4.4 Exercises

For each of the inequalities in Exercises 1-10, perform each of the following tasks.

i. Set up a coordinate system on a sheet of graph paper. Label and scale each axis.

ii. Sketch the graph of each side of the inequality without the aid of a calculator. Label each graph with its equation.

iii. Shade the solution of the inequality on the $x$-axis (if any) in the manner shown in Figures 4 and 8 in the narrative. That is, drop dashed lines from the points of intersection to the axis, then shade and label the solution set on the $x$-axis. Use set-builder and interval notation (when possible) to describe your solution set.

1. $|x| > -2$

2. $|x| > 0$

3. $|x| < 3$

4. $|x| > 2$

5. $|x| > 1$

6. $|x| < 4$

7. $|x| \leq 0$

8. $|x| \leq -2$

9. $|x| \leq 2$

10. $|x| \geq 1$

11. $|3 - 2x| > 5$

12. $|2x + 7| < 4$

13. $|4x + 5| < 7$

14. $|5x - 7| > 8$

15. $|4x + 5| > -2$

16. $|3x - 5| < -3$

17. $|2x - 9| \geq 6$

18. $|3x + 25| \geq 8$

For each of the inequalities in Exercises 11-22, perform each of the following tasks.

i. Load each side of the inequality into the $Y=$ menu of your calculator. Adjust the viewing window so that all points of intersection of the two graphs are visible in the viewing window.

ii. Copy the image in your viewing screen onto your homework paper. Label each axis and scale each axis with $\text{xmin}$, $\text{xmax}$, $\text{ymin}$, and $\text{ymax}$. Label each graph with its equation.

iii. Use the intersect utility in the CALC menu to determine the points of intersection. Shade the solution of the inequality on the $x$-axis (if any) in the manner shown in Figures 4 and 8 in the narrative. That is, drop dashed lines from the points of intersection to the axis, then shade and label the solution set on the $x$-axis. Use set-builder and interval notation (when appropriate) to describe your solution set.

1 Copyrighted material. See: http://msenux.redwoods.edu/IntAlgText/
19. $|13 - 2x| \leq 7$
20. $|2x + 15| \leq 7$
21. $|3x - 11| > 0$
22. $|4x + 19| \leq 0$

For each of the inequalities in Exercises 23-32, provide a purely algebraic solution without the use of a calculator. Show all of your work that leads to the solution, shade your solution set on a number line, then use set-builder and interval notation (if possible) to describe your solution set.

23. $|4x + 3| < 8$
24. $|3x - 5| > 11$
25. $|2x - 3| \leq 10$
26. $|3 - 5x| \geq 15$
27. $|3x - 4| < 7$
28. $|5 - 2x| > 10$
29. $|3 - 7x| \geq 5$
30. $|2 - 11x| \leq 6$
31. $|x + 2| \geq -3$
32. $|x + 5| < -4$

For each of the inequalities in Exercises 33-38, perform each of the following tasks.

i. Arrange each of the following parts on your homework paper in the same location. Do not do place the algebraic work on one page and the graphical work on another.
ii. Follow each of the directions given for Exercises 11-22 to find and record a solution with your graphing calculator.
iii. Provide a purely algebraic solution, showing all the steps of your work. Sketch your solution on a number line, then use set-builder and interval notation to describe your solution set. Do these solutions compare favorably with those found using your graphing calculator in part (ii)? If not, look for a mistake in your work.

33. $|x - 8| < 7$
34. $|2x - 15| > 5$
35. $|2x + 11| \geq 6$
36. $|5x - 21| \leq 7$
37. $|x - 12| > 6$
38. $|x + 11| < 5$

Use a strictly algebraic technique to solve each of the equations in Exercises 39-46. Do not use a calculator. Shade the solution set on a number line and describe the solution set using both set-builder and interval notation.

39. $|x + 2| - 3 > 4$
40. $3|x + 5| < 6$
41. $-2|3 - 2x| \leq -6$
42. $|4 - x| + 5 \geq 12$
43. $3|x + 2| - 5 > |x + 2| + 7$
44. $4 - 3|4 - x| > 2|4 - x| - 1$
45. $\frac{|x - 1|}{3} \leq \frac{1}{12}$
46. $\frac{|x - 1|}{4} \geq \frac{2}{3}$
Use the technique of distance on the number line demonstrated in Examples 21 and 22 to solve each of the inequalities in Exercises 47-50. Provide number line sketches as in Example 17 in the narrative. Describe the solution set using both set-builder and interval notation.

