4.1 Exercises

1. Given the function defined by the rule \( f(x) = 3 \), evaluate \( f(-3) \), \( f(0) \) and \( f(4) \), then sketch the graph of \( f \).

2. Given the function defined by the rule \( g(x) = 2 \), evaluate \( g(-2) \), \( g(0) \) and \( g(4) \), then draw the graph of \( g \).

3. Given the function defined by the rule \( h(x) = -4 \), evaluate \( h(-2) \), \( h(a) \), and \( h(2x + 3) \), then draw the graph of \( h \).

4. Given the function defined by the rule \( f(x) = -2 \), evaluate \( f(0) \), \( f(b) \), and \( f(5 - 4x) \), then draw the graph of \( f \).

5. The speed of an automobile traveling on the highway is a function of time and is described by the constant function \( v(t) = 30 \), where \( t \) is measured in hours and \( v \) is measured in miles per hour. Draw the graph of \( v \) versus \( t \). Be sure to label each axis with appropriate units. Shade the area under the graph of \( v \) over the time interval \([0, 5]\) hours. What is the area under the graph of \( v \) over this time interval and what does it represent?

6. The speed of a skateboarder as she travels down a slope is a function of time and is described by the constant function \( v(t) = 8 \), where \( t \) is measured in seconds and \( v \) is measured in feet per second. Draw the graph of \( v \) versus \( t \). Be sure to label each axis with the appropriate units. Shade the area under the graph of \( v \) over the time interval \([0, 60]\) seconds. What is the area under the graph of \( v \) over this time interval and what does it represent?

7. An unlicensed plumber charges 15 dollars for each hour of labor. Let’s define this rate as a function of time by \( r(t) = 15 \), where \( t \) is measured in hours and \( r \) is measured in dollars per hour. Draw the graph of \( r \) versus \( t \). Be sure to label each axis with appropriate units. Shade the area under the graph of \( r \) over the time interval \([0, 4]\) hours. What is the area under the graph of \( r \) over this time interval and what does it represent?

8. A carpenter charges a fixed rate for each hour of labor. Let’s describe this rate as a function of time by \( r(t) = 25 \), where \( t \) is measured in hours and \( r \) is measured in dollars per hour. Draw the graph of \( r \) versus \( t \). Be sure to label each axis with appropriate units. Shade the area under the graph of \( r \) over the time interval \([0, 5]\) hours. What is the area under the graph of \( r \) over this time interval and what does it represent?

9. Given the function defined by the rule

\[
 f(x) = \begin{cases} 
 0, & \text{if } x < 0 \\
 2, & \text{if } x \geq 0,
\end{cases}
\]

evaluate \( f(-2) \), \( f(0) \), and \( f(3) \), then draw the graph of \( f \) on a sheet of graph paper. State the domain and range of \( f \).

10. Given the function defined by the rule

\[
 f(x) = \begin{cases} 
 2, & \text{if } x < 0 \\
 0, & \text{if } x \geq 0,
\end{cases}
\]

evaluate \( f(-2) \), \( f(0) \), and \( f(3) \), then draw the graph of \( f \) on sheet of graph paper. State the domain and range of \( f \).
11. Given the function defined by the rule

\[ g(x) = \begin{cases} 
-3, & \text{if } x < -2, \\
1, & \text{if } -2 \leq x < 2, \\
3, & \text{if } x \geq 2, 
\end{cases} \]

evaluate \( g(-3) \), \( g(-2) \), and \( g(5) \), then draw the graph of \( g \) on a sheet of graph paper. State the domain and range of \( g \).

12. Given the function defined by the rule

\[ g(x) = \begin{cases} 
4, & \text{if } x \leq -1, \\
2, & \text{if } -1 < x \leq 2, \\
-3, & \text{if } x > 2, 
\end{cases} \]

evaluate \( g(-1) \), \( g(2) \), and \( g(3) \), then draw the graph of \( g \) on a sheet of graph paper. State the domain and range of \( g \).

