2.3 Exercises

For Exercises 1-6, perform each of the following tasks.

i. Make a copy of the graph on a sheet of graph paper and apply the vertical line test.

ii. Write a complete sentence stating whether or not the graph represents a function. Explain the reason for your response.

1.

2.

3.

4.

5.

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In Exercises 7-12, perform each of the following tasks.

i. Make an exact copy of the graph of the function \( f \) on a sheet of graph paper. Label and scale each axis. Remember to draw all lines with a ruler.

ii. Use the technique of Examples 3 and 4 in the narrative to evaluate the function at the given value. Draw and label the arrows as shown in Figures 4 and 5 in the narrative.

7. Use the graph of \( f \) to determine \( f(2) \).

8. Use the graph of \( f \) to determine \( f(3) \).

9. Use the graph of \( f \) to determine \( f(-2) \).

10. Use the graph of \( f \) to determine \( f(1) \).
11. Use the graph of $f$ to determine $f(1)$.

12. Use the graph of $f$ to determine $f(-2)$.

13. Use the graph of $f$ to solve the equation $f(x) = -2$.

14. Use the graph of $f$ to solve the equation $f(x) = 1$.

15. Use the graph of $f$ to solve the equation $f(x) = 2$.

In Exercises 13-18, perform each of the following tasks.

i. Make an exact copy of the graph of the function $f$ on a sheet of graph paper. Label and scale each axis. Remember to draw all lines with a ruler.

ii. Use the technique of Example 5 in the narrative to find the value of $x$ that maps onto the given value. Draw and label the arrows as shown in Figure 6 in the narrative.
16. Use the graph of $f$ to solve the equation $f(x) = -2$.

17. Use the graph of $f$ to solve the equation $f(x) = 2$.

18. Use the graph of $f$ to solve the equation $f(x) = -3$.

In the Exercises 19-22, perform each of the following tasks.

i. Make a copy of the graph of $f$ on a sheet of graph paper. Label and scale each axis.

ii. Using a different colored pen or pencil, project each point on the graph of $f$ onto the $x$-axis. Shade the resulting domain on the $x$-axis.

iii. Use both set-builder and interval notation to describe the domain.

19.

20.
In Exercises 23-26, perform each of the following tasks.

i. Make a copy of the graph of $f$ on a sheet of graph paper. Label and scale each axis.

ii. Using a different colored pen or pencil, project each point on the graph of $f$ onto the $y$-axis. Shade the resulting range on the $y$-axis.

iii. Use both set-builder and interval notation to describe the range.
26. 

In Exercises 27-30, perform each of the following tasks.

i. Use your graphing calculator to draw the graph of the given function. Make a reasonably accurate copy of the image in your viewing screen on your homework paper. Label and scale each axis with the WINDOW parameters xmin, xmax, ymin, and ymax. Label the graph with its equation.

ii. Using a colored pencil, project each point on the graph onto the x-axis; i.e., shade the domain on the x-axis. Use interval and set-builder notation to describe the domain.

iii. Use a purely algebraic technique, as demonstrated in Example 8 in the narrative, to find the domain. Compare this result with that found in part (ii).

iv. Using a different colored pencil, project each point on the graph onto the y-axis; i.e., shade the range on the y-axis. Use interval and set-builder notation to describe the range.

27. \( f(x) = \sqrt{x + 5} \).

28. \( f(x) = \sqrt{5 - x} \).

29. \( f(x) = -\sqrt{4 - x} \).

30. \( f(x) = -\sqrt{x + 4} \).
2.3 Solutions

1. Note that in the figure below a vertical line cuts the graph more than once. Therefore, the graph does not represent the graph of a function.

3. No vertical line cuts the graph more than once (see figure below). Therefore, the graph represents a function.

5. Note that in the figure below a vertical line cuts the graph more than once. Therefore, the graph does not represent the graph of a function.
7. Locate $x = 2$ on the $x$-axis (see figure below), draw a vertical arrow to the graph of $f$, then a horizontal arrow to the $y$-axis. Thus, $f(2) = -1$.

9. Locate $x = -2$ on the $x$-axis (see figure below), draw a vertical arrow to the graph of $f$, then a horizontal arrow to the $y$-axis. Thus, $f(-2) = 1$.