47. \(|x - 5| < 8\)

48. \(|x - 2| > 4\)

49. \(|x + 4| \geq 4\)

50. \(|x + 2| \leq 11\)

Use the instructions provided in Exercises 11-22 to solve the inequalities in Exercises 51-52. Describe the solution set using both set-builder and interval notation.

51. \(|x + 2| < \frac{1}{3}x + 5\)

52. \(|x - 3| > 5 - \frac{1}{2}x\)

In Exercises 53-54, perform each of the following tasks.

i. Set up a coordinate system on graph paper. Label and scale each axis.

ii. Without the use of a calculator, sketch the graphs of the left- and right-hand sides of the given inequality. Label each graph with its equation.

iii. Shade the solution of the inequality on the x-axis (if any) in the manner shown in Figures 4 and 8 in the narrative. That is, drop dashed lines from the points of intersection to the axis, then shade and label the solution set on the x-axis (you will have to approximate). Describe the solution set using both set-builder and interval notation.

53. \(|x - 2| > \frac{1}{3}x + 2\)

54. \(|x + 4| < \frac{1}{3}x + 4\)
1. Sketch the graph of $y = |x|$ on graph paper. Plotting points such as $(-5, 5)$, $(0, 0)$, and $(5, 5)$ will help provide accuracy. Sketch the graph of $y = -2$.

The question asks us to solve the inequality $|x| > -2$. Hence, we need to locate where the graph of $y = |x|$ lies above the graph of $y = -2$. This is true for all values of $x$. Thus, the solution is $\mathbb{R} = (-\infty, \infty)$. Note that this solution is shaded on the $x$-axis.

3. Sketch the graph of $y = |x|$ on graph paper. Plotting points such as $(-5, 5)$, $(0, 0)$, and $(5, 5)$ will help provide accuracy. Sketch the graph of $y = 3$. Drop dashed lines from the points of intersection to the $x$-axis and label these points with their $x$-values.

To find the solution of $|x| < 3$, note where the graph of $y = |x|$ lies below the graph of $y = 3$. This occurs for all values of $x$ between $-3$ and $3$. This set is shaded on the $x$-axis and described with the following notation: $(-3, 3) = \{x : -3 < x < 3\}$.

5. Sketch the graph of $y = |x|$ on graph paper. Plotting points such as $(-5, 5)$, $(0, 0)$, and $(5, 5)$ will help provide accuracy. Sketch the graph of $y = 1$. Drop dashed lines from the points of intersection to the $x$-axis and label these points with their $x$-values.
To find the solution of $|x| > 1$, note where the graph of $y = |x|$ lies above the graph of $y = 1$. This occurs for values of $x$ that lie to the left of $-1$ or to the right of $1$. This set is shaded on the $x$-axis and described with the following notation: $(-\infty, -1) \cup (1, \infty)$ or \{ $x : x < -1$ or $x > 1$ \}.

7. Sketch the graph of $y = |x|$ on graph paper. Plotting points such as $(-5, 5)$, $(0, 0)$, and $(5, 5)$ will help provide accuracy. Sketch the graph of $y = 0$. This is a somewhat unusual case as the two graphs intersect at only one point, namely $x = 0$.

To find the solution of $|x| \leq 0$, we need to find where the graph of $y = |x|$ lies below the line $y = 0$ (this never happens) or where the graph of $y = |x|$ intersects the graph of $y = 0$ (this happens at only one place, $x = 0$). Thus, the solution of $|x| \leq 0$ is $x = 0$. That is why you only see $x = 0$ shaded on the $x$-axis. In set-builder notation, the solution is \{ $x : x = 0$ \}.

