C and 3.14 for π.

$$20 = 3.14d$$

Divide both sides of the equation by 3.14.

$$\begin{aligned} \frac{20}{3.14} &= \frac{3.14d}{3.14} \\ \frac{20}{3.14} &= d \end{aligned}$$

Move the decimal point to the end of the divisor, then move the decimal point in the dividend an equal number of places (two places) to the right. Note that we must add two trailing zeros in the dividend.

$$3.14\overline{)20.00}$$

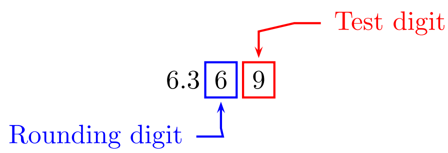
Thus, the problem becomes:

$$314\overline{)2000}$$

We need to round to the nearest hundredth. This requires that we carry the division one additional place to the right of the hundredths place (i.e., to the thousandths place).

$$\begin{array}{r} 6.369 \\ 314\overline{)2000.000} \\ \underline{1884} \\ 116\ 0 \\ \underline{94\ 2} \\ 21\ 80 \\ \underline{18\ 84} \\ 2\ 960 \\ \underline{2\ 826} \\ 134 \end{array}$$

For the final step, we must round 6.369 to the nearest hundredth. In the schematic that follows, we've boxed the hundredths digit (the "rounding digit") and the "test digit" that follows the "rounding digit."



Because the "test digit" is greater than or equal to 5, we add 1 to the "rounding digit," then truncate. Therefore, to the nearest hundredth of a foot, the diameter of the circle is approximately

$$d \approx 6.37\,\text{ft}.$$

Answer: 15.9 feet

□

Exercises

In Exercises 1-16, solve the equation.

1. $5.57x - 2.45x = 5.46$

2. $-0.3x - 6.5x = 3.4$

3. $-5.8x + 0.32 + 0.2x = -6.96$

4. $-2.2x - 0.8 - 7.8x = -3.3$

5. $-4.9x + 88.2 = 24.5$

6. $-0.2x - 32.71 = 57.61$

7. $0.35x - 63.58 = 55.14$

8. $-0.2x - 67.3 = 93.5$

9. $-10.3x + 82.4 = 0$

10. $-1.33x - 45.22 = 0$

11. $-12.5x + 13.5 = 0$

12. $44.15x - 8.83 = 0$

13. $7.3x - 8.9 - 8.34x = 2.8$

14. $0.9x + 4.5 - 0.5x = 3.5$

15. $-0.2x + 2.2x = 6.8$

16. $-7.9x + 2.9x = 8.6$

In Exercises 17-34, solve the equation.

17. $6.24x - 5.2 = 5.2x$

18. $-0.6x + 6.3 = 1.5x$

19. $-0.7x - 2.4 = -3.7x - 8.91$

20. $3.4x - 4.89 = 2.9x + 3.6$

21. $-4.9x = -5.4x + 8.4$

22. $2.5x = 4.5x + 5.8$

23. $-2.8x = -2.3x - 6.5$

24. $1.2x = 0.35x - 1.36$

25. $-2.97x - 2.6 = -3.47x + 7.47$

26. $-8.6x - 2.62 = -7.1x + 8.54$

27. $-1.7x = -0.2x - 0.6$

28. $3.89x = -5.11x + 5.4$

29. $-1.02x + 7.08 = -2.79x$

30. $1.5x - 2.4 = 0.3x$

31. $-4.75x - 6.77 = -7.45x + 3.49$

32. $-1.2x - 2.8 = -0.7x - 5.6$

33. $-4.06x - 7.38 = 4.94x$

34. $-4.22x + 7.8 = -6.3x$

In Exercises 35-52, solve the equation.

35. $2.3 + 0.1(x + 2.9) = 6.9$

36. $-6.37 + 6.3(x + 4.9) = -1.33$

37. $0.5(1.5x - 6.58) = 6.88$

38. $0.5(-2.5x - 4.7) = 16.9$

39. $-6.3x - 0.4(x - 1.8) = -16.03$

40. $-2.8x + 5.08(x - 4.84) = 19.85$

41. $2.4(0.3x + 3.2) = -11.4$

42. $-0.7(0.2x + 5.48) = 16.45$

43. $-0.8(0.3x + 0.4) = -11.3$

44. $7.5(4.4x + 7.88) = 17.19$

45. $-7.57 - 2.42(x + 5.54) = 6.95$

46. $5.9 - 0.5(x + 5.8) = 12.15$

47. $-1.7 - 5.56(x + 6.1) = 12.2$

48. $-7.93 + 0.01(x + 7.9) = 14.2$

49. $4.3x - 0.7(x + 2.1) = 8.61$

50. $1.5x - 4.5(x + 4.92) = 15.6$

51. $-4.8x + 3.3(x - 0.4) = -7.05$

52. $-1.1x + 1.3(x + 1.3) = 19.88$

In Exercises 53-58, solve the equation.

53. $0.9(6.2x - 5.9) = 3.4(3.7x + 4.3) - 1.8$

54. $0.4(-4.6x+4.7) = -1.6(-2.2x+6.9)-4.5$

55. $-1.8(-1.6x + 1.7) = -1.8(-3.6x - 4.1)$

56. $-3.3(-6.3x + 4.2) - 5.3 = 1.7(6.2x + 3.2)$

57. $0.9(0.4x + 2.5) - 2.5 = -1.9(0.8x + 3.1)$

58. $5.5(6.7x + 7.3) = -5.5(-4.2x + 2.2)$

59. Stacy runs a business out of her home making bird houses. Each month she has fixed costs of \$200. In addition, for each bird house she makes, she incurs an additional cost of \$3.00. If her total costs for the month were \$296.00, how many bird houses did she make?

