5.1 Exercises

In Exercises 1–6, sketch the image of your calculator screen on your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax. Label each graph with its equation. Remember to use a ruler to draw all lines, including axes.

1. Use your graphing calculator to sketch the graphs of \( f(x) = x^2 \), \( g(x) = 2x^2 \), and \( h(x) = 4x^2 \) on one screen. Write a short sentence explaining what you learned in this exercise.

2. Use your graphing calculator to sketch the graphs of \( f(x) = -x^2 \), \( g(x) = -2x^2 \), and \( h(x) = -4x^2 \) on one screen. Write a short sentence explaining what you learned in this exercise.

3. Use your graphing calculator to sketch the graphs of \( f(x) = x^2 \), \( g(x) = (x - 2)^2 \), and \( h(x) = (x - 4)^2 \) on one screen. Write a short sentence explaining what you learned in this exercise.

4. Use your graphing calculator to sketch the graphs of \( f(x) = x^2 \), \( g(x) = (x + 2)^2 \), and \( h(x) = (x + 4)^2 \) on one screen. Write a short sentence explaining what you learned in this exercise.

5. Use your graphing calculator to sketch the graphs of \( f(x) = x^2 \), \( g(x) = x^2 + 2 \), and \( h(x) = x^2 + 4 \) on one screen. Write a short sentence explaining what you learned in this exercise.

6. Use your graphing calculator to sketch the graphs of \( f(x) = x^2 \), \( g(x) = x^2 - 2 \), and \( h(x) = x^2 - 4 \) on one screen. Write a short sentence explaining what you learned in this exercise.

In Exercises 7–14, write down the given quadratic function on your homework paper, then state the coordinates of the vertex.

7. \( f(x) = -5(x - 4)^2 - 5 \)

8. \( f(x) = 5(x + 3)^2 - 7 \)

9. \( f(x) = 3(x + 1)^2 \)

10. \( f(x) = \frac{7}{5}(x + \frac{5}{9})^2 - \frac{3}{4} \)

11. \( f(x) = -7(x - 4)^2 + 6 \)

12. \( f(x) = -\frac{1}{2}(x - \frac{8}{9})^2 + \frac{2}{9} \)

13. \( f(x) = \frac{1}{6}(x + \frac{7}{3})^2 + \frac{3}{8} \)

14. \( f(x) = -\frac{3}{2}(x + \frac{1}{2})^2 - \frac{8}{9} \)

In Exercises 15–22, state the equation of the axis of symmetry of the graph of the given quadratic function.

15. \( f(x) = -7(x - 3)^2 + 1 \)

16. \( f(x) = -6(x + 8)^2 + 1 \)

---

1 Copyrighted material. See: http://msenux.redwoods.edu/IntAlgText/

Version: Fall 2007
In Exercises 23–36, perform each of the following tasks for the given quadratic function.

i. Set up a coordinate system on graph paper. Label and scale each axis.
ii. Plot the vertex of the parabola and label it with its coordinates.
iii. Draw the axis of symmetry and label it with its equation.
iv. Set up a table near your coordinate system that contains exact coordinates of two points on either side of the axis of symmetry. Plot them on your coordinate system and their “mirror images” across the axis of symmetry.
v. Sketch the parabola and label it with its equation.
vi. Use interval notation to describe both the domain and range of the quadratic function.

23. \( f(x) = (x + 2)^2 - 3 \)
24. \( f(x) = (x - 3)^2 - 4 \)
25. \( f(x) = -(x - 2)^2 + 5 \)
26. \( f(x) = -(x + 4)^2 + 4 \)
27. \( f(x) = (x - 3)^2 \)
28. \( f(x) = -(x + 2)^2 \)
29. \( f(x) = -x^2 + 7 \)
30. \( f(x) = -x^2 + 7 \)
31. \( f(x) = 2(x - 1)^2 - 6 \)
32. \( f(x) = -2(x + 1)^2 + 5 \)
33. \( f(x) = -\frac{1}{2}(x + 1)^2 + 5 \)
34. \( f(x) = \frac{1}{2}(x - 3)^2 - 6 \)
35. \( f(x) = 2(x - 5/2)^2 - 15/2 \)
36. \( f(x) = -3(x + 7/2)^2 + 15/4 \)

In Exercises 37–44, write the given quadratic function on your homework paper, then use set-builder and interval notation to describe the domain and the range of the function.

37. \( f(x) = 7(x + 6)^2 - 6 \)
38. \( f(x) = 8(x + 1)^2 + 7 \)
39. \( f(x) = -3(x + 4)^2 - 7 \)
40. \( f(x) = -6(x - 7)^2 + 9 \)
41. \( f(x) = -7(x + 5)^2 - 7 \)
42. \( f(x) = 8(x - 4)^2 + 3 \)
43. \( f(x) = -4(x - 1)^2 + 2 \)
44. \( f(x) = 7(x - 2)^2 - 3 \)
In Exercises 45-52, using the given value of \( a \), find the specific quadratic function of the form \( f(x) = a(x - h)^2 + k \) that has the graph shown. Note: \( h \) and \( k \) are integers. Check your solution with your graphing calculator.

45. \( a = -2 \)

46. \( a = 0.5 \)

47. \( a = 2 \)

48. \( a = 0.5 \)

49. \( a = 2 \)

50. \( a = -0.5 \)
In Exercises 53-54, use the graph to determine the range of the function \( f(x) = ax^2 + bx + c \). The arrows on the graph are meant to indicate that the graph continues indefinitely in the continuing pattern and direction of each arrow. Describe your solution using interval notation.

**53.**

**54.**

In Exercises 55-56, use the graph to determine the domain of the function \( f(x) = ax^2 + bx + c \). The arrows on the graph are meant to indicate that the graph continues indefinitely in the continuing pattern and direction of each arrow. Use interval notation to describe your solution.

**55.**

**56.**
5.1 Solutions

1. First, enter the functions into the Y= menu. Then press GRAPH to view a comparison of the three graphs.

Note how the graph of \( y = 2x^2 \) is narrower and taller than the graph of \( y = x^2 \), and the graph of \( y = 4x^2 \) is narrower and taller still. We see therefore that multiplying \( x^2 \) by a positive number such as 2 or 4 stretches or scales the graph vertically, making it taller (and narrower).

3. First, enter the functions into the Y= menu. Then press GRAPH to view a comparison of the three graphs.

Note how the graph of \( g(x) = (x-2)^2 \) has the same shape as the graph of \( f(x) = x^2 \) but it is shifted 2 units to the right; and the graph of \( h(x) = (x-4)^2 \) also has the same shape, but is shifted 4 units to the right. It thus appears that \( y = (x-c)^2 \), for positive \( c \), has the same shape as \( f(x) = x^2 \) but is shifted \( c \) units to the right.
5. First, enter the functions into the Y= menu. Then press GRAPH to view a comparison of the three graphs.

Note how the graph of \( g(x) = x^2 + 2 \) has the same shape as the graph of \( f(x) = x^2 \) but it is shifted 2 units up; and the graph of \( h(x) = x^2 + 4 \) also has the same shape, but is shifted 4 units up.

It thus appears that \( y = x^2 + k \), for a positive \( k \), has the same shape as \( f(x) = x^2 \) but is shifted \( k \) units up.

7. The function \( f(x) = -5(x - 4)^2 - 5 \) is given in vertex form \( f(x) = a(x - h)^2 + k \), where \( a = -5 \), \( h = 4 \), and \( k = -5 \). The vertex is \((h, k) = (4, -5)\).

9. The function \( f(x) = 3(x + 1)^2 \) is given in vertex form \( f(x) = a(x - h)^2 + k \), where \( a = 3 \), \( h = -1 \), and \( k = 0 \). The vertex is \((h, k) = (-1, 0)\).