9. Sketch the graph of $y = |x|$ on graph paper. Plotting points such as $(-5, 5)$, $(0, 0)$, and $(5, 5)$ will help provide accuracy. Sketch the graph of $y = 2$. Drop dashed lines from the points of intersection to the $x$-axis and label these points with their $x$-values.
To find the solution of $|x| \leq 2$, we need to find where the graph of $y = |x|$ lies below the graph of $y = 2$ (this happens for all values of $x$ between $-2$ and $2$) or where the graph of $y = |x|$ intersects the graph of $y = 2$ (this happens at $x = -2$ and $x = 2$). Hence, we shade on the $x$-axis all points that lie between $-2$ and $2$, then we shade the points $-2$ and $2$ as well. This solution set is described with the following notation: $[-2, 2] = \{x : -2 \leq x \leq 2\}$.

11. Load $y = |3 - 2x|$ into $Y1$ and $y = 5$ into $Y2$, as shown in (a). Use the intersect utility from the CALC menu to determine the points of intersection shown in (b) and (c).

Copy the image onto your homework paper. Drop dashed lines from the points of intersection to the $x$-axis and label the $x$-values.
We’re asked to solve $|3 - 2x| > 5$, so we must find where the graph of $y = |3 - 2x|$ lies **above** the graph of $y = 5$. This happens for all values of $x$ that lie to the left of $-1$ or to the right of $4$, which we’ve shaded on the $x$-axis. This solution set is described with the following notation: $(-\infty, -1) \cup (4, \infty)$ or $\{x : x < -1 \text{ or } x > 4\}$.

13. Load $y = |4x + 5|$ into Y1 and $y = 7$ into Y2, as shown in (a). Use the `intersect` utility from the `CALC` menu to determine the points of intersection shown in (b) and (c).

Copy the image onto your homework paper. Drop dashed lines from the points of intersection to the $x$-axis and label the $x$-values.

We’re asked to solve $|4x + 5| < 7$, so we must find where the graph of $y = |4x + 5|$ lies **below** the graph of $y = 7$. This happens for all values of $x$ that lie between $-3$ and $0.5$, which we’ve shaded on the $x$-axis. This solution set is described with the following notation: $(-3, 0.5) = \{x : -3 < x < 0.5\}$. 
15. Load \( y = |4x + 5| \) into Y1 and \( y = -2 \) into Y2, as shown in (a). Note that the graph of \( y = |4x + 5| \) does not intersect the graph of \( y = -2 \), as shown in (b).

![Graphs](image1)

Copy the image onto your homework paper.

We’re asked to solve \( |4x + 5| > -2 \), so we must find where the graph of \( y = |4x + 5| \) lies above the graph of \( y = -2 \). This is true for all values of \( x \), which we’ve shaded on the x-axis. This solution set is best described with \( \mathbb{R} = (-\infty, \infty) \).

17. Load \( y = |2x - 9| \) into Y1 and \( y = 6 \) into Y2, as shown in (a). Adjust the WINDOW parameters as shown in (b). Use the intersect utility from the CALC menu to determine the points of intersection shown in (c) and (d).

![Graphs](image2)

Copy the image onto your homework paper. Drop dashed lines from the points of intersection to the x-axis and label the x-values.
We’re asked to solve \(|2x - 9| \geq 6\), so we must find where the graph of \(y = |2x - 9|\) lies above the graph of \(y = 6\) (this happens for all values of \(x\) that lie to the left of 1.5 or to the right of 7.5), or where the graph of \(y = |2x - 9|\) intersects the graph of \(y = 6\) (this happens at \(x = 1.5\) and \(x = 7.5\)). We’ve shaded these on the \(x\)-axis. This solution set is described with the following notation: \((-\infty, 1.5] \cup [7.5, \infty)\) or \(\{x : x \leq 1.5 \text{ or } x \geq 7.5\}\).

19. Load \(y = |13 - 2x|\) into \(Y1\) and \(y = 7\) into \(Y2\), as shown in (a). Adjust the WINDOW parameters as shown in (b). Use the intersect utility from the CALC menu to determine the points of intersection shown in (c) and (d).

Copy the image onto your homework paper. Drop dashed lines from the points of intersection to the \(x\)-axis and label the \(x\)-values.
We’re asked to solve $|13 - 2x| \leq 7$, so we must find where the graph of $y = |13 - 2x|$ lies below the graph of $y = 7$ (this happens for all values of $x$ that lie between 3 and 10), or where the graph of $y = |13 - 2x|$ intersects the graph of $y = 7$ (this happens at $x = 3$ and $x = 10$). We’ve shaded these on the $x$-axis. This solution set is described with the following notation: $[3, 10] = \{x : 3 \leq x \leq 10\}$.