In Exercises 13-16, determine a piecewise definition of the function described by the graphs, then state the domain and range of the function.

13.

14.

15.

16.
17. Given the piecewise definition
\[ f(x) = \begin{cases} 
-x - 3, & \text{if } x < -3, \\
  x + 3, & \text{if } x \geq -3, 
\end{cases} \]
evaluate \( f(-4) \) and \( f(0) \), then draw the graph of \( f \) on a sheet of graph paper. State the domain and range of the function.

18. Given the piecewise definition
\[ f(x) = \begin{cases} 
-x + 1, & \text{if } x < 1, \\
 x - 1, & \text{if } x \geq 1, 
\end{cases} \]
evaluate \( f(-2) \) and \( f(3) \), then draw the graph of \( f \) on a sheet of graph paper. State the domain and range of the function.

19. Given the piecewise definition
\[ g(x) = \begin{cases} 
-2x + 3, & \text{if } x < 3/2, \\
 2x - 3, & \text{if } x \geq 3/2, 
\end{cases} \]
evaluate \( g(0) \) and \( g(3) \), then draw the graph of \( g \) on a sheet of graph paper. State the domain and range of the function.

20. Given the piecewise definition
\[ g(x) = \begin{cases} 
-3x - 4, & \text{if } x < -4/3, \\
 3x + 4, & \text{if } x \geq -4/3, 
\end{cases} \]
evaluate \( g(-2) \) and \( g(3) \), then draw the graph of \( g \) on a sheet of graph paper. State the domain and range of the function.

21. A battery supplies voltage to an electric circuit in the following manner. Before time \( t = 0 \) seconds, a switch is open, so the voltage supplied by the battery is zero volts. At time \( t = 0 \) seconds, the switch is closed and the battery begins to supply a constant 3 volts to the circuit. At time \( t = 2 \) seconds, the switch is opened again, and the voltage supplied by the battery drops immediately to zero volts. Sketch a graph of the voltage \( v \) versus time \( t \), label each axis with the appropriate units, then provide a piecewise definition of the voltage \( v \) supplied by the battery as a function of time \( t \).

22. Prior to time \( t = 0 \) minutes, a drum is empty. At time \( t = 0 \) minutes a hose is turned on and the water level in the drum begins to rise at a constant rate of 2 inches every minute. Let \( h \) represent water level (in inches) at time \( t \) (in minutes). Sketch the graph of \( h \) versus \( t \), label the axes with appropriate units, then provide a piecewise definition of \( h \) as a function of \( t \).
4.1 Solutions

1. Because \( f(x) = 3 \), we know that \( f \) maps any number to the number 3. Thus, \( f(-3) = 3 \), \( f(0) = 3 \), and \( f(4) = 3 \).

The graph of a constant function is always a horizontal line. In this case, \( f(x) = 3 \), so the function values are constantly equal to 3. Hence, the graph is a horizontal line 3 units up in the \( y \)-direction.

![Graph of \( f(x) = 3 \)](image)

3. Because \( h(x) = -4 \), we know that \( h \) maps any number to the number \(-4\). Thus, \( h(-2) = -4 \), \( h(a) = -4 \), and \( h(2x + 3) = -4 \).

The graph of a constant function is always a horizontal line. In this case, \( h(x) = -4 \), so the function values are constantly equal to \(-4\). Hence, the graph is a horizontal line 4 units down in the \( y \)-direction.

![Graph of \( h(x) = -4 \)](image)
5. The graph of the constant function \( v(t) = 30 \) is the horizontal line shown in the following figure.

![Graph of v(t) = 30](image)

The area under \( v(t) = 30 \) is

\[
\text{Area} = 30 \text{ mi/h} \times 5 \text{ h} = 150 \text{ mi}.
\]

This is the distance traveled by the car over the 5-hour time period.