11. The function \( f(x) = -7(x - 4)^2 + 6 \) is given in vertex form \( f(x) = a(x - h)^2 + k \), where \( a = -7 \), \( h = 4 \), and \( k = 6 \). The vertex is \((h, k) = (4, 6)\).

13. The function \( f(x) = \frac{1}{6}(x + \frac{7}{3})^2 + \frac{3}{2} \) is given in vertex form \( f(x) = a(x - h)^2 + k \), where \( a = 1/6 \), \( h = -7/3 \), and \( k = 3/8 \). The vertex is \((h, k) = (-7/3, 3/8)\).

15. The function \( f(x) = -7(x - 3)^2 + 1 \) is given in vertex form \( f(x) = a(x - h)^2 + k \), where \( a = -7 \), \( h = 3 \), and \( k = 1 \). The axis of symmetry is the vertical line through the vertex \( h = 3 \), so the axis of symmetry is \( x = 3 \).

17. The function \( f(x) = -\frac{7}{8}(x + \frac{1}{4})^2 + \frac{2}{3} \) is given in vertex form \( f(x) = a(x - h)^2 + k \), where \( a = -7/8 \), \( h = -1/4 \), and \( k = 2/3 \). The axis of symmetry is the vertical line through the vertex \( h = -1/4 \), so the axis of symmetry is \( x = -1/4 \).

19. The function \( f(x) = -\frac{2}{9}(x + \frac{3}{2})^2 - \frac{4}{5} \) is given in vertex form \( f(x) = a(x - h)^2 + k \), where \( a = -2/9 \), \( h = -2/3 \), and \( k = -4/5 \). The axis of symmetry is the vertical line through the vertex \( h = -2/3 \), so the axis of symmetry is \( x = -2/3 \).

21. The function \( f(x) = -\frac{8}{7}(x + \frac{3}{2})^2 + \frac{6}{5} \) is given in vertex form \( f(x) = a(x - h)^2 + k \), where \( a = -8/7 \), \( h = -2/9 \), and \( k = 6/5 \). The axis of symmetry is the vertical line through the vertex \( h = -2/9 \), so the axis of symmetry is \( x = -2/9 \).
23. First, sketch your coordinate system. Compare the quadratic function \( f(x) = (x + 2)^2 - 3 \) with \( f(x) = a(x - h)^2 + k \) and note that \( h = -2 \) and \( k = -3 \). Hence, the vertex is located at \((h, k) = (-2, -3)\). The axis of symmetry is a vertical line through the vertex with equation \( x = -2 \). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \( f(x) = (x + 2)^2 - 3 \).

\[
\begin{array}{|c|c|}
\hline
x & y = (x + 2)^2 - 3 \\
\hline
1 & -2 \\
0 & 1 \\
\hline
\end{array}
\]

To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain is \(( -\infty, \infty )\). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range \( [ -3, \infty ) \).
25. First, sketch your coordinate system. Compare the quadratic function \( f(x) = -(x - 2)^2 + 5 \) with \( f(x) = a(x - h)^2 + k \) and note that \( h = 2 \) and \( k = 5 \). Hence, the vertex is located at \((h, k) = (2, 5)\). The axis of symmetry is a vertical line through the vertex with equation \( x = 2 \).

Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \( f(x) = -(x - 2)^2 + 5 \).

\[
\begin{array}{c|c}
  x & y = -(x - 2)^2 + 5 \\
  0 & 1 \\
  1 & 4 \\
\end{array}
\]

To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain = \((-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range = \((-\infty, 5]\).
27. First, sketch your coordinate system. Compare the quadratic function \( f(x) = (x - 3)^2 \) with \( f(x) = a(x - h)^2 + k \) and note that \( h = 3 \) and \( k = 0 \). Hence, the vertex is located at \( (h, k) = (3, 0) \). The axis of symmetry is a vertical line through the vertex with equation \( x = 3 \). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \( f(x) = (x - 3)^2 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = (x - 3)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain = \((-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range = \([0, \infty)\).
29. First, sketch your coordinate system. Compare the quadratic function \( f(x) = -x^2 + 7 \) with \( f(x) = a(x - h)^2 + k \) and note that \( h = 0 \) and \( k = 7 \). Hence, the vertex is located at \((h, k) = (0, 7)\). The axis of symmetry is a vertical line through the vertex with equation \( x = 0 \). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \( f(x) = -x^2 + 7 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = -x^2 + 7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain is \((-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range = \((-\infty, 7]\).
31. First, sketch your coordinate system. Compare the quadratic function \( f(x) = 2(x - 1)^2 - 6 \) with \( f(x) = a(x - h)^2 + k \) and note that \( h = 1 \) and \( k = -6 \). Hence, the vertex is located at \((h,k) = (1,-6)\). The axis of symmetry is a vertical line through the vertex with equation \( x = 1 \). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \( f(x) = 2(x - 1)^2 - 6 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 2(x - 1)^2 - 6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-4</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain = \((-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range = \([-6, \infty)\).
33. First, sketch your coordinate system. Compare the quadratic function \( f(x) = -\frac{1}{2}(x + 1)^2 + 5 \) with \( f(x) = a(x - h)^2 + k \) and note that \( h = -1 \) and \( k = 5 \). Hence, the vertex is located at \((h, k) = (-1, 5)\). The axis of symmetry is a vertical line through the vertex with equation \( x = -1 \). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \( f(x) = -\frac{1}{2}(x + 1)^2 + 5 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = -\frac{1}{2}(x+1)^2 + 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9/2</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain = \((-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range = \((-\infty, 5]\).
35. First, sketch your coordinate system. Compare the quadratic function $f(x) = 2(x - 5/2)^2 - 15/2$ with $f(x) = a(x - h)^2 + k$ and note that $h = 5/2$ and $k = -15/2$. Hence, the vertex is located at $(h, k) = (5/2, -15/2)$. The axis of symmetry is a vertical line through the vertex with equation $x = 5/2$. Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of $f(x) = 2(x - 5/2)^2 - 15/2$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = 2(x - 5/2)^2 - 15/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>-7</td>
</tr>
</tbody>
</table>

To find the domain of $f$, mentally project every point of the graph onto the $x$-axis, as shown on the left below. This covers the entire $x$-axis, so the domain $= (-\infty, \infty)$. To find the range, mentally project every point of the graph onto the $y$-axis, as shown on the right below. The shaded interval on the $y$-axis is range $= [-15/2, \infty)$. 
37. The graph opens upward since \( a = 7 > 0 \), and the vertex is at \((h, k) = (-6, -6)\). Thus the domain is \([-\infty, \infty)\) and the range is \(\{y : y \geq -6\} = [-6, \infty)\).

39. The graph opens downward since \( a = -3 < 0 \), and the vertex is at \((h, k) = (-4, -7)\). Thus the domain is \([-\infty, \infty)\) and the range is \(\{y : y \leq -7\} = [-7, \infty)\).

41. The graph opens downward since \( a = -7 < 0 \), and the vertex is at \((h, k) = (-5, -7)\). Thus the domain is \([-\infty, \infty)\) and the range is \(\{y : y \leq -7\} = [-7, \infty)\).

43. The graph opens downward since \( a = -4 < 0 \), and the vertex is at \((h, k) = (1, 2)\). Thus the domain is \([-\infty, \infty)\) and the range is \(\{y : y \leq 2\} = [2, \infty)\).

45. Note that the parabola opens downward (see figure below). Hence, let’s start with the form \( f(x) = -2x^2 \), which is a parabola that opens downward, with vertex at the origin. Next, the parabola in the image has been shifted 3 units to the right, so we must replace \( x \) with \( x-3 \) in \( f(x) = -2x^2 \), arriving at \( f(x) = -2(x-3)^2 \). Finally, we see that the graph has been shifted 1 unit up, so we add 1 to our last form to arrive at the final answer, \( f(x) = -2(x-3)^2 + 1 \).