21. Load $y = |3x - 11|$ into Y1, then select 6:ZStandard on the ZOOM menu to produce the image shown in (b).

![Graph of |3x-11| and y=7](image)

Copy the image onto your homework paper.

We’re asked to solve $|3x - 11| > 0$, so we must find where the graph of $y = |3x - 11|$ lies above the $x$-axis. This is true for all values of $x$ except where the vertex touches the $x$-axis. This point is easily found with this calculation.

\[
3x - 11 = 0
\]
\[
3x = 11
\]
\[
x = \frac{11}{3}
\]

Thus, the graph of $y = |3x - 11|$ lies above the $x$-axis for all values of $x$ except $11/3$. This solution set is described with the following notation: $\{x : x \neq 11/3\}$.

23. To solve $|4x + 3| < 8$, set

\[-8 < 4x + 3 < 8,\]

subtract 3 from all three members, then divide all three members of the resulting inequality by 4.
25. To solve $|2x - 3| \leq 10$, set

\[-10 \leq 2x - 3 \leq 10,\]

add 3 to all three members, then divide all three members of the resulting inequality by 2.

\[-7 \leq 2x \leq 13\]

\[-\frac{7}{2} \leq x \leq \frac{13}{2}\]

Sketch the solution on a number line.

\[\frac{-7}{2} \quad \frac{13}{2}\]

The solution set is described with the following notation: $[-7/2, 13/2] = \{x : -7/2 \leq x \leq 13/2\}$.

27. To solve $|3x - 4| < 7$, set

\[-7 < 3x - 4 < 7,\]

add 4 to all three members, then divide all three members of the resulting inequality by 3.

\[-3 < 3x < 11\]

\[-1 < x < \frac{11}{3}\]

Sketch the solution on a number line.

\[\frac{-1}{1} \quad \frac{11}{3}\]

The solution set is described with the following notation: $(-1, 11/3) = \{x : -1 < x < 11/3\}$.
29. To solve $|3 - 7x| \geq 5$, set

$$3 - 7x \leq -5 \quad \text{or} \quad 3 - 7x \geq 5.$$ 

Solve each inequality independently by first subtracting 3 from each side of each inequality, then dividing both sides of each inequality by $-7$, reversing the inequality symbols as we do so.

$$-7x \leq -8 \quad \text{or} \quad -7x \geq 2$$

$$x \geq \frac{8}{7} \quad \text{or} \quad x \leq -\frac{2}{7}$$

We write the last inequality in the more natural order $x \leq -2/7$ or $x \geq 8/7$ and sketch the solution on a number line.

$$\frac{-2}{7} \quad \frac{8}{7}$$

We describe the solution with the following notation: $(-\infty, -2/7] \cup [8/7, \infty)$ or \{ $x : x \leq -2/7$ or $x \geq 8/7$ \}

31. To solve the inequality $|x + 2| \geq -3$, it is easiest to reason that the absolute value will be greater than or equal to $-3$ for all values of $x$. Sketch the solution on a number line.

$$-\infty \quad \infty$$

Hence, the following notation is used to describe the solution set: $\mathbb{R} = (-\infty, \infty)$.

33. Load $y = |x - 8|$ in $Y1$ and $y = 7$ in $Y2$, as shown in (a). Set the WINDOW parameters as shown in (b). Use the intersect utility from the CALC menu to find the points of intersection shown in (c) and (d).

Copy the image onto your homework paper. Drop dashed lines from the points of intersection to the $x$-axis and label the $x$-values.
To find the solution of $|x - 8| < 7$ note that the graph of $y = |x - 8|$ falls below the graph of $y = 7$ for all values of $x$ between 1 and 15. We’ve shaded these solutions on the $x$-axis.

To solve $|x - 8| < 7$ algebraically, set

$$-7 < x - 8 < 7,$$

then add 8 to all three members of the inequality.

$$1 < x < 15$$

Note that this solution matches the graphical solution found above. We describe the solution using the following notation: $(1, 15) = \{x : 1 < x < 15\}$

35. Load $y = |2x + 11|$ in $Y_1$ and $y = 6$ in $Y_2$, as shown in (a). Set the WINDOW parameters as shown in (b). Use the intersect utility from the CALC menu to find the points of intersection shown in (c) and (d).