7. The graph of the constant function \( r(t) = 15 \) is the horizontal line shown in the following figure.

![Graph of r(t) = 15](image)

The area under \( r(t) = 15 \) is

\[
\text{Area} = 15 \text{ dollars/h} \times 4 \text{ h} = 60 \text{ dollars}.
\]

This is the bill for labor charged by the plumber for 4 hours of work.

9. Because \(-2\) is less than 0, we use the first piece of the function to determine that \( f(-2) = 0 \). Because 0 is greater than or equal to zero, we use the second piece of the function to determine that \( f(0) = 2 \). Finally, because 3 is greater than or equal to zero, we use the second piece of the function to determine that \( f(3) = 2 \). The graph follows.
Chapter 4 Absolute Value Functions

The domain of \( f \) is the set of all real numbers, easily seen by examining the piecewise definition or by projecting all points on the graph onto the \( x \)-axis. The range has only a finite number of possibilities, so the range is best described by listing each member.

\[
\text{Range} = \{0, 2\}
\]

11. Because \(-3\) is less than \(-2\), we use the first piece of the function to determine that \( g(-3) = -3 \). Because \(-2\) is greater than or equal to \(-2\) and less than 2, we use the second piece of the function to determine that \( g(-2) = 1 \). Finally, because 5 is greater than or equal to 2, we use the third piece of the function to determine that \( g(5) = 3 \). The graph follows.

The domain of \( g \) is the set of all real numbers, easily seen by examining the piecewise definition or by projecting all points on the graph onto the \( x \)-axis. The range has only a finite number of possibilities, so the range is best described by listing each member.

\[
\text{Range} = \{-3, 1, 3\}
\]
13. Here is the graph of $f$.

From the graph of $f$, if $x < 0$, then $f(x) = 3$. On the other hand, if $x \geq 0$, then $f(x) = -2$. Consequently,

$$f(x) = \begin{cases} 3, & \text{if } x < 0, \\ -2, & \text{if } x \geq 0. \end{cases}$$

The domain of $f$ is the set of all real numbers. The range of $f$ is $\{-2, 3\}$.

15. Here is the graph of $f$.

From the graph, if $x < 0$, then $g(x) = 2$. Secondly, if $0 \leq x < 2$, then $g(x) = -2$. Thirdly, if $x \geq 2$, then $g(x) = 2$. Consequently,

$$g(x) = \begin{cases} 2, & \text{if } x < 0, \\ -2, & \text{if } 0 \leq x < 2, \\ 2, & \text{if } x \geq 2. \end{cases}$$

The domain of $f$ is the set of all real numbers. The range of $f$ is $\{-2, 2\}$.

17. We’re given the following piecewise definition.

$$f(x) = \begin{cases} -x - 3, & \text{if } x < -3, \\ x + 3, & \text{if } x \geq -3. \end{cases}$$
Note that $-4 < -3$, so to evaluate $f(-4)$, we should substitute into the first piece of this function, namely

\[ f(x) = -x - 3 \]

\[ f(-4) = -(4) - 3 = 1. \] (1)

Note that $0 \geq -3$, so to evaluate $f(0)$, we should substitute into the second piece of this function, namely

\[ f(x) = x + 3 \]

\[ f(0) = 0 + 3 = 3. \]

The first part of the function is $f(x) = -x - 3$, but only for $x < -3$. Hence, this is a ray, starting at the point where $x = -3$ and moving to the left. At $x = -3$, $f(-3) = -(3) - 3 = 0$, so the starting point of the ray is at $(-3, 0)$. We have already found that $f(-4) = 1$, so this gives us a second point on the ray, namely $(-4, 1)$. Plot these two points, then draw the ray starting at $(-3, 0)$ and passing through $(-4, 1)$ as it moves to the left, as shown in (a) below. Note that the point at $(-3, 0)$ is empty, because $f(x) = -x - 3$ only if $x < -3$.