47. Note that the parabola opens upward (see figure below). Hence, let’s start with the form \( f(x) = 2x^2 \), which is a parabola that opens upward, with vertex at the origin. Next, the parabola in the image has been shifted 1 unit to the left, so we must replace \( x \) with \( x+1 \) in \( f(x) = 2x^2 \), arriving at \( f(x) = 2(x+1)^2 \). Finally, we see that the graph has been shifted 1 unit down, so we add -1 to our last form to arrive at the final answer, \( f(x) = 2(x+1)^2 - 1 \).
49. Note that the parabola opens upward (see figure below). Hence, let’s start with the form \( f(x) = 2x^2 \), which is a parabola that opens upward, with vertex at the origin. Next, the parabola in the image has been shifted 2 units to the left, so we must replace \( x \) with \( x + 2 \) in \( f(x) = 2x^2 \), arriving at \( f(x) = 2(x + 2)^2 \). Finally, we see that the graph has been shifted 1 unit up, so we add 1 to our last form to arrive at the final answer, \( f(x) = 2(x + 2)^2 + 1 \).

51. Note that the parabola opens upward (see figure below). Hence, let’s start with the form \( f(x) = 2x^2 \), which is a parabola that opens upward, with vertex at the origin. Next, the parabola in the image has been shifted 3 units to the right, so we must replace \( x \) with \( x - 3 \) in \( f(x) = 2x^2 \), arriving at \( f(x) = 2(x - 3)^2 \). Finally, we see that the graph has been shifted 1 unit down, so we add -1 to our last form to arrive at the final answer, \( f(x) = 2(x - 3)^2 - 1 \).
53. To find the range of \( f(x) = ax^2 + bx + c \), examine the graph and mentally project each point of the graph onto the \( y \)-axis (see figure below). Note that the arrows on the ends of the blue graph imply that the blue graph opens downward and to the left and right indefinitely. Thus, the range is all real numbers less than or equal to \(-2\), or in interval notation, \((-\infty, -2]\).

55. To find the domain of \( f(x) = ax^2 + bx + c \), examine the graph and mentally project each point of the graph onto the \( x \)-axis (see figure below). Note that the arrows on the ends of the blue graph imply that the blue graph opens downward and to the left and right indefinitely. Thus, the domain is all real numbers, or in interval notation, \((-\infty, \infty)\).
5.2 Exercises

In Exercises 1-8, expand the binomial.

1. \((x + \frac{4}{5})^2\)

2. \((x - \frac{4}{5})^2\)

3. \((x + 3)^2\)

4. \((x + 5)^2\)

5. \((x - 7)^2\)

6. \((x - \frac{2}{5})^2\)

7. \((x - 6)^2\)

8. \((x - \frac{5}{2})^2\)

In Exercises 9-16, factor the perfect square trinomial.

9. \(x^2 - \frac{6}{5}x + \frac{9}{25}\)

10. \(x^2 + 5x + \frac{25}{4}\)

11. \(x^2 - 12x + 36\)

12. \(x^2 + 3x + \frac{9}{4}\)

13. \(x^2 + 12x + 36\)

14. \(x^2 - \frac{3}{2}x + \frac{9}{16}\)

15. \(x^2 + 18x + 81\)

16. \(x^2 + 10x + 25\)

In Exercises 17-24, transform the given quadratic function into vertex form \(f(x) = (x - h)^2 + k\) by completing the square.

17. \(f(x) = x^2 - x + 8\)

18. \(f(x) = x^2 + x - 7\)

19. \(f(x) = x^2 - 5x - 4\)

20. \(f(x) = x^2 + 7x - 1\)

21. \(f(x) = x^2 + 2x - 6\)

22. \(f(x) = x^2 + 4x + 8\)

23. \(f(x) = x^2 - 9x + 3\)

24. \(f(x) = x^2 - 7x + 8\)

In Exercises 25-32, transform the given quadratic function into vertex form \(f(x) = a(x - h)^2 + k\) by completing the square.

25. \(f(x) = -2x^2 - 9x - 3\)

26. \(f(x) = -4x^2 - 6x + 1\)

27. \(f(x) = 5x^2 + 5x + 5\)

28. \(f(x) = 3x^2 - 4x - 6\)

29. \(f(x) = 5x^2 + 7x - 3\)

30. \(f(x) = 5x^2 + 6x + 4\)

31. \(f(x) = -x^2 - x + 4\)

32. \(f(x) = -3x^2 - 6x + 4\)

---

1 Copyrighted material. See: http://msenux.redwoods.edu/IntAlgText/
In Exercises 33-38, find the vertex of the graph of the given quadratic function.

33. \( f(x) = -2x^2 + 5x + 3 \)
34. \( f(x) = x^2 + 5x + 8 \)
35. \( f(x) = -4x^2 - 4x + 1 \)
36. \( f(x) = 5x^2 + 7x + 8 \)
37. \( f(x) = 4x^2 + 2x + 8 \)
38. \( f(x) = x^2 + x - 7 \)

In Exercises 39-44, find the axis of symmetry of the graph of the given quadratic function.

39. \( f(x) = -5x^2 - 7x - 8 \)
40. \( f(x) = x^2 + 6x + 3 \)
41. \( f(x) = -2x^2 - 5x - 8 \)
42. \( f(x) = -x^2 - 6x + 2 \)
43. \( f(x) = -5x^2 + x + 6 \)
44. \( f(x) = x^2 - 9x - 6 \)

For each of the quadratic functions in Exercises 45-66, perform each of the following tasks.

i. Use the technique of completing the square to place the given quadratic function in vertex form.

ii. Set up a coordinate system on a sheet of graph paper. Label and scale each axis.

iii. Draw the axis of symmetry and label it with its equation. Plot the vertex and label it with its coordinates.

iv. Set up a table near your coordinate system that calculates the coordinates of two points on either side of the axis of symmetry. Plot these points and their mirror images across the axis of symmetry. Draw the parabola and label it with its equation.

v. Use the graph of the parabola to determine the domain and range of the quadratic function. Describe the domain and range using interval notation.

45. \( f(x) = x^2 - 8x + 12 \)
46. \( f(x) = x^2 + 4x - 1 \)
47. \( f(x) = x^2 + 6x + 3 \)
48. \( f(x) = x^2 - 4x + 1 \)
49. \( f(x) = x^2 - 2x - 6 \)
50. \( f(x) = x^2 + 10x + 23 \)
51. \( f(x) = -x^2 + 6x - 4 \)
52. \( f(x) = -x^2 - 6x - 3 \)
53. \( f(x) = -x^2 - 10x - 21 \)
54. \( f(x) = -x^2 + 12x - 33 \)
55. \( f(x) = 2x^2 - 8x + 3 \)
56. \( f(x) = 2x^2 + 8x + 4 \)
57. \( f(x) = -2x^2 - 12x - 13 \)
58. \( f(x) = -2x^2 + 24x - 70 \)
59. \( f(x) = (1/2)x^2 - 4x + 5 \)
60. \( f(x) = (1/2)x^2 + 4x + 6 \)
61. \( f(x) = (-1/2)x^2 - 3x + 1/2 \)
62. \( f(x) = (-1/2)x^2 + 4x - 2 \)
63. \( f(x) = 2x^2 + 7x - 2 \)
64. \( f(x) = -2x^2 - 5x - 4 \)
65. \( f(x) = -3x^2 + 8x - 3 \)
66. \( f(x) = 3x^2 + 4x - 6 \)

67. \( f(x) = -2x^2 + 4x + 3 \)
68. \( f(x) = x^2 + 4x + 8 \)
69. \( f(x) = 5x^2 + 4x + 4 \)
70. \( f(x) = 3x^2 - 8x + 3 \)
71. \( f(x) = -x^2 - 2x - 7 \)
72. \( f(x) = x^2 + x + 9 \)

In **Exercises 67-72**, find the range of the given quadratic function. Express your answer in both interval and set notation.

