2.1 Exercises

In Exercises 1-6, state the domain and range of the given relation.

1. \( R = \{(1,3), (2,4), (3,4)\} \)
2. \( R = \{(1,3), (2,4), (2,5)\} \)
3. \( R = \{(1,4), (2,5), (2,6)\} \)
4. \( R = \{(1,5), (2,4), (3,6)\} \)

In Exercises 7-12, create a mapping diagram for the given relation and state whether or not it is a function.

7. The relation in Exercise 1.
8. The relation in Exercise 2.
11. The relation in Exercise 5.

13. Given that \( g \) takes a real number and doubles it, then \( g : x \rightarrow ? \).
14. Given that \( f \) takes a real number and divides it by 3, then \( f : x \rightarrow ? \).
15. Given that \( g \) takes a real number and adds 3 to it, then \( g : x \rightarrow ? \).
16. Given that \( h \) takes a real number and subtracts 4 from it, then \( h : x \rightarrow ? \).
17. Given that \( g \) takes a real number, doubles it, then adds 5, then \( g : x \rightarrow ? \).
18. Given that \( h \) takes a real number, subtracts 3 from it, then divides the result by 4, then \( h : x \rightarrow ? \).

Given that the function \( f \) is defined by the rule \( f : x \rightarrow 3x - 5 \), determine where the input number is mapped in Exercises 19-22.

19. \( f : 3 \rightarrow ? \)

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Given that the function $f$ is defined by the rule $f: x \rightarrow 4 - 5x$, determine where the input number is mapped in Exercises 23-26.

20. $f: -5 \rightarrow ?$
21. $f: a \rightarrow ?$
22. $f: 2a + 3 \rightarrow ?$

23. $f: 2 \rightarrow ?$
24. $f: -3 \rightarrow ?$
25. $f: a \rightarrow ?$
26. $f: 2a + 11 \rightarrow ?$

Given that the function $f$ is defined by the rule $f: x \rightarrow x^2 - 4x - 6$, determine where the input number is mapped in Exercises 27-30.

27. $f: 1 \rightarrow ?$
28. $f: -2 \rightarrow ?$
29. $f: -1 \rightarrow ?$
30. $f: a \rightarrow ?$

31. $f: a \rightarrow ?$
32. $f: a + 1 \rightarrow ?$
33. $f: 2a - 5 \rightarrow ?$
34. $f: a + h \rightarrow ?$

35. $f: 2 \rightarrow ?$
36. $g: 2 \rightarrow ?$
37. $f: a + 1 \rightarrow ?$
38. $g: a - 3 \rightarrow ?$

39. Given that $g$ takes a real number and triples it, then $g(x) = ?$.
40. Given that $f$ takes a real number and divides it by 5, then $f(x) = ?$.
41. Given that $g$ takes a real number and subtracts it from 10, then $g(x) = ?$.
42. Given that $f$ takes a real number, multiplies it by 5 and then adds 4 to the result, then $f(x) = ?$.
43. Given that $f$ takes a real number, doubles it, then subtracts the result from 11, then $f(x) = ?$.
44. Given that $h$ takes a real number, doubles it, adds 5, then takes the square root of the result, then $h(x) = ?$.

In Exercises 45-54, evaluate the given function at the given value $b$.

45. $f(x) = 12x + 2$ for $b = 6$.
46. $f(x) = -11x - 4$ for $b = -3$.
47. $f(x) = -9x - 1$ for $b = -5$.
48. $f(x) = 11x + 4$ for $b = -4$. 

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49. \( f(x) = 4 \) for \( b = -12 \).

50. \( f(x) = 7 \) for \( b = -7 \).

51. \( f(x) = 0 \) for \( b = -7 \).

52. \( f(x) = 12x + 8 \) for \( b = -3 \).

53. \( f(x) = -9x + 3 \) for \( b = -1 \).

54. \( f(x) = 6x - 3 \) for \( b = 3 \).

In Exercises 55-58, given that the function \( f \) is defined by the rule \( f(x) = 2x + 7 \), determine where the input number is mapped.

55. \( f(a) = ? \)

56. \( f(a + 1) = ? \)

57. \( f(3a - 2) = ? \)

58. \( f(a + h) = ? \)

In Exercises 59-62, given that the function \( g \) is defined by the rule \( g(x) = 3 - 2x \), determine where the input number is mapped.

59. \( g(a) = ? \)

60. \( g(a + 3) = ? \)

61. \( g(2 - 5a) = ? \)

62. \( g(a + h) = ? \)

Given that the functions \( f \) and \( g \) are defined by the rules \( f(x) = 3x + 4 \) and \( g(x) = 2x - 5 \), determine where the input number is mapped in Exercises 67-70.