60. Stella runs a business out of her home making curtains. Each month she has fixed costs of \$175. In addition, for each curtain she makes, she incurs an additional cost of \$2.75. If her total costs for the month were \$274.00, how many curtains did she make?

61. A stationary store has staplers on sale for \$1.50 apiece. A business purchases an unknown number of these and the total cost of their purchase is \$36.00. How many were purchased?

62. A stationary store has CD packs on sale for \$2.50 apiece. A business purchases an unknown number of these and the total cost of their purchase is \$40.00. How many were purchased?

63. Julie runs a business out of her home making table cloths. Each month she has fixed costs of \$100. In addition, for each table cloth she makes, she incurs an additional cost of \$2.75. If her total costs for the month were \$221.00, how many table cloths did she make?

64. Stella runs a business out of her home making quilts. Each month she has fixed costs of \$200. In addition, for each quilt she makes, she incurs an additional cost of \$1.75. If her total costs for the month were \$280.50, how many quilts did she make?

65. Marta has 60 feet of decorative fencing which she will use for the border of a small circular garden. Find the diameter of the circular garden, correct to the nearest hundredth of a foot. Use $\pi \approx 3.14$.

66. Trinity has 44 feet of decorative fencing which she will use for the border of a small circular garden. Find the diameter of the circular garden, correct to the nearest hundredth of a foot. Use $\pi \approx 3.14$.

67. Children's tickets to the ice capades go on sale for \$4.25. The YMCA of Sacramento has \$1,000 set aside to purchase these tickets. Approximately how many tickets can the YMCA of Sacramento purchase?

68. Children's tickets to the ice capades go on sale for \$5. The Knights of Columbus has \$1,200 set aside to purchase these tickets. Approximately how many tickets can the Knights of Columbus purchase?

69. A stationary store has mechanical pencils on sale for \$2.25 apiece. A business purchases an unknown number of these and the total cost of their purchase is \$65.25. How many were purchased?

70. A stationary store has engineering templates on sale for \$2.50 apiece. A business purchases an unknown number of these and the total cost of their purchase is \$60.00. How many were purchased?

71. Marta has 61 feet of decorative fencing which she will use for the border of a small circular garden. Find the diameter of the circular garden, correct to the nearest hundredth of a foot. Use $\pi \approx 3.14$.

72. Kathy has 86 feet of decorative fencing which she will use for the border of a small circular garden. Find the diameter of the circular garden, correct to the nearest hundredth of a foot. Use $\pi \approx 3.14$.

73. Kathy needs to create a rectangular garden plot covering 100 square meters (100 m^2). If the width of the plot is 7.5 meters, find the length of the plot correct to the nearest tenth of a meter.

74. Marianne needs to create a rectangular garden plot covering 223 square meters (223 m^2). If the width of the plot is 8.3 meters, find the length of the plot correct to the nearest tenth of a meter.

75. Children's tickets to the stock car races go on sale for \$4.5. The Boys and Girls club of Eureka has \$1,300 set aside to purchase these tickets. Approximately how many tickets can the Boys and Girls club of Eureka purchase?

76. Children's tickets to the movies go on sale for \$4.75. The Lions club of Alameda has \$800 set aside to purchase these tickets. Approximately how many tickets can the Lions club of Alameda purchase?

77. Ashley needs to create a rectangular garden plot covering 115 square meters (115 m^2). If the width of the plot is 6.8 meters, find the length of the plot correct to the nearest tenth of a meter.

78. Molly needs to create a rectangular garden plot covering 268 square meters (268 m^2). If the width of the plot is 6.1 meters, find the length of the plot correct to the nearest tenth of a meter.

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79. **Crude Inventory.** US commercial crude oil inventories decreased by 3.8 million barrels in the week ending June 19. If there were 353.9 million barrels the following week, what were crude oil inventories before the decline? *rttnews.com 06/24/09*

80. **Undocumented.** In 2008, California had 2.7 million undocumented residents. This is double the number in 1990. How many undocumented residents were in California in 1990? *Associated Press Times-Standard 4/15/09*

81. **Diamonds Shining.** The *index of refraction* n indicates the number of times slower that a light wave travels in a particular medium than it travels in a vacuum. A diamond has an index of refraction of 2.4. This is about one and one-quarter times greater than the index of refraction of a zircon. What is the index of refraction of a zircon? Round your result to the nearest tenth.

❧ ❧ ❧ Answers ☙ ☙ ☙

1. 1.75

3. 1.3

5. 13

7. 339.2

9. 8

11. 1.08

13. −11.25

15. 3.4

17. 5

19. −2.17

21. 16.8

23. 13

25. 20.14

27. 0.4

29. −4

31. 3.8

33. −0.82

35. 43.1

37. 13.56

39. 2.5

41. −26.5

43. 45.75

45. −11.54

47. −8.6

49. 2.8

51. 3.82

53. −2.59

55. −2.9

57. −3

59. 32

61. 24

63. 44

65. 19.11 feet

67. 235 tickets

69. 29

71. 19.43 feet

73. 13.3 meters

75. 288 tickets

77. 16.9 meters

79. 357.7 million barrels

81. 1.9

5.7 Introduction to Square Roots

Recall that

$$x^2 = x \cdot x.$$

The Square of a Number. The number x^2 is called the *square* of the number x.

Thus, for example:

- $9^2 = 9 \cdot 9 = 81$. Therefore, the number 81 is the square of the number 9.
- $(-4)^2 = (-4)(-4) = 16$. Therefore, the number 16 is the square of the number -4.

In the margin, we've placed a "List of Squares" of the whole numbers ranging from 0 through 25, inclusive.