73. \( f(x) = 9x^2 - 9x + 4; \ b = -6 \)
74. \( f(x) = -12x^2 + 5x + 2; \ b = -3 \)
75. \( f(x) = 4x^2 - 6x - 4; \ b = 11 \)
76. \( f(x) = -2x^2 - 11x - 10; \ b = -12 \)

**Drill for Skill.** In **Exercises 73-76**, evaluate the function at the given value \( b \).

77. Evaluate \( f(x+4) \) if \( f(x) = -5x^2 + 4x + 2 \).
78. Evaluate \( f(-4x-5) \) if \( f(x) = 4x^2 + x + 1 \).
5.2 Solutions

1. \((x + \frac{4}{5})^2 = x^2 + 2(x)(\frac{4}{5}) + \left(\frac{4}{5}\right)^2 = x^2 + \frac{8}{5}x + \frac{16}{25}\)

3. \((x + 3)^2 = x^2 + 2(x)(3) + 3^2 = x^2 + 6x + 9\)

5. \((x - 7)^2 = x^2 + 2(x)(-7) + (-7)^2 = x^2 - 14x + 49\)

7. \((x - 6)^2 = x^2 + 2(x)(-6) + (-6)^2 = x^2 - 12x + 36\)

9. \(x^2 - \frac{6}{5}x + \frac{9}{25} = \left(x - \frac{3}{5}\right)^2\)

11. \(x^2 - 12x + 36 = (x - 6)^2\)

13. \(x^2 + 12x + 36 = (x + 6)^2\)

15. \(x^2 + 18x + 81 = (x + 9)^2\)

17.

\[f(x) = x^2 - x + 8\]
\[= x^2 - x + \frac{1}{4} - \frac{1}{4} + 8\]
\[= \left(x^2 - x + \frac{1}{4}\right) - \frac{1}{4} + 8\]
\[= \left(x - \frac{1}{2}\right)^2 + \frac{31}{4}\]

19.

\[f(x) = x^2 - 5x - 4\]
\[= x^2 - 5x + \frac{25}{4} - \frac{25}{4} - 4\]
\[= \left(x^2 - 5x + \frac{25}{4}\right) - \frac{25}{4} - 4\]
\[= \left(x - \frac{5}{2}\right)^2 - \frac{41}{4}\]
21. 

\[ f(x) = x^2 + 2x - 6 \]
\[ = x^2 + 2x + 1 - 1 - 6 \]
\[ = (x^2 + 2x + 1) - 1 - 6 \]
\[ = (x + 1)^2 - 7 \]

23. 

\[ f(x) = x^2 - 9x + 3 \]
\[ = x^2 - 9x + \frac{81}{4} - \frac{81}{4} + 3 \]
\[ = \left(x^2 - 9x + \frac{81}{4}\right) - \frac{81}{4} + 3 \]
\[ = \left(x - \frac{9}{2}\right)^2 - \frac{69}{4} \]

25. 

\[ f(x) = -2x^2 - 9x - 3 \]
\[ = -2 \left(x^2 + \frac{9}{2}x + \frac{3}{2}\right) \]
\[ = -2 \left(x^2 + \frac{9}{2}x + \frac{81}{16} - \frac{81}{16} + \frac{3}{2}\right) \]
\[ = -2 \left(x^2 + \frac{9}{2}x + \frac{81}{16} - \frac{81}{16} + \frac{3}{2}\right) \]
\[ = -2 \left(x + \frac{9}{4}\right)^2 - \frac{57}{16} \]
\[ = -2 \left(x + \frac{9}{4}\right)^2 + \frac{57}{8} \]
27. 

\[ f(x) = 5x^2 + 5x + 5 \]
\[ = 5 \left( x^2 + x + 1 \right) \]
\[ = 5 \left( x^2 + x + \frac{1}{4} - \frac{1}{4} + 1 \right) \]
\[ = 5 \left( \left( x^2 + x + \frac{1}{4} \right) - \frac{1}{4} + 1 \right) \]
\[ = 5 \left( \left( x + \frac{1}{2} \right)^2 - \frac{3}{4} \right) \]
\[ = 5 \left( x + \frac{1}{2} \right)^2 + \frac{15}{4} \]

29. 

\[ f(x) = 5x^2 + 7x - 3 \]
\[ = 5 \left( x^2 + \frac{7}{5}x - \frac{3}{5} \right) \]
\[ = 5 \left( x^2 + \frac{7}{5}x + \frac{49}{100} - \frac{49}{100} - \frac{3}{5} \right) \]
\[ = 5 \left( \left( x^2 + \frac{7}{5}x + \frac{49}{100} \right) - \frac{49}{100} - \frac{3}{5} \right) \]
\[ = 5 \left( \left( x + \frac{7}{10} \right)^2 - \frac{109}{100} \right) \]
\[ = 5 \left( x + \frac{7}{10} \right)^2 - \frac{109}{20} \]
31. 
\[ f(x) = -x^2 - x + 4 \]
\[ = -1 \left( x^2 + x - 4 \right) \]
\[ = -1 \left( x^2 + x + \frac{1}{4} - \frac{1}{4} - 4 \right) \]
\[ = -1 \left( \left( x + \frac{1}{2} \right)^2 - \frac{17}{4} \right) \]
\[ = -1 \left( x + \frac{1}{2} \right)^2 + \frac{17}{4} \]

33. First complete the square to transform the function into vertex form \( a(x - h)^2 + k \):
\[ f(x) = -2x^2 + 5x + 3 \]
\[ = -2 \left( x^2 - \frac{5}{2}x - \frac{3}{2} \right) \]
\[ = -2 \left( x^2 - \frac{5}{2}x + \frac{25}{16} - \frac{25}{16} - \frac{3}{2} \right) \]
\[ = -2 \left( \left( x - \frac{5}{4} \right)^2 - \frac{49}{16} \right) \]
\[ = -2 \left( x - \frac{5}{4} \right)^2 + \frac{49}{8} \]

Thus, \( h = \frac{5}{4} \) and \( k = \frac{49}{8} \), so the vertex is \( \left( \frac{5}{4}, \frac{49}{8} \right) \).
35. First complete the square to transform the function into vertex form \(a(x-h)^2+k\):

\[
f(x) = -4x^2 - 4x + 1
\]

\[
= -4 \left( x^2 + x - \frac{1}{4} \right)
\]

\[
= -4 \left( x^2 + x + \frac{1}{4} - \frac{1}{4} - \frac{1}{4} \right)
\]

\[
= -4 \left( \left(x + \frac{1}{2}\right)^2 - \frac{1}{4} \right)
\]

\[
= -4 \left( x + \frac{1}{2} \right)^2 + 2
\]

Thus, \(h = -\frac{1}{2}\) and \(k = 2\), so the vertex is \((-\frac{1}{2}, 2)\).