Copy the image onto your homework paper. Drop dashed lines from the points of intersection to the $x$-axis and label the $x$-values.
To find the solution of $|2x + 11| \geq 6$, note where the graph of $y = |2x + 11|$ falls above the graph of $y = 6$ (this happens for values of $x$ that lie to the left of $-8.5$ or to the right of $-2.5$), then note where the graph of $y = |2x + 11|$ intersects the graph of $y = 6$ (this happens at $x = -8.5$ and $x = -2.5$). We’ve shaded these solutions on the $x$-axis. To solve $|2x + 11| \geq 6$ algebraically, set

$$2x + 11 \leq -6 \quad \text{or} \quad 2x + 11 \geq 6.$$

Solve each inequality independently by first subtracting 11 from each side of each inequality, then dividing both sides of each inequality by 2.

$$2x \leq -17 \quad \text{or} \quad 2x \geq -5$$

$$x \leq -\frac{17}{2} \quad \text{or} \quad x \geq -\frac{5}{2}$$

Note that this solution matches the graphical solution found above. We describe the solution using the following notation: $(-\infty, -17/2] \cup [-5/2, \infty)$ or $\{x : x \leq -17/2 \text{ or } x \geq -5/2\}$.

37. Load $y = |x - 12|$ in Y1 and $y = 6$ in Y2, as shown in (a). Set the WINDOW parameters as shown in (b). Use the intersect utility from the CALC menu to find the points of intersection shown in (c) and (d).
To find the solution of $|x - 12| > 6$, note where the graph of $y = |x - 12|$ lies above the graph of $y = 6$ (this happens for values of $x$ that lie to the left of 6 or to the right of 18). We’ve shaded these solutions on the $x$-axis.

To solve $|x - 12| > 6$ algebraically, set

$$x - 12 < -6 \quad \text{or} \quad x - 12 > 6.$$ 

Solve each inequality independently by adding 12 to each side of each inequality.

$$x < 6 \quad \text{or} \quad x > 18$$

Note that this solution matches the graphical solution found above. We describe the solution using the following notation:

$$(-\infty, 6) \cup (18, \infty)$$

or

$$\{x : x < 6 \text{ or } x > 18\}.$$

39. To solve $|x + 2| - 3 > 4$, start by adding 3 to both sides of the inequality to produce the equivalent inequality

$$|x + 2| > 7.$$ 

Next, set

$$x + 2 < -7 \quad \text{or} \quad x + 2 > 7.$$ 

Solve each inequality independently by subtracting 2 from each side of each inequality.

$$x < -9 \quad \text{or} \quad x > 5$$

Sketch the solution on a number line.

We describe the solution set with the following notation:

$$(-\infty, -9) \cup (5, \infty)$$

or

$$\{x : x < -9 \text{ or } x > 5\}.$$

41. To solve the inequality $-2|3 - 2x| \leq -6$, start by dividing both sides of the inequality by $-2$, reversing the inequality symbol.

$$|3 - 2x| \geq 3$$

Set
Chapter 4  Absolute Value Functions

\[ 3 - 2x \leq -3 \quad \text{or} \quad 3 - 2x \geq 3. \]

Solve each inequality independently by first subtracting 3 from each side of each inequality, then dividing both sides of each inequality by \(-2\), reversing the inequality symbols as we do so.

\[ -2x \leq -6 \quad \text{or} \quad -2x \geq 0 \]
\[ x \geq 3 \quad \quad x \leq 0 \]

We write this solution in a more natural order using the following notation: \((-\infty, 0] \cup [3, \infty)\) or \(\{x : x \leq 0 \text{ or } x \geq 3\}\).

Sketch this solution on a number line.

43. To solve the inequality \(3|x + 2| - 5 > |x + 2| + 7\), first add 5 to both sides of the inequality, then subtract \(|x + 2|\) from both sides of the inequality.

\[ 3|x + 2| - |x + 2| > 7 + 5 \]
\[ 2|x + 2| > 12 \]

Divide both sides of the last inequality by 2.

\[ |x + 2| > 6 \]

Set

\[ x + 2 < -6 \quad \text{or} \quad x + 2 > 6. \]

Solve each inequality independently by subtracting 2 from each side of each inequality.

\[ x < -8 \quad \text{or} \quad x > 4 \]

Sketch the solution on a number line.

