2.4 Exercises

In Exercises 1-6, you are given the definition of two functions $f$ and $g$. Compare the functions, as in Example 1 of the narrative, at the given values of $x$.

1. $f(x) = x+2$, $g(x) = 4-x$ at $x = -3, 1, \text{and } 2$.

2. $f(x) = 2x - 3$, $g(x) = 3 - x$ at $x = -4, 2, \text{and } 5$.

3. $f(x) = 3-x$, $g(x) = x+9$ at $x = -4, -3, \text{and } -2$.

4. $f(x) = x^2$, $g(x) = 4x + 5$ at $x = -2, 1, \text{and } 6$.

5. $f(x) = x^2$, $g(x) = -3x - 2$ at $x = -3, -1, \text{and } 0$.

6. $f(x) = |x|$, $g(x) = 4 - x$ at $x = 1, 2, \text{and } 3$.

In Exercises 7-12, perform each of the following tasks. Remember to use a ruler to draw all lines.

i. Make an accurate copy of the image on graph paper (label each equation, label and scale each axis), drop a dashed vertical line through the point of intersection, then label and shade the solution of $f(x) < g(x)$ on the $x$-axis. Use set-builder and interval notation to describe your solution set.

ii. Make a second copy of the image on graph paper, drop a dashed, vertical line through the point of intersection, then label and shade the solution of $f(x) > g(x)$ on the $x$-axis. Use set-builder and interval notation to describe your solution set.

iii. Make a third copy of the image on graph paper, drop a dashed, vertical line through the point of intersection, then label and shade the solution of $f(x) = g(x)$ on the $x$-axis. Use set-builder and interval notation to describe your solution set.

\[ \text{Graphs for Exercises 7-8.} \]

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1 Copyrighted material. See: http://msenux.redwoods.edu/IntAlgText/
In Exercises 13-16, perform each of the following tasks. Remember to use a ruler to draw all lines.

i. Make an accurate copy of the image on graph paper, drop dashed, vertical lines through the points of intersection, then label and shade the solution of $f(x) \geq g(x)$ on the $x$-axis. Use set-builder and interval notation to describe your solution set.

ii. Make a second copy of the image on graph paper, drop dashed, vertical lines through the points of intersection, then label and shade the solution of $f(x) < g(x)$ on the $x$-axis. Use set-builder and interval notation to describe your solution set.
In Exercises 17-20, perform each of the following tasks. *Remember to use a ruler to draw all lines.*

i. Load each side of the equation into the Y= menu of your calculator. Adjust the WINDOW parameters so that the point of intersection of the graphs is visible in the viewing window. Use the intersect utility in the CALC menu of your calculator to determine the x-coordinate of the point of intersection.

ii. Make an accurate copy of the image in your viewing window on your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax, and label each graph with its equation.

iii. Draw a dashed, vertical line through the point of intersection. Shade and label the solution of the equation on the x-axis.

17. \[1.23x - 4.56 = 3.46 - 2.3x\]
18. \[2.23x - 1.56 = 5.46 - 3.3x\]
19. \[5.46 - 1.3x = 2.2x - 5.66\]
20. \[2.46 - 1.4x = 1.2x - 2.66\]
In Exercises 21-26, perform each of the following tasks. Remember to use a ruler to draw all lines.

i. Load each side of the inequality into the Y= menu of your calculator. Adjust the WINDOW parameters so that the point(s) of intersection of the graphs is visible in the viewing window. Use the intersect utility in the CALC menu of your calculator to determine the coordinates of the point(s) of intersection.

ii. Make an accurate copy of the image in your viewing window on your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax, and label each graph with its equation.

iii. Draw a dashed, vertical line through the point(s) of intersection. Shade and label the solution of the inequality on the x-axis. Use both set-builder and interval notation to describe the solution set.