37. First complete the square to transform the function into vertex form \(a(x-h)^2+k\):

\[
f(x) = 4x^2 + 2x + 8
\]

\[
= 4 \left( x^2 + \frac{1}{2}x + 2 \right)
\]

\[
= 4 \left( x^2 + \frac{1}{2}x + \frac{1}{16} - \frac{1}{16} + 2 \right)
\]

\[
= 4 \left( \left(x + \frac{1}{4}\right)^2 + \frac{31}{16} \right)
\]

\[
= 4 \left( x + \frac{1}{4} \right)^2 + \frac{31}{4}
\]

Thus, \(h = -\frac{1}{4}\) and \(k = \frac{31}{4}\), so the vertex is \((-\frac{1}{4}, \frac{31}{4})\).
39. First complete the square to transform the function into vertex form $a(x-h)^2+k$:

\[
f(x) = -5x^2 - 7x - 8
\]

\[
= -5 \left( x^2 + \frac{7}{5}x + \frac{8}{5} \right)
\]

\[
= -5 \left( x^2 + \frac{7}{5}x + \frac{49}{100} - \frac{49}{100} + \frac{8}{5} \right)
\]

\[
= -5 \left( \left( x + \frac{7}{10} \right)^2 + \frac{111}{100} \right)
\]

\[
= -5 \left( x + \frac{7}{10} \right)^2 - \frac{111}{20}
\]

Thus, $h = -\frac{7}{10}$, so the axis of symmetry is $x = -\frac{7}{10}$.

41. First complete the square to transform the function into vertex form $a(x-h)^2+k$:

\[
f(x) = -2x^2 - 5x - 8
\]

\[
= -2 \left( x^2 + \frac{5}{2}x + 4 \right)
\]

\[
= -2 \left( x^2 + \frac{5}{2}x + \frac{25}{16} - \frac{25}{16} + 4 \right)
\]

\[
= -2 \left( \left( x + \frac{5}{4} \right)^2 + \frac{39}{16} \right)
\]

\[
= -2 \left( x + \frac{5}{4} \right)^2 - \frac{39}{8}
\]

Thus, $h = -\frac{5}{4}$, so the axis of symmetry is $x = -\frac{5}{4}$.
43. First complete the square to transform the function into vertex form $a(x-h)^2+k$:

$$f(x) = -5x^2 + x + 6$$

$$= -5 \left( x^2 - \frac{1}{5}x - \frac{6}{5} \right)$$

$$= -5 \left( x^2 - \frac{1}{5}x + \frac{1}{100} - \frac{1}{100} - \frac{6}{5} \right)$$

$$= -5 \left( x^2 - \frac{1}{5}x + \frac{1}{100} \right) - \frac{1}{100} - \frac{6}{5}$$

$$= -5 \left( x - \frac{1}{10} \right)^2 - \frac{121}{100}$$

$$= -5 \left( x - \frac{1}{10} \right)^2 + \frac{121}{20}$$

Thus, $h = \frac{1}{10}$, so the axis of symmetry is $x = \frac{1}{10}$. 
45. First complete the square to transform the function into vertex form \(a(x-h)^2+k\):

\[
f(x) = x^2 - 8x + 12
= x^2 - 8x + 16 - 16 + 12
= (x^2 - 8x + 16) - 16 + 12
= (x - 4)^2 - 4
\]

Compare the quadratic function \(f(x) = (x - 4)^2 - 4\) with \(f(x) = a(x-h)^2 + k\) and note that \(h = 4\) and \(k = -4\). Hence, the vertex is located at \((h,k) = (4, -4)\). The axis of symmetry is a vertical line through the vertex with equation \(x = 4\). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \(f(x) = (x - 4)^2 - 4\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y = (x - 4)^2 - 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
</tr>
</tbody>
</table>

To find the domain of \(f\), mentally project every point of the graph onto the \(x\)-axis, as shown on the left below. This covers the entire \(x\)-axis, so the domain = \((-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \(y\)-axis, as shown on the right below. The shaded interval on the \(y\)-axis is range = \([-4, \infty)\).
47. First complete the square to transform the function into vertex form \( a(x-h)^2+k \):

\[
f(x) = x^2 + 6x + 3
= x^2 + 6x + 9 - 9 + 3
= (x^2 + 6x + 9) - 9 + 3
= (x + 3)^2 - 9 + 3
= (x + 3)^2 - 6
\]

Compare the quadratic function \( f(x) = (x + 3)^2 - 6 \) with \( f(x) = a(x - h)^2 + k \) and note that \( h = -3 \) and \( k = -6 \). Hence the vertex is located at \((h,k) = (-3,-6)\). The axis of symmetry is a vertical line through the vertex with equation \( x = -3 \). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \( f(x) = (x + 3)^2 - 6 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = (x + 3)^2 - 6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-5</td>
</tr>
<tr>
<td>-1</td>
<td>-2</td>
</tr>
</tbody>
</table>

To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain= \((−∞, ∞)\). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range= \([-6, ∞)\).
49. First complete the square to transform the function into vertex form \( a(x-h)^2 + k \):

\[
f(x) = x^2 - 2x - 6 \\
= x^2 - 2x + 1 - 1 - 6 \\
= (x^2 - 2x + 1) - 1 - 6 \\
= (x - 1)^2 - 1 - 6 \\
= (x - 1)^2 - 7
\]

Compare the quadratic function \( f(x) = (x - 1)^2 - 7 \) with \( f(x) = a(x - h)^2 + k \) and note that \( h = 1 \) and \( k = -7 \). Hence, the vertex is located at \((h, k) = (1, -7)\). The axis of symmetry is a vertical line through the vertex with equation \( x = 1 \). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \( f(x) = (x - 1)^2 - 7 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = (x - 1)^2 - 7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-6</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
</tr>
</tbody>
</table>

To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain \( = (-\infty, \infty) \). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range \( = [-7, \infty) \).
51. First complete the square to transform the function into vertex form \(a(x-h)^2+k\):

\[
f(x) = -x^2 + 6x - 4
\]

\[
= -(x^2 - 6x + 4)
\]

\[
= -[x^2 - 6x + 9 - 9 + 4]
\]

\[
= -[(x^2 - 6x + 9) - 9 + 4]
\]

\[
= -[(x - 3)^2 - 9 + 4]
\]

\[
= -[(x - 3)^2 - 5]
\]

\[
= -(x - 3)^2 + 5
\]

Compare the quadratic function \(f(x) = -(x - 3)^2 + 5\) with \(f(x) = a(x - h)^2 + k\) and note that \(h = 3\) and \(k = 5\). Hence, the vertex is located at \((h, k) = (3, 5)\). The axis of symmetry is a vertical line through the vertex with equation \(x = 3\). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \(f(x) = -(x - 3)^2 + 5\).

\[
\begin{array}{c|c}
  x & y = -(x - 3)^2 + 5 \\
  \hline
  2 & 4 \\
  1 & 1 \\
\end{array}
\]

To find the domain of \(f\), mentally project every point of the graph onto the \(x\)-axis, as shown on the left below. This covers the entire \(x\)-axis, so the domain= \((-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \(y\)-axis, as shown on the right below. The shaded interval on the \(y\)-axis is range= \((-\infty, 5]\).
53. First complete the square to transform the function into vertex form \( a(x-h)^2+k \):

\[
f(x) = -x^2 - 10x - 21
= -(x^2 + 10x + 21)
= -(x^2 + 10x + 25 - 25 + 21)
= -(x^2 + 10x + 25) - 25 + 21
= -(x + 5)^2 - 25 + 21
= -(x + 5)^2 - 4
= -(x + 5)^2 + 4
\]

Compare the quadratic function \( f(x) = -(x + 5)^2 + 4 \) with \( f(x) = a(x-h)^2 + k \) and note that \( h = -5 \) and \( k = 4 \). Hence, the vertex is located at \((h, k) = (-5, 4)\). The axis of symmetry is a vertical line through the vertex with equation \( x = -5 \). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \( f(x) = -(x + 5)^2 + 4 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = -(x+5)^2 + 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>3</td>
</tr>
<tr>
<td>-3</td>
<td>0</td>
</tr>
</tbody>
</table>
To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain is \( (-\infty, \infty) \). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range is \( (-\infty, 4] \).