The second part of the function is $f(x) = x + 3$, but only for $x \geq -3$. Hence, this is a ray, starting at the point where $x = -3$ and moving to the right. At $x = -3$, $f(-3) = (-3) + 3 = 0$, so the starting point of the ray is at $(-3, 0)$. We have already found that $f(0) = 3$, so this gives us a second point on the ray, namely $(0, 3)$. Plot these two points, then draw the ray starting at $(-3, 0)$ and passing through $(0, 3)$ as it moves to the right, as shown in (b) below. Note that the point at $(-3, 0)$ is filled, because $f(x) = x + 3$ if $x \geq -3$.

Finally, put these two pieces together to form the graph of $f$ shown in (c) below.

The domain of $f$ is the set of all real numbers. The range of $f$ is $\{y : y \geq 0\}$. 

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19. We’re given the following piecewise definition.

\[ g(x) = \begin{cases} 
-2x + 3, & \text{if } x < 3/2, \\
2x - 3, & \text{if } x \geq 3/2, 
\end{cases} \]

Note that \(0 < 3/2\), so to evaluate \(g(0)\), we should substitute into the first piece of this function, namely

\[ g(x) = -2x + 3 \]
\[ g(0) = -2(0) + 3 \]
\[ g(0) = 3. \] (2)

Note that \(3 \geq 3/2\), so to evaluate \(g(3)\), we should substitute into the second piece of this function, namely

\[ g(x) = 2x - 3 \]
\[ g(3) = 2(3) - 3 \]
\[ g(3) = 3. \]

The first part of the function is \(g(x) = -2x + 3\), but only for \(x < 3/2\). Hence, this is a ray, starting at the point where \(x = 3/2\) and moving to the left. At \(x = 3/2\), 
\[ g(3/2) = -2(3/2) + 3 = 0, \] so the starting point of the ray is at \((3/2,0)\). We have already found that \(g(0) = 3\), so this gives us a second point on the ray, namely \((0,3)\). Plot these two points, then draw the ray starting at \((3/2,0)\) and passing through \((3,0)\) as it moves to the left, as shown in (a) below. Note that the point at \((3/2,0)\) is empty, because \(g(x) = -2x + 3\) only if \(x < 3/2\).

The second part of the function is \(g(x) = 2x - 3\), but only for \(x \geq 3/2\). Hence, this is a ray, starting at the point where \(x = 3/2\) and moving to the right. At \(x = 3/2\), 
\[ g(3/2) = 2(3/2) - 3 = 0, \] so the starting point of the ray is at \((3/2,0)\). We have already found that \(g(3) = 3\), so this gives us a second point on the ray, namely \((3,3)\). Plot these two points, then draw the ray starting at \((-3,0)\) and passing through \((3,3)\) as it moves to the right, as shown in (b) below. Note that the point at \((3/2,0)\) is filled, because \(g(x) = 2x - 3\) for \(x \geq 3/2\).

Finally, put these two pieces together to form the graph of \(g\) shown in (c) below.
The domain of $g$ is the set of all real numbers. The range of $g$ is $\{y : y \geq 0\}$.

21. Three facts lead to the development of the piecewise function and its graph.

- Before time $t = 0$, the switch is open and the voltage is zero. That is, $V(t) = 0$ if $t < 0$. The graph of this piece is shown in (a) below.
- At time $t = 0$ the switch is closed and remains closed until time $t = 2$ when it is again opened. During this time, the voltage is a constant 3 volts. That is, $V(t) = 3$ for $0 \leq t < 2$. The graph of this piece is shown in (b) below.
- Finally, at time $t = 2$ and thereafter, the switch remains open and the voltage is zero. That is, $V(t) = 0$ for $t \geq 2$. The graph of this piece is shown in (c).

Putting the pieces together that are described above gives the following piecewise definition.

$$V(t) = \begin{cases} 
0, & \text{if } t < 0, \\
3, & \text{if } 0 \leq t < 2, \\
0, & \text{if } t \geq 2.
\end{cases}$$

The complete graph of $V$ follows.