List of Squares

x	x^2
0	0
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225
16	256
17	289
18	324
19	361
20	400
21	441
22	484
23	529
24	576
25	625

Square Roots

Once you've mastered the process of squaring a whole number, then you are ready for the inverse of the squaring process, taking the *square root* of a whole number.

- Above, we saw that $9^2 = 81$. We called the number 81 the *square* of the number 9. Conversely, we call the number 9 a *square root* of the number 81.
- Above, we saw that $(-4)^2 = 16$. We called the number 16 the *square* of the number -4. Conversely, we call the number -4 a *square root* of the number 16.

Square Root. If $a^2 = b$, then a is called a *square root* of the number b.

You Try It!

EXAMPLE 1. Find the square roots of the number 49.

Solution. To find a square root of 49, we must think of a number a such that $a^2 = 49$. Two numbers come to mind.

- $(-7)^2 = 49$. Therefore, -7 is a square root of 49.
- $7^2 = 49$. Therefore, 7 is a square root of 49.

Note that 49 has two square roots, one of which is positive and the other one is negative.

Find the square roots of 256.

Answer: $-16, 16$

□

Square Roots

x	$\sqrt{x}$
0	0
1	1
4	2
9	3
16	4
25	5
36	6
49	7
64	8
81	9
100	10
121	11
144	12
169	13
196	14
225	15
256	16
289	17
324	18
361	19
400	20
441	21
484	22
529	23
576	24
625	25

You Try It!

Find the square roots of 625.

EXAMPLE 2. Find the square roots of the number 196.

Solution. To find a square root of 196, we must think of a number a such that $a^2 = 196$. With help from the "List of Squares," two numbers come to mind.

- $(-14)^2 = 196$. Therefore, -14 is a square root of 196.
- $14^2 = 196$. Therefore, 14 is a square root of 196.

Note that 196 has two square roots, one of which is positive and the other one is negative.

Answer: -25, 25

□

You Try It!

Find the square roots of 9.

EXAMPLE 3. Find the square roots of the number 0.

Solution. To find a square root of 0, we must think of a number a such that $a^2 = 0$. There is only one such number, namely zero. Hence, 0 is the square root of 0.

Answer: -3, 3

□

You Try It!

Find the square roots of -81.

EXAMPLE 4. Find the square roots of the number -25.

Solution. To find a square root of -25, we must think of a number a such that $a^2 = -25$. This is impossible because no square of a real number (whole number, integer, fraction, or decimal) can be negative. Positive times positive is positive and negative times negative is also positive. You cannot square and get a negative answer. Therefore, -25 has no square roots[2].

Answer: There are none.

□

Radical Notation

Because $(-3)^2 = 9$ and $3^2 = 9$, both -3 and 3 are square roots of 9. Special notation, called *radical notation*, is used to request these square roots.

[2] At least not in Prealgebra. In later courses, you will be introduced to the set of complex numbers, where -25 will have two square roots

- The radical notation $\sqrt{9}$, pronounced "the nonnegative square root of 9," calls for the nonnegative[3] square root of 9. Hence,

$$\sqrt{9} = 3.$$

- The radical notation $-\sqrt{9}$, pronounced "the negative square root of 9," calls for the negative square root of 9. Hence,

$$-\sqrt{9} = -3.$$

Radical Notation. In the expression $\sqrt{9}$, the symbol $\sqrt{\ }$ is called a *radical* and the number within the radical, in this case the number 9, is called the *radicand.*

List of Squares

x	x^2
0	0
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225
16	256
17	289
18	324
19	361
20	400
21	441
22	484
23	529
24	576
25	625

For example,

- In the expression $\sqrt{529}$, the number 529 is the radicand.
- In the expression $\sqrt{a^2 + b^2}$, the expression $a^2 + b^2$ is the radicand.

Radical Notation and Square Root. If b is a positive number, then

1. $\sqrt{b}$ calls for the *nonnegative* square root of b.
2. $-\sqrt{b}$ calls for the *negative* square root of b.

Note: Nonnegative is equivalent to saying "not negative;" i.e., positive or zero.

You Try It!

Simplify:

a) $\sqrt{144}$

b) $-\sqrt{324}$

EXAMPLE 5. Simplify: (a) $\sqrt{121}$, (b) $-\sqrt{625}$, and (c) $\sqrt{0}$.

Solution.

(a) Referring to the list of squares, we note that $11^2 = 121$ and $(-11)^2 = 121$. Therefore, both 11 and -11 are square roots of 121. However, $\sqrt{121}$ calls for the nonnegative square root of 121. Thus,

$$\sqrt{121} = 11.$$

(b) Referring to the list of squares, we note that $25^2 = 625$ and $(-25)^2 = 625$. Therefore, both 25 and -25 are square roots of 625. However, $-\sqrt{625}$ calls for the negative square root of 625. Thus,

$$-\sqrt{625} = -25.$$

[3] Nonnegative is equivalent to saying "not negative;" i.e., positive or zero.

(c) There is only one square root of zero. Therefore,

$$\sqrt{0} = 0.$$

Answer: (a) 12 (b) −18

You Try It!

Simplify:

a) $-\sqrt{36}$

b) $\sqrt{-36}$

EXAMPLE 6. Simplify: (a) $-\sqrt{25}$, and (b) $\sqrt{-25}$.

Solution.

(a) Because $5^2 = 25$ and $(-5)^2 = 25$, both 5 and −5 are square roots of 25. However, the notation $-\sqrt{25}$ calls for the negative square root of 25. Thus,

$$-\sqrt{25} = -5.$$

(b) It is not possible to square a real number (whole number, integer, fraction, or decimal) and get −25. Therefore, there is no real square root of −25. That is,

$$\sqrt{-25}$$

is not a real number. It is undefined[4].