55. First complete the square to transform the function into vertex form \( a(x-h)^2+k \):

\[
f(x) = 2x^2 - 8x + 3 \\
= 2(x^2 - 4x + \frac{3}{2}) \\
= 2\left(x^2 - 4x + 4 - 4 + \frac{3}{2}\right) \\
= 2\left((x^2 - 4x + 4) - 4 + \frac{3}{2}\right) \\
= 2\left((x - 2)^2 - \frac{8}{2} + \frac{3}{2}\right) \\
= 2\left((x - 2)^2 - \frac{5}{2}\right) \\
= 2(x - 2)^2 - 5
\]

Compare the quadratic function \( f(x) = 2(x-2)^2 - 5 \) with \( f(x) = a(x-h)^2 + k \) and note that \( h = 2 \) and \( k = -5 \). Hence, the vertex is located at \( (h, k) = (2, -5) \). The axis of symmetry is a vertical line through the vertex with equation \( x = 2 \). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \( f(x) = 2(x-2)^2 - 5 \).
57. First complete the square to transform the function into vertex form \( a(x - h)^2 + k \):

\[
f(x) = -2x^2 - 12x - 13
\]

\[
= -2 \left( x^2 + 6x + \frac{13}{2} \right)
\]

\[
= -2 \left( x^2 + 6x + 9 - 9 + \frac{13}{2} \right)
\]

\[
= -2 \left( (x + 3)^2 - 9 + \frac{13}{2} \right)
\]

\[
= -2 \left( (x + 3)^2 - \frac{18}{2} + \frac{13}{2} \right)
\]

\[
= -2 \left( (x + 3)^2 - \frac{5}{2} \right)
\]

\[
= -2 (x + 3)^2 + 5
\]
Compare the quadratic function \( f(x) = -2(x + 3)^2 + 5 \) with \( f(x) = a(x - h)^2 + k \) and note that \( h = -3 \) and \( k = 5 \). Hence, the vertex is located at \((h, k) = (-3, 5)\). The axis of symmetry is a vertical line through the vertex with equation \( x = -3 \). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \( f(x) = -2(x + 3)^2 + 5 \).

\[
\begin{array}{|c|c|}
\hline
x & y = -2(x+3)^2+5 \\
\hline
-2 & 3 \\
-1 & -3 \\
\hline
\end{array}
\]

To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain = \((-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range = \((-\infty, 5]\).
59. First complete the square to transform the function into vertex form \(a(x-h)^2+k\):

\[
f(x) = \frac{1}{2}x^2 - 4x + 5 \\
= \frac{1}{2}(x^2 - 8x + 10) \\
= \frac{1}{2}(x^2 - 8x + 16 - 16 + 10) \\
= \frac{1}{2}((x^2 - 8x + 16) - 16 + 10) \\
= \frac{1}{2}((x - 4)^2 - 6) \\
= \frac{1}{2}(x - 4)^2 - 3
\]

Compare the quadratic function \(f(x) = \frac{1}{2}(x - 4)^2 - 3\) with \(f(x) = a(x-h)^2 + k\) and note that \(h = 4\) and \(k = -3\). Hence, the vertex is located at \((h, k) = (4, -3)\). The axis of symmetry is a vertical line through the vertex with equation \(x = 4\). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \(f(x) = \frac{1}{2}(x - 4)^2 - 3\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y = \frac{1}{2}(x - 4)^2 - 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-2.5</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
</tbody>
</table>

To find the domain of \(f\), mentally project every point of the graph onto the \(x\)-axis, as shown on the left below. This covers the entire \(x\)-axis, so the domain = \((-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \(y\)-axis, as shown on the right below. The shaded interval on the \(y\)-axis is range = \([-3, \infty)\).
61. First complete the square to transform the function into vertex form \( a(x - h)^2 + k \):

\[
f(x) = (-1/2)x^2 - 3x + 1/2
\]

\[
= -\frac{1}{2} (x^2 + 6x - 1)
\]

\[
= -\frac{1}{2} (x^2 + 6x + 9 - 9 - 1)
\]

\[
= -\frac{1}{2} ((x^2 + 6x + 9) - 9 - 1)
\]

\[
= -\frac{1}{2} (x + 3)^2 - 10
\]

\[
= -\frac{1}{2} (x + 3)^2 + 5
\]

Compare the quadratic function \( f(x) = -\frac{1}{2} (x + 3)^2 + 5 \) with \( f(x) = a(x - h)^2 + k \) and note that \( h = -3 \) and \( k = 5 \). Hence, the vertex is located at \( (h, k) = (-3, 5) \). The axis of symmetry is a vertical line through the vertex with equation \( x = -3 \). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \( f(x) = -\frac{1}{2} (x + 3)^2 + 5 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = -\frac{1}{2} (x + 3)^2 + 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>4.5</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
</tr>
</tbody>
</table>
To find the domain of $f$, mentally project every point of the graph onto the $x$-axis, as shown on the left below. This covers the entire $x$-axis, so the domain = $(-\infty, \infty)$. To find the range, mentally project every point of the graph onto the $y$-axis, as shown on the right below. The shaded interval on the $y$-axis is range = $(-\infty, 5]$.

63. First complete the square to transform the function into vertex form $a(x-h)^2+k$:

$$f(x) = 2x^2 + 7x - 2$$

$$= 2 \left( x^2 + \frac{7}{2}x - 1 \right)$$

$$= 2 \left( x^2 + \frac{7}{2}x + \frac{49}{16} - \frac{49}{16} - 1 \right)$$

$$= 2 \left( \left( x + \frac{7}{4} \right)^2 - \frac{49}{16} - \frac{16}{16} \right)$$

$$= 2 \left( \left( x + \frac{7}{4} \right)^2 - \frac{65}{16} \right)$$

$$= 2 \left( x + \frac{7}{4} \right)^2 - \frac{65}{8}$$

Compare the quadratic function $f(x) = 2 \left( x + \frac{7}{4} \right)^2 - \frac{65}{8}$ with $f(x) = a(x-h)^2 + k$ and note that $h = -7/4$ and $k = -65/8$. Hence, the vertex is located at $(h,k) = (-7/4,-65/8)$. The axis of symmetry is a vertical line through the vertex with equation $x = -7/4$. Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of $f(x) = 2 \left( x + \frac{7}{4} \right)^2 - \frac{65}{8}$. 
To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain = \((−\infty, \infty)\). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range = \([-\frac{65}{8}, \infty)\).

\[
\begin{array}{|c|c|}
\hline
x & y = 2(x + \frac{7}{4})^2 - \frac{65}{8} \\
\hline
-\frac{1}{2} & -5 \\
\frac{1}{2} & 2 \\
\hline
\end{array}
\]
65. First complete the square to transform the function into vertex form \( a(x-h)^2 + k \):

\[
f(x) = -3x^2 + 8x - 3
= -3 \left( x^2 - \frac{8}{3}x + \frac{16}{9} + \frac{16}{9} + 1 \right)
= -3 \left( \left( x - \frac{4}{3} \right)^2 - \frac{16}{9} + \frac{9}{9} \right)
= -3 \left( x - \frac{4}{3} \right)^2 + \frac{7}{3}
\]

Compare the quadratic function \( f(x) = -3 \left( x - \frac{4}{3} \right)^2 + \frac{7}{3} \) with \( f(x) = a(x-h)^2 + k \) and note that \( h = 4/3 \) and \( k = 7/3 \). Hence, the vertex is located at \((h, k) = (4/3, 7/3)\). The axis of symmetry is a vertical line through the vertex with equation \( x = 4/3 \). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \( f(x) = -3 \left( x - \frac{4}{3} \right)^2 + \frac{7}{3} \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y ) = (-3(x-4/3)^2+7/3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>-3</td>
</tr>
</tbody>
</table>

To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain = \((-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range = \((-\infty, 7/3]\).
67. First complete the square to transform the function into vertex form \(a(x-h)^2+k\):

\[
f(x) = -2x^2 + 4x + 3
\]

\[
= -2 \left( x^2 - 2x - \frac{3}{2} \right)
\]

\[
= -2 \left( x^2 - 2x + 1 - 1 - \frac{3}{2} \right)
\]

\[
= -2 \left( (x^2 - 2x + 1) - 1 - \frac{3}{2} \right)
\]

\[
= -2 \left( (x - 1)^2 - \frac{5}{2} \right)
\]

\[
= -2 (x - 1)^2 + 5
\]

The graph opens downward since \(a = -2 < 0\), and the vertex is at \((h, k)\), where \(h = 1\) and \(k = 5\). Thus, the range is \((-\infty, k] = (-\infty, 5]\).