21. \(1.6x + 1.23 \geq -2.3x - 4.2\)

22. \(1.24x + 5.6 < 1.2 - 0.52x\)

23. \(0.15x - 0.23 > 8.2 - 0.6x\)

24. \(-1.23x - 9.76 \leq 1.44x + 22.8\)

25. \(0.5x^2 - 5 < 1.23 - 0.75x\)

26. \(4 - 0.5x^2 \leq 0.72x - 1.34\)

In Exercises 27-30, perform each of the following tasks. Remember to use a ruler to draw all lines.

i. Make an accurate copy of the image on graph paper (label the graph with the letter \(f\) and label and scale each axis), drop a dashed vertical line through the x-intercept of the graph of \(f\), then label and shade the solution of \(f(x) = 0\) on the x-axis. Use set-builder notation to describe your solution.

ii. Make a second copy of the image on graph paper, drop a dashed, vertical line through the x-intercept of the graph of \(f\), then label and shade the solution of \(f(x) > 0\) on the x-axis. Use set-builder and interval notation to describe your solution set.

iii. Make a third copy of the image on graph paper, drop a dashed, vertical line through the x-intercept of the graph of \(f\), then label and shade the solution of \(f(x) < 0\) on the x-axis. Use set-builder and interval notation to describe your solution set.

27.
In Exercises 31-34, perform each of the following tasks. Remember to use a ruler to draw all lines.

i. Make an accurate copy of the image on graph paper, drop dashed, vertical lines through the $x$-intercepts, then label and shade the solution of $f(x) \geq 0$ on the $x$-axis. Use set-builder and interval notation to describe your solution set.

ii. Make a second copy of the image on graph paper, drop dashed, vertical lines through the $x$-intercepts, then label and shade the solution of $f(x) < 0$ on the $x$-axis. Use set-builder and interval notation to describe your solution set.
In Exercises 35-38, perform each of the following tasks. *Remember to use a ruler to draw all lines.*

i. Load the given function \( f \) into the \( Y= \) menu of your calculator. Adjust the \( \text{WINDOW} \) parameters so that the \( x \)-intercept(s) of the graph of \( f \) is visible in the viewing window. Use the \textbf{zero} utility in the \textsc{calc} menu of your calculator to determine the coordinates of the \( x \)-intercept(s) of the graph of \( f \).

ii. Make an accurate copy of the image in your viewing window on your homework paper. Label and scale each axis with \( \text{xmin}, \text{xmax}, \text{ymin}, \) and \( \text{ymax} \), and label the graph with its equation.

iii. Draw a dashed, vertical line through the \( x \)-intercept(s). Shade and label the solution of the inequality \( f(x) > 0 \) on the \( x \)-axis. Use both set-builder and interval notation to describe the solution set.

35. \( f(x) = -1.25x + 3.58 \)

36. \( f(x) = 1.34x - 4.52 \)

37. \( f(x) = 1.25x^2 + 4x - 5.9125 \)

38. \( f(x) = -1.32x^2 - 3.96x + 5.9532 \)

In Exercises 39-42, perform each of the following tasks. *Remember to use a ruler to draw all lines.*

i. Load the given function \( f \) into the \( Y= \) menu of your calculator. Adjust the \( \text{WINDOW} \) parameters so that the \( x \)-intercept(s) of the graph of \( f \) is visible in the viewing window. Use the \textbf{zero} utility in the \textsc{calc} menu of your calculator to determine the coordinates of the \( x \)-intercept(s) of the graph of \( f \).

ii. Make an accurate copy of the image...
in your viewing window on your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax, and label the graph with its equation.

iii. Draw a dashed, vertical line through the x-intercept(s). Shade and label the solution of the inequality $f(x) \leq 0$ on the x-axis. Use both set-builder and interval notation to describe the solution set.

39. $f(x) = -1.45x - 5.6$

40. $f(x) = 1.35x + 8.6$

41. $f(x) = -1.11x^2 - 5.9940x + 1.2432$

42. $f(x) = 1.22x^2 - 6.3440x + 1.3176$
2.4 Solutions

1. We’re given that $f(x) = x + 2$ and $g(x) = 4 - x$. At $x = -3$,
   
   $f(-3) = -3 + 2 = -1$
   $g(-3) = 4 - (-3) = 7$.