Answer: (a) −6 (b) undefined

Square Roots

x	$\sqrt{x}$
0	0
1	1
4	2
9	3
16	4
25	5
36	6
49	7
64	8
81	9
100	10
121	11
144	12
169	13
196	14
225	15
256	16
289	17
324	18
361	19
400	20
441	21
484	22
529	23
576	24
625	25

Order of Operations

With the addition of radical notation, the *Rules Guiding Order of Operations* change slightly.

Rules Guiding Order of Operations. When evaluating expressions, proceed in the following order.

1. Evaluate expressions contained in grouping symbols first. If grouping symbols are nested, evaluate the expression in the innermost pair of grouping symbols first.
2. Evaluate all exponents and radicals that appear in the expression.
3. Perform all multiplications and divisions in the order that they appear in the expression, moving left to right.
4. Perform all additions and subtractions in the order that they appear in the expression, moving left to right.

The only change in the rules is in item #2, which says: "Evaluate all exponents and radicals that appear in the expression," putting radicals on the same level as exponents.

[4] At least in Prealgebra. In later courses you will be introduced to the set of complex numbers, where $\sqrt{-25}$ will take on a new meaning.

You Try It!

EXAMPLE 7. Simplify: $-3\sqrt{9} + 12\sqrt{4}$.

Solution. According to the *Rules Guiding Order of Operations*, we must evaluate the radicals in this expression first.

$$\begin{aligned} -3\sqrt{9} + 12\sqrt{4} &= -3(3) + 12(2) && \text{Evaluate radicals first: } \sqrt{9} = 3 \text{ and } \sqrt{4} = 2. \\ &= -9 + 24 && \text{Multiply: } -3(3) = -9 \text{ and } 12(2) = 24. \\ &= 15 && \text{Add: } -9 + 24 = 15. \end{aligned}$$

□

Simplify:

$2\sqrt{4} - 3\sqrt{9}$

Answer: -5

List of Squares

x	x^2
0	0
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225
16	256
17	289
18	324
19	361
20	400
21	441
22	484
23	529
24	576
25	625

You Try It!

EXAMPLE 8. Simplify: $-2 - 3\sqrt{36}$.

Solution. According to the *Rules Guiding Order of Operations*, we must evaluate the radicals in this expression first, moving left to right.

$$\begin{aligned} -2 - 3\sqrt{36} &= -2 - 3(6) && \text{Evaluate radicals first: } \sqrt{36} = 6 \\ &= -2 - 18 && \text{Multiply: } 3(6) = 18. \\ &= -20 && \text{Subtract: } -2 - 18 = -2 + (-18) = -20. \end{aligned}$$

□

Simplify:

$5 - 8\sqrt{169}$

Answer: -99

You Try It!

EXAMPLE 9. Simplify: (a) $\sqrt{9+16}$ and (b) $\sqrt{9} + \sqrt{16}$.

Solution. Apply the *Rules Guiding Order of Operations.*

a) In this case, the radical acts like grouping symbols, so we must evaluate what is inside the radical first.

$$\begin{aligned} \sqrt{9+16} &= \sqrt{25} && \text{Add: } 9 + 16 = 25. \\ &= 5 && \text{Take nonnegative square root: } \sqrt{25} = 5. \end{aligned}$$

b) In this example, we must evaluate the square roots first.

$$\begin{aligned} \sqrt{9} + \sqrt{16} &= 3 + 4 && \text{Square root: } \sqrt{9} = 3 \text{ and } \sqrt{16} = 4. \\ &= 7 && \text{Add: } 3 + 4 = 7. \end{aligned}$$

□

Simplify:

a) $\sqrt{25+144}$

b) $\sqrt{25} + \sqrt{144}$

Answer: (a) 13 (b) 17

Fractions and Decimals

We can also find square roots of fractions and decimals.

You Try It!

Simplify:

a) $\sqrt{\frac{25}{49}}$

b) $\sqrt{0.36}$

EXAMPLE 10. Simplify: (a) $\sqrt{\dfrac{4}{9}}$, and (b) $-\sqrt{0.49}$.

Solution.

(a) Because $\left(\dfrac{2}{3}\right)^2 = \left(\dfrac{2}{3}\right)\left(\dfrac{2}{3}\right) = \dfrac{4}{9}$, then

$$\sqrt{\frac{4}{9}} = \frac{2}{3}.$$

(b) Because $(0.7)^2 = (0.7)(0.7) = 0.49$ and $(-0.7)^2 = (-0.7)(-0.7) = 0.49$, both 0.7 and -0.7 are square roots of 0.49. However, $-\sqrt{0.49}$ calls for the negative square root of 0.49. Hence,

$$-\sqrt{0.49} = -0.7.$$

Answer: (a) 5/7 (b) 0.6

Square Roots

x	$\sqrt{x}$
0	0
1	1
4	2
9	3
16	4
25	5
36	6
49	7
64	8
81	9
100	10
121	11
144	12
169	13
196	14
225	15
256	16
289	17
324	18
361	19
400	20
441	21
484	22
529	23
576	24
625	25

Estimating Square Roots

The squares in the "List of Squares" are called *perfect squares*. Each is the square of a whole number. Not all numbers are perfect squares. For example, in the case of $\sqrt{24}$, there is no whole number whose square is equal to 24. However, this does not prevent $\sqrt{24}$ from being a perfectly good number.

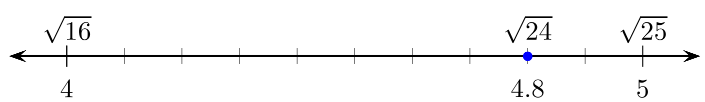
We can use the "List of Squares" to find decimal approximations when the radicand is not a perfect square.