69. First complete the square to transform the function into vertex form \(a(x-h)^2+k\):

\[
f(x) = 5x^2 + 4x + 4
\]

\[
= 5 \left( x^2 + \frac{4}{5}x + \frac{4}{5} \right)
\]

\[
= 5 \left( x^2 + \frac{4}{5}x + \frac{4}{25} - \frac{4}{25} + \frac{4}{5} \right)
\]

\[
= 5 \left( \left( x + \frac{2}{5} \right)^2 - \frac{4}{25} + \frac{4}{5} \right)
\]

\[
= 5 \left( \left( x + \frac{2}{5} \right)^2 + \frac{16}{25} \right)
\]

\[
= 5 \left( x + \frac{2}{5} \right)^2 + \frac{16}{5}
\]
The graph opens upward since \( a = 5 > 0 \), and the vertex is at \((h, k)\), where \( h = -\frac{2}{5} \) and \( k = \frac{16}{5} \). Thus, the range is \([k, \infty) = \left[ \frac{16}{5}, \infty \right)\).

71. First complete the square to transform the function into vertex form \( a(x-h)^2 + k \):

\[
f(x) = -x^2 - 2x - 7
  = -1 \left( x^2 + 2x + 7 \right)
  = -1 \left( x^2 + 2x + 1 - 1 + 7 \right)
  = -1 \left( (x^2 + 2x + 1) - 1 + 7 \right)
  = -1 \left( (x + 1)^2 + 6 \right)
  = -1 (x + 1)^2 - 6
\]

The graph opens downward since \( a = -1 < 0 \), and the vertex is at \((h, k)\), where \( h = -1 \) and \( k = -6 \). Thus, the range is \((-\infty, k] \) = \((-\infty, -6]\).

73. Substitute \(-6\) for \( x \) in \( 9x^2 - 9x + 4 \) and simplify to get 382:

\[
f(-6) = 9(-6)^2 - 9(-6) + 4 = 382
\]

75. Substitute \(11\) for \( x \) in \( 4x^2 - 6x - 4 \) and simplify to get 414:

\[
f(11) = 4(11)^2 - 6(11) - 4 = 414
\]

77. Substitute \( x + 4 \) for \( x \) in \( -5x^2 + 4x + 2 \) and simplify:

\[
f(x + 4) = -5(x + 4)^2 + 4(x + 4) + 2
  = -5(x^2 + 8x + 16) + 4x + 16 + 2
  = -5x^2 - 36x - 62
\]

79. Substitute \( 4x - 1 \) for \( x \) in \( 4x^2 + 3x - 3 \) and simplify:

\[
f(4x - 1) = 4(4x - 1)^2 + 3(4x - 1) - 3
  = 4(16x^2 - 8x + 1) + 12x - 3 - 3
  = 64x^2 - 20x - 2
\]
5.3 Exercises

In Exercises 1-8, factor the given quadratic polynomial.

1. \( x^2 + 9x + 14 \)
2. \( x^2 + 6x + 5 \)
3. \( x^2 + 10x + 9 \)
4. \( x^2 + 4x - 9 \)
5. \( x^2 - 4x - 5 \)
6. \( x^2 + 7x - 8 \)
7. \( x^2 - 7x + 12 \)
8. \( x^2 + 5x - 24 \)

In Exercises 9-16, find the zeros of the given quadratic function.

9. \( f(x) = x^2 - 2x - 15 \)
10. \( f(x) = x^2 + 4x - 32 \)
11. \( f(x) = x^2 + 10x - 39 \)
12. \( f(x) = x^2 + 4x - 45 \)
13. \( f(x) = x^2 - 14x + 40 \)
14. \( f(x) = x^2 - 5x - 14 \)
15. \( f(x) = x^2 + 9x - 36 \)
16. \( f(x) = x^2 + 11x - 26 \)

In Exercises 17-22, perform each of the following tasks for the quadratic functions.

i. Load the function into Y1 of the Y= of your graphing calculator. Adjust the window parameters so that the vertex is visible in the viewing window.

ii. Set up a coordinate system on your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax. Make a reasonable copy of the image in the viewing window of your calculator on this coordinate system and label it with its equation.

iii. Use the zero utility on your graphing calculator to find the zeros of the function. Use these results to plot the x-intercepts on your coordinate system and label them with their coordinates.

iv. Use a strictly algebraic technique (no calculator) to find the zeros of the given quadratic function. Show your work next to your coordinate system. Be stubborn! Work the problem until your algebraic and graphically zeros are a reasonable match.

17. \( f(x) = x^2 + 5x - 14 \)
18. \( f(x) = x^2 + x - 20 \)
19. \( f(x) = -x^2 + 3x + 18 \)
20. \( f(x) = -x^2 + 3x + 40 \)
21. \( f(x) = x^2 - 16x - 36 \)
22. \( f(x) = x^2 + 4x - 96 \)

\(^1\) Copyrighted material. See: http://msenux.redwoods.edu/IntAlgText/
In Exercises 23–30, perform each of the following tasks for the given quadratic function.

i. Set up a coordinate system on graph paper. Label and scale each axis. Remember to draw all lines with a ruler.

ii. Use the technique of completing the square to place the quadratic function in vertex form. Plot the vertex on your coordinate system and label it with its coordinates. Draw the axis of symmetry on your coordinate system and label it with its equation.

iii. Use a strictly algebraic technique (no calculators) to find the $x$-intercepts of the graph of the given quadratic function. Plot them on your coordinate system and label them with their coordinates.

iv. Find the $y$-intercept of the graph of the quadratic function. Plot the $y$-intercept on your coordinate system and its mirror image across the axis of symmetry, then label these points with their coordinates.

v. Using all the information plotted, draw the graph of the quadratic function and label it with the vertex form of its equation. Use interval notation to describe the domain and range of the quadratic function.

23. $f(x) = x^2 + 2x - 8$
24. $f(x) = x^2 - 6x + 8$
25. $f(x) = x^2 + 4x - 12$
26. $f(x) = x^2 + 8x + 12$
27. $f(x) = -x^2 - 2x + 8$
28. $f(x) = -x^2 - 2x + 24$
29. $f(x) = -x^2 - 8x + 48$
30. $f(x) = -x^2 - 8x + 20$

In Exercises 31–38, factor the given quadratic polynomial.

31. $42x^2 + 5x - 2$
32. $3x^2 + 7x - 20$
33. $5x^2 - 19x + 12$
34. $54x^2 - 3x - 1$
35. $-4x^2 + 9x - 5$
36. $3x^2 - 5x - 12$
37. $2x^2 - 3x - 35$
38. $-6x^2 + 25x + 9$

In Exercises 39–46, find the zeros of the given quadratic functions.