11. Locate $x = 1$ on the $x$-axis (see figure below), draw a vertical arrow to the graph of $f$, then a horizontal arrow to the $y$-axis. Thus, $f(1) = 3$. 
13. Locate \( y = -2 \) on the \( y \)-axis (see figure below), draw a horizontal arrow to the graph of \( f \), then a vertical arrow to the \( y \)-axis. Thus, the solution of \( f(x) = -2 \) is \( x = -3 \).

![Graph of a function with points on the y-axis and solution]

15. Locate \( y = 2 \) on the \( y \)-axis (see figure below), draw a horizontal arrow to the graph of \( f \), then a vertical arrow to the \( y \)-axis. Thus, the solution of \( f(x) = 2 \) is \( x = -2 \).

![Graph of a function with points on the y-axis and solution]

17. Locate \( y = 2 \) on the \( y \)-axis (see figure below), draw a horizontal arrow to the graph of \( f \), then a vertical arrow to the \( y \)-axis. Thus, the solution of \( f(x) = 2 \) is \( x = -1 \).

![Graph of a function with points on the y-axis and solution]
19. To find the domain of the function, project the graph of $f$ onto the $x$-axis. Note that all values of $x$ that lie to the right of $-3$ lie in shadow and are hence in the domain of $f$. Therefore, the domain is best described with the notation $\{x : x > -3\} = (-3, \infty)$.

21. To find the domain of the function, project the graph of $f$ onto the $x$-axis. Note that all values of $x$ that lie to the left of 0 lie in shadow and are hence in the domain of $f$. Therefore, the domain is best described with the interval notation $\{x : x < 0\} = (-\infty, 0)$.

23. To find the range of the function, project the graph of $f$ onto the $y$-axis. Note that all values of $y$ that lie below 1 lie in shadow and are hence in the range of $f$. Therefore, the range is best described with the interval notation $\{y : y < 1\} = (-\infty, 1)$.
25. To find the range of the function, project the graph of \( f \) onto the \( y \)-axis. Note that all values of \( y \) that lie above \(-2\) lie in shadow and are hence in the range of \( f \). Therefore, the range is best described with the interval notation \( \{ y : y > -2 \} = (-2, \infty) \).

![Graph of a function](image)

27. Load the function \( f(x) = \sqrt{x+5} \) into \( Y1 \) as shown in (a). Select 6:ZStandard from the ZOOM menu to produce the graph in (b).

![Graph of a function](image)

Copy the image in (b) onto your homework paper, then project the domain and range onto the \( x \)- and \( y \)-axes, as shown in (c) and (d), respectively.

![Graph of a function](image)

(c) \( D = [-5, \infty) = \{ x : x \geq -5 \} \)  
(d) \( R = [0, \infty) = \{ y : y \geq 0 \} \)

To find the domain algebraically, note that you cannot take the square root of a negative number, so the expression under the radical in \( f(x) = \sqrt{x+5} \), namely \( x+5 \), must either be positive or zero (nonnegative). That is,
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\[ x + 5 \geq 0, \]

or equivalently,

\[ x \geq -5. \]

Thus, the domain of \( f \) is Domain = \([-5, \infty)\), or in set-builder notation, Domain = \( \{ x : x \geq -5 \} \).

29. Load the function \( f(x) = -\sqrt{4-x} \) into \( Y_1 \) as shown in (a). Select 6:ZStandard from the ZOOM menu to produce the graph in (b).

COPY THE IMAGE IN (B) ONTO YOUR HOMEWORK PAPER, THEN PROJECT THE DOMAIN AND RANGE ONTO THE \( x \)- AND \( y \)-AXES, AS SHOWN IN (C) AND (D), RESPECTIVELY.

To find the domain algebraically, note that you cannot take the square root of a negative number, so the expression under the radical in \( f(x) = \sqrt{4-x} \), namely \( 4-x \), must either be positive or zero (nonnegative). That is,

\[ 4 - x \geq 0, \]

or equivalently,

\[ -x \geq -4 \]

\[ x \leq 4 \]

Thus, the domain of \( f \) is Domain = \((\infty, 4]\), or in set-builder notation, Domain = \( \{ x : x \leq 4 \} \).

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