You Try It!

Estimate: $\sqrt{83}$

EXAMPLE 11. Estimate $\sqrt{24}$ by guessing. Use a calculator to find a more accurate result and compare this result with your guess.

Solution. From the "List of Squares," note that 24 lies betwen 16 and 25, so $\sqrt{24}$ will lie between 4 and 5, with $\sqrt{24}$ much closer to 5 than it is to 4.



Let's guess

$$\sqrt{24} \approx 4.8.$$

As a check, let's square 4.8.

$$(4.8)^2 = (4.8)(4.8) = 23.04$$

Not quite 24! Clearly, $\sqrt{24}$ must be a little bit bigger than 4.8.

Let's use a scientific calculator to get a better approximation. From our calculator, using the square root button, we find

$$\sqrt{24} \approx 4.89897948557.$$

Even though this is better than our estimate of 4.8, it is still only an approximation. Our calculator was only capable of providing 11 decimal places. However, the exact decimal representation of $\sqrt{24}$ is an infinite decimal that never terminates and never establishes a pattern of repetition.

Just for fun, here is a decimal approximation of $\sqrt{24}$ that is accurate to 1000 places, courtesy of http://www.wolframalpha.com/.

4.8989794855663561963945681494117827839318949613133402568653851
3450192075491463005307971886620928046963718920245322837824971773
09196755146832515679024745571056578254950553531424952602105418235
40446962621357973381707264886705091208067617617878749171135693149
44872260828854054043234840367660016317961567602617940145738798726
16743161888016008874773750983290293078782900240894528962666325870
21889483627026570990088932343453262850995296636249008023132090729
18018687172335863967331332533818263813071727532210516312358732472
35822058934417670915102576710597966482011173804100128309322482347
06798820862115985796934679065105574720836593103436607820735600767
24633259464660565809954782094852720141025275395093777354012819859
11851434656929005776183028851492605205905926474151050068455119830
90852562596006129344159884850604575685241068135895720093193879959
87119508123342717309306912496416512553772738561882612744867017729
60314496926744648947590909762887695867274018394820295570465751182
126319692156620734019070649453

If you were to multiply this number by itself (square the number), you would get a number that is extremely close to 24, but it would not be exactly 24. There would still be a little discrepancy.

Answer: 9.1

□

Important Observation. A calculator can only produce a finite number of decimal places. If the decimal representation of your number does not terminate within this limited number of places, then the number in your calculator window is only an approximation.

- The decimal representation of 1/8 will terminate within three places, so most calculators will report the exact answer, 0.125.

- For contrast, 2/3 does not terminate. A calculator capable of reporting 11 places of accuracy produces the number 0.666666666667. However, the exact decimal representation of 2/3 is $0.\overline{6}$. Note that the calculator has rounded in the last place and only provides an approximation of 2/3. If your instructor asks for an exact answer on an exam or quiz then 0.666666666667, being an approximation, is not acceptable. You must give the exact answer 2/3.

❧ ❧ ❧ Exercises ☙ ☙ ☙

In Exercises 1-16, list all square roots of the given number. If the number has no square roots, write "none".

1. 256

2. 361

3. -289

4. -400

5. 441

6. 36

7. 324

8. 0

9. 144

10. 100

11. -144

12. -100

13. 121

14. -196

15. 529

16. 400

In Exercises 17-32, compute the exact square root. If the square root is undefined, write "undefined".

17. $\sqrt{-9}$

18. $-\sqrt{-196}$

19. $\sqrt{576}$

20. $\sqrt{289}$

21. $\sqrt{-529}$

22. $\sqrt{-256}$

23. $-\sqrt{25}$

24. $\sqrt{225}$

25. $-\sqrt{484}$

26. $-\sqrt{36}$

27. $-\sqrt{196}$

28. $-\sqrt{289}$

29. $\sqrt{441}$

30. $\sqrt{324}$

31. $-\sqrt{4}$

32. $\sqrt{100}$

In Exercises 33-52, compute the exact square root.

33. $\sqrt{0.81}$

34. $\sqrt{5.29}$

35. $\sqrt{3.61}$

36. $\sqrt{0.09}$

37. $\sqrt{\dfrac{225}{16}}$

38. $\sqrt{\dfrac{100}{81}}$

39. $\sqrt{3.24}$

40. $\sqrt{5.76}$

41. $\sqrt{\dfrac{121}{49}}$

42. $\sqrt{\dfrac{625}{324}}$

43. $\sqrt{\dfrac{529}{121}}$

44. $\sqrt{\dfrac{4}{121}}$

45. $\sqrt{2.89}$

46. $\sqrt{4.41}$

47. $\sqrt{\dfrac{144}{25}}$

48. $\sqrt{\dfrac{49}{36}}$

49. $\sqrt{\dfrac{256}{361}}$

50. $\sqrt{\dfrac{529}{16}}$

51. $\sqrt{0.49}$

52. $\sqrt{4.84}$

In Exercises 53-70, compute the exact value of the given expression.

53. $6-\sqrt{576}$

54. $-2-7\sqrt{576}$

55. $\sqrt{8^2+15^2}$

56. $\sqrt{7^2+24^2}$

57. $6\sqrt{16}-9\sqrt{49}$

58. $3\sqrt{441}+6\sqrt{484}$

59. $\sqrt{5^2+12^2}$

60. $\sqrt{15^2+20^2}$

61. $\sqrt{3^2+4^2}$

62. $\sqrt{6^2+8^2}$

63. $-2\sqrt{324}-6\sqrt{361}$

64. $-6\sqrt{576}-8\sqrt{121}$

65. $-4-3\sqrt{529}$

66. $-1+\sqrt{625}$

67. $-9\sqrt{484}+7\sqrt{81}$

68. $-\sqrt{625}-5\sqrt{576}$

69. $2-\sqrt{16}$

70. $8-6\sqrt{400}$

In Exercises 71-76, complete the following tasks to estimate the given square root.

a) Determine the two integers that the square root lies between.

b) Draw a number line, and locate the approximate location of the square root between the two integers found in part (a).

c) Without using a calculator, estimate the square root to the nearest tenth.