   Therefore, $f(-3) < g(-3)$. At $x = 1$,
   
   $f(1) = 1 + 2 = 3$
   $g(1) = 4 - 1 = 3$.

   Therefore, $f(1) = g(1)$. At $x = 2$,
   
   $f(2) = 2 + 2 = 4$
   $g(2) = 4 - 2 = 2$.

   Therefore, $f(2) > g(2)$.

3. We’re given that $f(x) = 3 - x$ and $g(x) = x + 9$. At $x = -4$,
   
   $f(-4) = 3 - (-4) = 7$
   $g(-4) = -4 + 9 = 5$.

   Therefore, $f(-4) > g(-4)$. At $x = -3$,
   
   $f(-3) = 3 - (-3) = 6$
   $g(-3) = -3 + 9 = 6$.

   Therefore, $f(-3) = g(-3)$. At $x = -2$,
   
   $f(-2) = 3 - (-2) = 5$
   $g(-2) = -2 + 9 = 7$.

   Therefore, $f(-2) < g(-2)$. 
5. We’re given that $f(x) = x^2$ and $g(x) = -3x - 2$. At $x = -3$,

$$f(-3) = (-3)^2 = 9$$
$$g(-3) = -3(-3) - 2 = 7.$$ 

Therefore, $f(-3) > g(-3)$. At $x = -1$,

$$f(-1) = (-1)^2 = 1$$
$$g(-1) = -3(-1) - 2 = 1.$$ 

Therefore, $f(-1) = g(-1)$. At $x = 0$,

$$f(0) = (0)^2 = 0$$
$$g(0) = -3(0) - 2 = -2.$$ 

Therefore, $f(0) > g(0)$.

7. The graph of $f$ intersects the graph of $g$ at $x = 3$. The solution of $f(x) = g(x)$ is $x = 3$.

The graph of $f$ lies above the graph of $g$ to the right of $x = 3$. The solution of $f(x) > g(x)$ is $(3, \infty) = \{x : x > 3\}$. 

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The graph of $f$ lies below the graph of $g$ to the left of $x = 3$. The solution of $f(x) < g(x)$ is $(\infty, 3) = \{x : x < 3\}$.

9. The graph of $f$ intersects the graph of $g$ at $x = -2$. The solution of $f(x) = g(x)$ is $x = -2$.

The graph of $f$ lies above the graph of $g$ to the left of $x = -2$. The solution of $f(x) > g(x)$ is $(\infty, -2) = \{x : x < -2\}$.
The graph of $f$ lies below the graph of $g$ to the right of $x = -2$. The solution of $f(x) < g(x)$ is $(-2, \infty) = \{x : x > -2\}$.

11. The graph of $f$ intersects the graph of $g$ at $x = 3$. The solution of $f(x) = g(x)$ is $x = 3$.

The graph of $f$ is above the graph of $g$ to the right of $x = 3$. The solution of $f(x) > g(x)$ is $(3, \infty) = \{x : x > 3\}$.
The graph of \( f \) is below the graph of \( g \) to the left of \( x = 3 \). The solution of \( f(x) < g(x) \) is \(( -\infty, 3) = \{ x : x < 3 \} \).

13. The graph of \( f \) intersects the graph of \( g \) at \( x = -3 \) and \( x = 3 \). The graph of \( f \) lies above the graph of \( g \) for values of \( x \) that lie between \(-3 \) and \( 3 \). Therefore, the solution of \( f(x) \geq g(x) \) is \([-3, 3] = \{ x : -3 \leq x \leq 3 \} \).
The graph of $f$ is below the graph of $g$ for values of $x$ that lie to the left of $-3$ or to the right of $3$. Therefore, the solution of $f(x) < g(x)$ is $(-\infty, -3) \cup (3, \infty)$ or \{x : x < -3 or x > 3\}.