67. \( f(g(2)) = ? \)

68. \( g(f(2)) = ? \)

69. \( f(g(a)) = ? \)

70. \( g(f(a)) = ? \)

Given that the functions \( f \) and \( g \) are defined by the rules \( f(x) = 2x - 9 \) and \( g(x) = 11 \), determine where the input number is mapped in Exercises 71-74.

71. \( f(g(2)) = ? \)

72. \( g(f(2)) = ? \)

73. \( f(g(a)) = ? \)

74. \( g(f(a)) = ? \)

Use set-builder notation to describe the domain of each of the functions defined in Exercises 75-78.

75. \( f(x) = \frac{93}{x + 98} \)

76. \( f(x) = \frac{54}{x + 65} \)

77. \( f(x) = -\frac{87}{x - 88} \)

78. \( f(x) = -\frac{30}{x - 52} \)
Use set-builder and interval notation to describe the domain of the functions defined in Exercises 79-82.

79. \( f(x) = \sqrt{x + 69} \)

80. \( f(x) = \sqrt{x + 62} \)

81. \( f(x) = \sqrt{x - 81} \)

82. \( f(x) = \sqrt{x - 98} \)

Two integers are said to be relatively prime if their greatest common divisor is 1. For example, the greatest common divisor of 6 and 35 is 1, so 6 and 35 are relatively prime. On the other hand, the greatest common divisor of 14 and 21 is not 1 (it is 7), so 14 and 21 are not relatively prime. The Euler \( \phi \)-function is defined as follows:

- If \( n = 1 \), then \( \phi(n) = 1 \).
- If \( n > 1 \), then \( \phi(n) \) is the number of positive integers less than \( n \) that are relatively prime to \( n \). In Exercises 83-84, evaluate the Euler \( \phi \)-function at the given input.

83. \( \phi(12) \)

84. \( \phi(36) \)
2.1 Answers

1. Domain = \{1, 2, 3\}, Range = \{3, 4\}  
23. \( f : 2 \rightarrow -6 \)

3. Domain = \{1, 2\}, Range = \{4, 5, 6\}  
25. \( f : a \rightarrow 4 - 5a \)

5. Domain = \{1, 2, 3\}, Range = \{1, 2, 3, 4\}  
27. \( f : 1 \rightarrow -9 \)

7.  
\[
\begin{array}{c}
R \\
1 \rightarrow 3 \\
2 \rightarrow 4 \\
3
\end{array}
\]
Function.

9.  
\[
\begin{array}{c}
R \\
1 \rightarrow 4 \\
2 \rightarrow 5 \\
3 \rightarrow 6
\end{array}
\]
Not a function.

11.  
\[
\begin{array}{c}
R \\
1 \rightarrow 1 \\
2 \rightarrow 2 \\
3 \rightarrow 3 \\
4
\end{array}
\]
Not a function.

13. \( g : x \rightarrow 2x \)

15. \( g : x \rightarrow x + 3 \)

17. \( g : x \rightarrow 2x + 5 \)

19. \( f : 3 \rightarrow 4 \)

21. \( f : a \rightarrow 3a - 5 \)

29. \( f : -1 \rightarrow -1 \)

31. \( f : a \rightarrow 3a - 9 \)

33. \( f : 2a - 5 \rightarrow 6a - 24 \)

35. \( f : 2 \rightarrow 7 \)

37. \( f : a + 1 \rightarrow 2a + 5 \)

39. \( g(x) = 3x \)

41. \( g(x) = 10 - x \)

43. \( f(x) = 11 - 2x \)

45. 74

47. 44

49. 4

51. 0

53. 12

55. \( f(a) = 2a + 7 \)

57. \( f(3a - 2) = 6a + 3 \)

59. \( g(a) = 3 - 2a \)

61. \( g(2 - 5a) = 10a - 1 \)

63. \( f(a) = 1 - a \)

65. \( f(a + 3) = -a - 2 \)
67. \( f(g(2)) = 1 \)

69. \( f(g(a)) = 6a - 11 \)

71. \( f(g(2)) = 13 \)

73. \( f(g(a)) = 13 \)

75. Domain = \( \{ x : x \neq -98 \} \)

77. Domain = \( \{ x : x \neq 88 \} \)

79. Domain = \([-69, \infty) = \{ x : x \geq -69 \}\)

81. Domain = \([81, \infty) = \{ x : x \geq 81 \}\)

83. \( \phi(12) = 4 \)