71. $\sqrt{58}$

72. $\sqrt{27}$

73. $\sqrt{79}$

74. $\sqrt{12}$

75. $\sqrt{44}$

76. $\sqrt{88}$

In Exercises 77-82, use a calculator to approximate the square root to the nearest tenth.

77. $\sqrt{469}$

78. $\sqrt{73}$

79. $\sqrt{615}$

80. $\sqrt{162}$

81. $\sqrt{444}$

82. $\sqrt{223}$

❧ ❧ ❧ Answers ☙ ☙ ☙

1. $16, -16$

3. none

5. $21, -21$

7. $18, -18$

9. $12, -12$

11. none

13. $11, -11$

15. $23, -23$

17. undefined

19. 24

21. undefined

23. -5

25. -22

27. -14

29. 21

31. -2

33. 0.9

35. 1.9

37. $\frac{15}{4}$

39. 1.8

41. $\frac{11}{7}$

43. $\frac{23}{11}$

45. 1.7

47. $\frac{12}{5}$

49. $\frac{16}{19}$

51. 0.7

53. -18

55. 17

57. -39

59. 13

61. 5

63. -150

65. -73

67. -135

69. -2

71. 7.6

73. 8.9

75. 6.6

77. 21.7

79. 24.8

81. 21.1

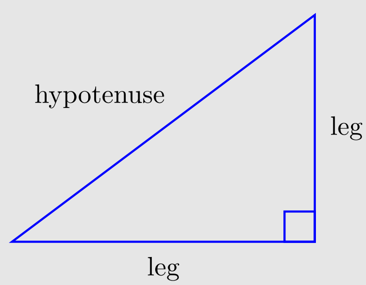
5.8 The Pythagorean Theorem

Pythagoras was a Greek mathematician and philosopher, born on the island of Samos (ca. 582 BC). He founded a number of schools, one in particular in a town in southern Italy called Crotone, whose members eventually became known as the Pythagoreans. The inner circle at the school, the *Mathematikoi*, lived at the school, rid themselves of all personal possessions, were vegetarians, and observed a strict vow of silence. They studied mathematics, philosophy, and music, and held the belief that numbers constitute the true nature of things, giving numbers a mystical or even spiritual quality.

Today, nothing is known of Pythagoras's writings, perhaps due to the secrecy and silence of the Pythagorean society. However, one of the most famous theorems in all of mathematics does bear his name, the *Pythagorean Theorem*.

Prior to revealing the contents of the Pythagorean Theorem, we pause to provide the definition of a right triangle and its constituent parts.

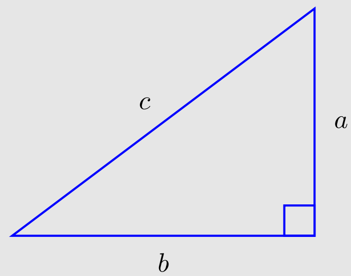
Right Triangle. A triangle with one right angle ($90°$) is called a *right triangle*. In the figure below, the right angle is marked with a little square.



The side of the triangle that is directly opposite the right angle is called the *hypotenuse*. The sides of the triangle that include the right angle are called the *legs* of the right triangle.

Now we can state one of the most ancient theorems of mathematics, the *Pythagorean Theorem*.

Pythagorean Theorem. Let c represent the length of the *hypotenuse* of a right triangle, and let a and b represent the lengths of its legs, as pictured in the image that follows.



The relationship involving the legs and hypotenuse of the right triangle, given by

$$a^2 + b^2 = c^2,$$

is called the *Pythagorean Theorem.*

Here are two important observations.

Observations Regarding the Hypotenuse. Two important facts regarding the hypotenuse of the right triangle are:

1. The hypotenuse is the longest side of the triangle and lies directly opposite the right angle.
2. In the Pythagorean equation $a^2 + b^2 = c^2$, the hypotenuse lies by itself on one side of the equation.

The Pythagorean Theorem can only be applied to right triangles.

Let's look at a simple application of the Pythagorean Theorem.

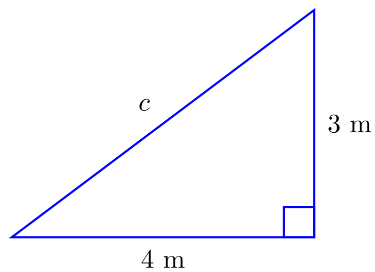
You Try It!

The legs of a right triangle measure 5 and 12 feet, respectively. Find the length of the hypotenuse.

EXAMPLE 1. The legs of a right triangle measure 3 and 4 meters, respectively. Find the length of the hypotenuse.

Solution. Let's follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let c represent the length of the hypotenuse, as pictured in the following sketch.



2. *Set up an Equation.* The Pythagorean Theorem says that

$$a^2 + b^2 = c^2.$$

In this example, the legs are known. Substitute 4 for a and 3 for b (3 for a and 4 for b works equally well) into the Pythagorean equation.

$$4^2 + 3^2 = c^2$$

3. *Solve the Equation.*

$4^2 + 3^2 = c^2$	The Pythagorean equation.
$16 + 9 = c^2$	Exponents first: $4^2 = 16$ and $3^2 = 9$.
$25 = c^2$	Add: $16 + 9 = 25$.
$5 = c$	Take the nonnegative square root.