The graph of $f$ intersects the graph of $g$ at $x = -2$ and at $x = 2$. The graph of $f$ lies above the graph of $g$ for all values of $x$ that lie to the left of $-2$ or to the right of $2$. Therefore, the solution of $f(x) \geq g(x)$ is $(-\infty, -2] \cup [2, \infty)$ or \{x : x \leq -2 or x \geq 2\}.
The graph of $f$ lies below the graph of $g$ for values of $x$ that lie between $-2$ and $2$. Therefore, the solution of $f(x) < g(x)$ is $(-2, 2) = \{x : -2 < x < 2\}$.

17. To solve the equation $1.23x - 4.56 = 3.46 - 2.3x$ graphically, start by loading the left- and right-hand sides of the equation into $Y1$ and $Y2$, respectively, as shown in (a). Use the intersect utility in the CALC menu to determine the point of intersection, as shown in (c).

Therefore, the solution of the equation is $x = 2.2719547$, which is shaded on the $x$-axis in the image that follows. Answers may vary due to roundoff error.
19. To solve the equation $5.46 - 1.3x = 2.2x - 5.66$ graphically, start by loading the left- and right-hand sides of the equation into Y1 and Y2, respectively, as shown in (a). Use the `intersect` utility in the `CALC` menu to determine the point of intersection, as shown in (c).

Therefore, the solution of the equation is $x = 3.1771429$, which is shaded on the $x$-axis in the image that follows. Answers may vary due to roundoff error.

21. To solve the inequality $1.6x + 1.23 \geq -2.3x - 4.2$ graphically, start by loading the left- and right-hand sides of the inequality into Y1 and Y2, respectively, as shown in (a). Use the `intersect` utility in the `CALC` menu to determine the point of intersection, as shown in (c).
The two graphs intersect at \( x = -1.392308 \). The graph of \( y = 1.6x + 1.23 \) is above the graph of \( y = -2.3x - 4.2 \) for all values of \( x \) that lie to the right of \(-1.392308\). Therefore, the solution of \( 1.6x + 1.23 \geq -2.3x - 4.2 \) is \([-1.392308, \infty) = \{x : x \geq -1.392308\}\).

23. To solve the inequality \( 0.15x - 0.23 > 8.2 - 0.6x \) graphically, start by loading the left- and right-hand sides of the inequality into \( Y_1 \) and \( Y_2 \), respectively, as shown in (a). Adjust the viewing window as shown in (b). Use the \texttt{intersect} utility in the \texttt{CALC} menu to determine the point of intersection, as shown in (c).

The graph of \( y = 0.15x - 0.23 \) is above the graph of \( y = 8.2 - 0.6x \) for all values of \( x \) that lie to the right of \( 11.24 \). Therefore, the solution of \( 0.15x - 0.23 > 8.2 - 0.6x \) is \((11.24, \infty) = \{x : x > 11.24\}\).
25. To solve the inequality \(0.5x^2 - 5 < 1.23 - 0.75x\) graphically, start by loading the left- and right-hand sides of the inequality into \(Y_1\) and \(Y_2\), respectively, as shown in (a). Use the intersect utility in the CALC menu to determine the points of intersection, as shown in (b) and (c).

The graph of \(y = 0.5x^2 - 5\) is below the graph of \(y = 1.23 - 0.75x\) for all values of \(x\) that lie between \(-4.35867\) and \(2.8586701\). Therefore, the solution of \(0.5x^2 - 5 < 1.23 - 0.75x\) is \((-4.35867, 2.8586701)\) or \(\{x : -4.35867 < x < 2.8586701\}\).

27. The graph of \(f\) intercepts the \(x\)-axis at \(x = -1\). Therefore, the solution of \(f(x) = 0\) is \(x = -1\).
The graph of \( f \) lies above the \( x \)-axis for all values of \( x \) that lie to the right of \(-1\). Therefore, the solution of \( f(x) > 0 \) is \((-1, \infty) = \{x : x > -1\}\).

The graph of \( f \) lies below the \( x \)-axis for all values of \( x \) that lie to the left of \(-1\). Therefore, the solution of \( f(x) < 0 \) is \((-\infty, -1) = \{x : x < -1\}\).