45. To solve the inequality \(|x/3 - 1/4| \leq 1/12\), first multiply both sides of the inequality by 12.
Section 4.4 Absolute Value Inequalities

\[ \left| \frac{x}{3} - \frac{1}{4} \right| \leq 12 \left( \frac{1}{12} \right) \]

\[ \left| 12 \left( \frac{x}{3} - \frac{1}{4} \right) \right| \leq 1 \]

Set

\[ -1 \leq 4x - 3 \leq 1, \]

add 3 to all three members, then divide all three members of the resulting inequality by 4.

\[ \frac{2}{4} \leq 4x \leq 4 \]

\[ \frac{1}{2} \leq x \leq 1 \]

Sketch the solution on a number line.

We describe this solution set using the notation: \([1/2, 1] = \{x : 1/2 \leq x \leq 1\}\).

47. The inequality \(|x - 5| < 8\) is pronounced “the distance between \(x\) and 5 is less than 8.” Draw a number line and mark 5 on the line. Next, mark -3 and 13, both of which are 8 units away from 5.

\[ -3 \quad 5 \quad 13 \]

We want the numbers that are less than 8 units away from 5. These are the numbers that lie between -3 and 13, which are shaded on the number line above. This solution set is described with the following notation: \((-3, 13) = \{x : -3 < x < 13\}\)

49. First, the inequality \(|x + 4| \geq 3\) is equivalent to the inequality \(|x - (-4)| \geq 3\). This latter inequality is pronounced “the distance between \(x\) and -4 is greater than or equal to 3.” Draw a number line and mark -4 on the line. Next, mark -7 and -1, both of which are 3 units away from -4.

\[ -7 \quad -4 \quad -1 \]
We want the numbers that are more than 3 units away from $-1$. These are the numbers that lie to the left of $-7$ or to the right of $-1$. We also need the numbers that are exactly 3 units away from $-4$, namely $-7$ and $-1$, which are shaded on the number line above. This solution set is described with the following notation: $(-\infty, -7] \cup [-1, \infty)$ or $\{x : x \leq -7 \text{ or } x \geq -1\}$.

51. Load the equations $y = |x + 2|$ into $Y1$ and $y = (1/3)x + 5$ into $Y2$, as shown in (a), then select 6:ZStandard from the ZOOM menu. Use the intersect utility from the CALC menu to determine the points of intersection, as shown in (b) and (c).

Copy the image onto your homework. Drop dashed vertical lines from the points of intersection to the $x$-axis and label the $x$-values.

Note that the graph of $y = |x + 2|$ lies below the graph of $y = (1/3)x + 5$ for all values of $x$ that lie between $-5.25$ and $4.5$. These solutions are shaded on the $x$-axis above and are described with the following notation: $(-5.25, 4.5) = \{x : -5.25 < x < 4.5\}$.

53. There are a number of ways that you can draw an accurate graph of $y = |x - 2|$. One, you can do a number line analysis.

This leads to the piecewise definition

$$y = \begin{cases} 
-x + 2, & \text{if } x < 2, \\
-x - 2, & \text{if } x \geq 2.
\end{cases}$$

This can be used to draw the graph of $y = |x - 2|$ in the figure that follows. Alternatively, we know that $y = |x - 2|$ is a “V” that is shifted 2 units to the right. Plotting a point on each side of the vertex point should lead to the graph shown below.

The graph of $y = (1/3)x + 2$ is a line having slope $m = 1/3$ and $y$-intercept $(0, 2)$. Plot the $y$-intercept at $(0, 2)$, then move 3 units to the right and 1 unit up to draw the line shown below.
The graphs intersect in two locations. Drop dashed lines from these points of intersection to the \(x\)-axis and label the \(x\)-values as shown in the figure that follows.

Finally, we need to state where the graph of \(y = |x - 2|\) lies above the graph of \(y = (1/3)x + 2\) (this happens for values of \(x\) that lie to the left of 0 or to the right of 6). These solutions are shaded on the \(x\)-axis in the figure above and can be described using the following notation: \((-\infty, 0) \cup (6, \infty)\) or \(\{x : x < 0 \text{ or } x > 6\}\).