Technically, there are two answers to $c^2 = 25$, i.e., $c = -5$ or $c = 5$. However, c represents the hypotenuse of the right triangle and must be nonnegative. Hence, we must choose $c = 5$.

4. *Answer the Question.* The hypotenuse has length 5 meters.

5. *Look Back.* Do the numbers satisfy the Pythagorean Theorem? The sum of the squares of the legs should equal the square of the hypotenuse. Let's check.

$$\begin{aligned} 4^2 + 3^2 &= 5^2 \\ 16 + 9 &= 25 \\ 25 &= 25 \end{aligned}$$

All is well!

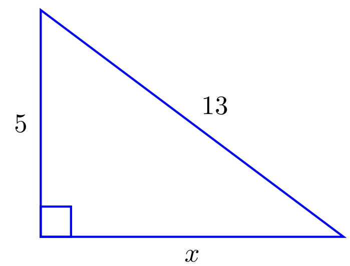
Answer: 13 feet

□

You Try It!

EXAMPLE 2. Given the following right triangle, find the length of the missing side.

The hypotenuse of a right triangle measures 25 centimeters. One leg of the right triangle measures 24 centimeters. Find the length of the remaining leg.



Solution. Note that the hypotenuse (across from the right angle) has length 13. This quantity should lie on one side of the Pythagorean equation all by itself. The sum of the squares of the legs go on the other side. Hence,

$$5^2 + x^2 = 13^2$$

Solve the equation for x.

$25 + x^2 = 169$	Exponents first: $5^2 = 25$ and $13^2 = 169$.
$25 + x^2 - 25 = 169 - 25$	Subtract 25 from both sides.
$x^2 = 144$	Simplify both sides.
$x = 12$	Take the nonnegative square root of 144.

Answer: 7 centimeters

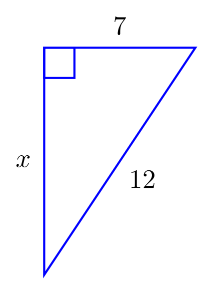
□

Perfect squares are nice, but not required.

You Try It!

The hypotenuse and one leg of a right triangle measure 9 and 7 inches, respectively. Find the length of the remaining leg.

EXAMPLE 3. Given the following right triangle, find the **exact** length of the missing side.



Solution. Note that the hypotenuse (across from the right angle) has length 12. This quantity should lie on one side of the Pythagorean equation all by itself. The sum of the squares of the legs go on the other side. Hence,

$$x^2 + 7^2 = 12^2$$

Solve the equation for x.

$$x^2 + 49 = 144 \quad \text{Exponents first: } 7^2 = 49 \text{ and } 12^2 = 144.$$
$$x^2 + 49 - 49 = 144 - 49 \quad \text{Subtract 49 from both sides.}$$
$$x^2 = 95 \quad \text{Simplify both sides.}$$
$$x = \sqrt{95} \quad \text{Take the nonnegative square root of 95.}$$

Hence, the **exact** length of the missing side is $\sqrt{95}$.

Answer: $\sqrt{32}$ inches

Important Observation. Any attempt to use your calculator to approximate $\sqrt{95}$ in Example 3 would be an error as the instructions asked for an **exact** answer.

Sometimes an approximate answer is desired, particularly in applications.

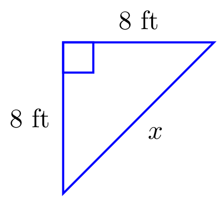
You Try It!

A 15 foot ladder leans against the wall of a building. The base of the ladder lies 5 feet from the base of the wall. How high up the wall does the top of the ladder reach? Round your answer to the nearest tenth of a foot.

EXAMPLE 4. Ginny want to create a vegetable garden in the corner of her yard in the shape of a right triangle. She cuts two boards of length 8 feet which will form the legs of her garden. Find the length of board she should cut to form the hypotenuse of her garden, correct to the nearest tenth of a foot.

Solution. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We begin with a labeled sketch. Let x represent the length of the unknown hypotenuse.



2. *Set Up an Equation.* The hypotenuse is isolated on one side of the Pythagorean equation.

$$x^2 = 8^2 + 8^2$$

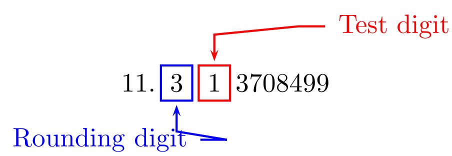
3. *Solve the Equation.*

$$x^2 = 8^2 + 8^2 \quad \text{The Pythagorean equation.}$$
$$x^2 = 64 + 64 \quad \text{Exponents first: } 8^2 = 64 \text{ and } 8^2 = 64.$$
$$x^2 = 128 \quad \text{Add: } 64 + 64 = 128.$$
$$x = \sqrt{128} \quad \text{Take the nonnegative square root.}$$

4. *Answer the Question.* The **exact** length of the hypotenuse is $\sqrt{128}$ feet, but we're asked to find the hypotenuse to the nearest tenth of a foot. Using a calculator, we find an approximation for $\sqrt{128}$.

$$\sqrt{128} \approx 11.313708499$$

To round to the nearest tenth, first identify the rounding and test digits.



The test digit is less than five. So we leave the rounding digit alone and truncate. Therefore, correct to the nearest tenth of a foot, the length of the hypotenuse is approximately 11.3 feet.

5. *Look Back.* The sum of the squares of the legs is

$$\begin{aligned} 8^2 + 8^2 &= 64 + 64 \\ &= 128. \end{aligned}$$

The square of the hypotenuse is

$$(11.3)^2 = 127.69$$

These are almost the same, the discrepancy due to the fact that we rounded to find an approximation for the hypotenuse.