29. The graph of \( f \) intercepts the \( x \)-axis at \( x = 2 \). Therefore, the solution of \( f(x) = 0 \) is \( x = 2 \).
The graph of \( f \) lies above the \( x \)-axis for all values of \( x \) that lie to the left of \( x = 2 \). Therefore, the solution of \( f(x) > 0 \) is \((-\infty, 2) = \{x : x < 2\}\).

\[ x \]
\[ y \]
\[ f \]

The graph of \( f \) lies below the \( x \)-axis for all values of \( x \) that lie to the right of \( x = 2 \). Therefore, the solution of \( f(x) < 0 \) is \((2, \infty) = \{x : x > 2\}\).
31. The graph of $f$ intercepts the $x$-axis at $x = -3$ and $x = 2$. The graph of $f$ lies above the $x$-axis for all values of $x$ that lie between $x = -3$ and $x = 2$. Therefore, the solution of $f(x) \geq 0$ is $[-3, 2] = \{x : -3 \leq x \leq 2\}$.

The graph of $f$ lies below the $x$-axis for all values of $x$ that lie to the left of $x = -3$ or to the right of $x = 2$. Therefore, the solution of $f(x) < 0$ is $(-\infty, -3) \cup (2, \infty) = \{x : x < -3 \text{ or } x > 2\}$. 
33. The graph of $f$ intercepts the $x$-axis at $x = -2$ and $x = 1$. The graph of $f$ lies above the $x$-axis for all values of $x$ that lie to the left $x = -2$ or to the right of $x = 1$. Therefore, the solution of $f(x) \geq 0$ is $(-\infty, -2] \cup [1, \infty) = \{x : x \leq -2 \text{ or } x \geq 1\}$.

35. To solve the inequality $f(x) > 0$ graphically, start by loading $f(x) = -1.25x+3.58$ into Y1. Use the zero utility in the CALC menu to determine the zero of $f$, as shown in (c).
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The graph of $f$ lies above the $x$-axis for all values of $x$ that lie to the left of $x = 2.864$. Therefore, the solution of $f(x) > 0$ is $(-\infty, 2.864) = \{x : x < 2.864\}$. Answers may vary due to roundoff error.

37. To solve the inequality $f(x) > 0$ graphically, start by loading $f(x) = 1.25x^2 + 4x - 5.9125$ into $Y_1$. Use the zero utility in the CALC menu to determine the zeros of $f$, as shown in (b) and (c).

The graph of $f$ lies above the $x$-axis for all values of $x$ that lie to the left of $x = -4.3$ or to the right of $x = 1.1$. Therefore, the solution of $f(x) > 0$ is $(-\infty, -4.3) \cup (1.1, \infty)$ or $\{x : x < -4.3 \text{ or } x > 1.1\}$. Answers may vary due to roundoff error.
39. To solve the inequality \( f(x) \leq 0 \) graphically, start by loading \( f(x) = -1.45x - 5.6 \) into \( Y_1 \). Use the zero utility in the CALC menu to determine the zero of \( f \), as shown in (c).

The graph of \( f \) intercepts the \( x \)-axis at \( x = -3.862069 \). The graph of \( f \) lies below the \( x \)-axis for all values of \( x \) that lie to the right of \( x = -3.862069 \). Therefore, the solution of \( f(x) \leq 0 \) is \([-3.862069, \infty) = \{x : x \geq -3.862069\}\). Answers may vary due to roundoff error.

41. To solve the inequality \( f(x) \leq 0 \) graphically, start by loading \( f(x) = -1.11x^2 - 5.9940x + 1.2432 \) into \( Y_1 \). Use the zero utility in the CALC menu to determine the zeros of \( f \), as shown in (b) and (c).
The graph of $f$ intercepts the $x$-axis at $x = -5.6$ and $x = 0.2$. The graph of $f$ lies below the $x$-axis for all values of $x$ that lie to the left of $x = -5.6$ or to the right of $x = 0.2$. Therefore, the solution of $f(x) \leq 0$ is $(-\infty, -5.6] \cup [0.2, \infty)$ or \{ $x : x \leq -5.6$ or $x \geq 0.2$ \}. Answers may vary due to roundoff error.