Answer: 14.1 feet

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❧ ❧ ❧ Exercises ☙ ☙ ☙

In Exercises 1-16, your solutions should include a well-labeled sketch.

1. The length of one leg of a right triangle is 15 meters, and the length of the hypotenuse is 25 meters. Find the exact length of the other leg.

2. The length of one leg of a right triangle is 7 meters, and the length of the hypotenuse is 25 meters. Find the exact length of the other leg.

3. The lengths of two legs of a right triangle are 12 meters and 16 meters. Find the exact length of the hypotenuse.

4. The lengths of two legs of a right triangle are 9 meters and 12 meters. Find the exact length of the hypotenuse.

5. The length of one leg of a right triangle is 13 meters, and the length of the hypotenuse is 22 meters. Find the exact length of the other leg.

6. The length of one leg of a right triangle is 6 meters, and the length of the hypotenuse is 15 meters. Find the exact length of the other leg.

7. The lengths of two legs of a right triangle are 2 meters and 21 meters. Find the exact length of the hypotenuse.

8. The lengths of two legs of a right triangle are 7 meters and 8 meters. Find the exact length of the hypotenuse.

9. The length of one leg of a right triangle is 12 meters, and the length of the hypotenuse is 19 meters. Find the exact length of the other leg.

10. The length of one leg of a right triangle is 5 meters, and the length of the hypotenuse is 10 meters. Find the exact length of the other leg.

11. The lengths of two legs of a right triangle are 6 meters and 8 meters. Find the exact length of the hypotenuse.

12. The lengths of two legs of a right triangle are 5 meters and 12 meters. Find the exact length of the hypotenuse.

13. The length of one leg of a right triangle is 6 meters, and the length of the hypotenuse is 10 meters. Find the exact length of the other leg.

14. The length of one leg of a right triangle is 9 meters, and the length of the hypotenuse is 15 meters. Find the exact length of the other leg.

15. The lengths of two legs of a right triangle are 6 meters and 22 meters. Find the exact length of the hypotenuse.

16. The lengths of two legs of a right triangle are 9 meters and 19 meters. Find the exact length of the hypotenuse.

In Exercises 17-24, your solutions should include a well-labeled sketch.

17. The lengths of two legs of a right triangle are 3 meters and 18 meters. Find the length of the hypotenuse. Round your answer to the nearest hundredth.

18. The lengths of two legs of a right triangle are 10 feet and 16 feet. Find the length of the hypotenuse. Round your answer to the nearest tenth.

19. The length of one leg of a right triangle is 2 meters, and the length of the hypotenuse is 17 meters. Find the length of the other leg. Round your answer to the nearest tenth.

20. The length of one leg of a right triangle is 4 meters, and the length of the hypotenuse is 12 meters. Find the length of the other leg. Round your answer to the nearest hundredth.

21. The lengths of two legs of a right triangle are 15 feet and 18 feet. Find the length of the hypotenuse. Round your answer to the nearest hundredth.

22. The lengths of two legs of a right triangle are 6 feet and 13 feet. Find the length of the hypotenuse. Round your answer to the nearest tenth.

23. The length of one leg of a right triangle is 4 meters, and the length of the hypotenuse is 8 meters. Find the length of the other leg. Round your answer to the nearest hundredth.

24. The length of one leg of a right triangle is 3 meters, and the length of the hypotenuse is 15 meters. Find the length of the other leg. Round your answer to the nearest tenth.

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25. Greta and Fritz are planting a 13-meter by 18-meter rectangular garden, and are laying it out using string. They would like to know the length of a diagonal to make sure that right angles are formed. Find the length of a diagonal. Round your answer to the nearest hundredth. Your solution should include a well-labeled sketch.

26. Markos and Angelina are planting an 11-meter by 19-meter rectangular garden, and are laying it out using string. They would like to know the length of a diagonal to make sure that right angles are formed. Find the length of a diagonal. Round your answer to the nearest tenth. Your solution should include a well-labeled sketch.

27. The base of a 24-meter long guy wire is located 10 meters from the base of the telephone pole that it is anchoring. How high up the pole does the guy wire reach? Round your answer to the nearest hundredth. Your solution should include a well-labeled sketch.

28. The base of a 30-foot long guy wire is located 9 feet from the base of the telephone pole that it is anchoring. How high up the pole does the guy wire reach? Round your answer to the nearest hundredth. Your solution should include a well-labeled sketch.

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29. **Hiking Trail.** A hiking trail runs due south for 8 kilometers, then turns west for about 15 kilometers, and then heads northeast on a direct path to the starting point. How long is the entire trail?

30. **Animal Trail.** An animal trail runs due east from a watering hole for 12 kilometers, then goes north for 5 kilometers. Then the trail turns southwest on a direct path back to the watering hole. How long is the entire trail?

31. **Upper Window.** A 10-foot ladder leans against the wall of a house. How close to the wall must the bottom of the ladder be in order to reach a window 8 feet above the ground?

32. **How high?** A 10-foot ladder leans against the wall of a house. How high will the ladder be if the bottom of the ladder is 4 feet from the wall? Round your answer to the nearest tenth.

❧ ❧ ❧ Answers ☙ ☙ ☙

1. 20 meters

3. 20 meters

5. $\sqrt{315}$ meters

7. $\sqrt{445}$ meters

9. $\sqrt{217}$ meters

11. 10 meters

13. 8 meters

15. $\sqrt{520}$ meters

17. 18.25 meters

19. 16.9 meters

21. 23.43 feet

23. 6.93 meters

25. 22.20 meters

27. 21.82 meters

29. 40 kilometers

31. 6 ft.

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