39. $f(x) = 2x^2 - 3x - 20$
40. $f(x) = 2x^2 - 7x - 30$
41. $f(x) = -2x^2 + x + 28$
42. $f(x) = -2x^2 + 15x - 22$
43. $f(x) = 3x^2 - 20x + 12$
44. $f(x) = 4x^2 + 11x - 20$
45. $f(x) = -4x^2 + 4x + 15$
46. $f(x) = -6x^2 - x + 12$
In **Exercises 47-52**, perform each of the following tasks for the given quadratic functions.

i. Load the function into Y1 of the Y= of your graphing calculator. Adjust the window parameters so that the vertex is visible in the viewing window.

ii. Set up a coordinate system on your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax. Make a reasonable copy of the image in the viewing window of your calculator on this coordinate system and label it with its equation.

iii. Use the zero utility on your graphing calculator to find the zeros of the function. Use these results to plot the x-intercepts on your coordinate system and label them with their coordinates.

iv. Use a strictly algebraic technique (no calculator) to find the zeros of the given quadratic function. Show your work next to your coordinate system. Be stubborn! Work the problem until your algebraic and graphically zeros are a reasonable match.

47. \( f(x) = 2x^2 + 3x - 35 \)

48. \( f(x) = 2x^2 - 5x - 42 \)

49. \( f(x) = -2x^2 + 5x + 33 \)

50. \( f(x) = -2x^2 - 5x + 52 \)

51. \( f(x) = 4x^2 - 24x - 13 \)

52. \( f(x) = 4x^2 + 24x - 45 \)

In **Exercises 53-60**, perform each of the following tasks for the given quadratic functions.

i. Set up a coordinate system on graph paper. Label and scale each axis. Remember to draw all lines with a ruler.

ii. Use the technique of completing the square to place the quadratic function in vertex form. Plot the vertex on your coordinate system and label it with its coordinates. Draw the axis of symmetry on your coordinate system and label it with its equation.

iii. Use a strictly algebraic method (no calculators) to find the x-intercepts of the graph of the quadratic function. Plot them on your coordinate system and label them with their coordinates.

iv. Find the y-intercept of the graph of the quadratic function. Plot the y-intercept on your coordinate system and its mirror image across the axis of symmetry, then label these points with their coordinates.

v. Using all the information plotted, draw the graph of the quadratic function and label it with the vertex form of its equation. Use interval notation to describe the domain and range of the quadratic function.

53. \( f(x) = 2x^2 - 8x - 24 \)

54. \( f(x) = 2x^2 - 4x - 6 \)

55. \( f(x) = -2x^2 - 4x + 16 \)

56. \( f(x) = -2x^2 - 16x + 40 \)

57. \( f(x) = 3x^2 + 18x - 48 \)

58. \( f(x) = 3x^2 + 18x - 216 \)

59. \( f(x) = 2x^2 + 10x - 48 \)

60. \( f(x) = 2x^2 - 10x - 100 \)
In Exercises 61-66, Use the graph of \( f(x) = ax^2 + bx + c \) shown to find all solutions of the equation \( f(x) = 0 \). (Note: Every solution is an integer.)

61.

62.

63.

64.

65.

66.
5.3 Solutions

1. Look for \( p \) and \( q \) such that \((x+p)(x+q) = x^2+9x+14\). It follows that \( p+q = 9 \) and \( pq = 14 \), so \( p = 2 \) and \( q = 7 \). Now verify the factorization by multiplying \((x+2)(x+7)\) to obtain \( x^2 + 9x + 14 \).

3. Look for \( p \) and \( q \) such that \((x+p)(x+q) = x^2+10x+9\). It follows that \( p+q = 10 \) and \( pq = 9 \), so \( p = 9 \) and \( q = 1 \). Now verify the factorization by multiplying \((x+9)(x+1)\) to obtain \( x^2 + 10x + 9 \).

5. Look for \( p \) and \( q \) such that \((x+p)(x+q) = x^2-4x-5\). It follows that \( p+q = -4 \) and \( pq = -5 \), so \( p = -5 \) and \( q = 1 \). Now verify the factorization by multiplying \((x-5)(x+1)\) to obtain \( x^2 - 4x - 5 \).

7. Look for \( p \) and \( q \) such that \((x+p)(x+q) = x^2-7x+12\). It follows that \( p+q = -7 \) and \( pq = 12 \), so \( p = -4 \) and \( q = -3 \). Now verify the factorization by multiplying \((x-4)(x-3)\) to obtain \( x^2 - 7x + 12 \).

9. To find the zeroes, set \( f(x) = 0 \) and factor.

\[
0 = x^2 - 2x - 15 \\
0 = (x - 5)(x + 3)
\]

By the zero product property, either

\[
x - 5 = 0 \quad \text{or} \quad x + 3 = 0.
\]

Solve these linear equations independently.

\[
x = 5 \quad \text{or} \quad x = -3
\]

11. To find the zeroes, set \( f(x) = 0 \) and factor.

\[
0 = x^2 + 10x - 39 \\
0 = (x + 13)(x - 3)
\]

By the zero product property, either

\[
x + 13 = 0 \quad \text{or} \quad x - 3 = 0.
\]

Solve these linear equations independently.

\[
x = -13 \quad \text{or} \quad x = 3
\]
13. To find the zeroes, set $f(x) = 0$ and factor.

$$0 = x^2 - 14x + 40$$

$$0 = (x - 4)(x - 10)$$

By the zero product property, either

$$x - 4 = 0 \quad \text{or} \quad x - 10 = 0.$$ 

Solve these linear equations independently.

$$x = 4 \quad \text{or} \quad x = 10$$

15. To find the zeroes, set $f(x) = 0$ and factor.

$$0 = x^2 + 9x - 36$$

$$0 = (x - 3)(x + 12)$$

By the zero product property, either

$$x - 3 = 0 \quad \text{or} \quad x + 12 = 0.$$ 

Solve these linear equations independently.

$$x = 3 \quad \text{or} \quad x = -12$$

17. To find the zeroes with your calculator, press 2nd TRACE to access the CALC menu and choose 2:zero.

Repeat this process for the second zero. We get $(-7,0)$ and $(2,0)$. 
To find the zeroes algebraically, set \( f(x) = 0 \) and factor.

\[
0 = x^2 + 5x - 14
\]

By the zero product property, either

\[
x + 7 = 0 \quad \text{or} \quad x - 2 = 0.
\]

Solve these linear equations independently.

\[
x = -7 \quad \text{or} \quad x = 2
\]

So the zeroes are \((-7, 0)\) and \((2, 0)\).

19. To find the zeroes with your calculator, press 2nd TRACE to access the CALC menu and choose 2:zero.

Repeat this process for the second zero. We get \((6, 0)\) and \((-3, 0)\).
To find the zeroes algebraically, set $f(x) = 0$ and factor.

$$0 = -x^2 + 3x + 18$$
$$0 = -(x^2 - 3x - 18)$$
$$0 = -(x - 6)(x + 3)$$

By the zero product property, either

$$x - 6 = 0 \quad \text{or} \quad x + 3 = 0.$$ 

Solve these linear equations independently.

$$x = 6 \quad \text{or} \quad x = -3$$

21. To find the zeroes with your calculator, press 2nd TRACE to access the CALC menu and choose 2:zero.

Repeat this process for the second zero. We get $(-2,0)$ and $(18,0)$. 

Version: Fall 2007
To find the zeroes algebraically, set \( f(x) = 0 \) and factor.

\[
0 = x^2 - 16x - 36
\]

By the *zero product property*, either

\[ x + 2 = 0 \quad \text{or} \quad x - 18 = 0. \]

Solve these linear equations independently.

\[ x = -2 \quad \text{or} \quad x = 18 \]

23. First, complete the square:

\[
f(x) = x^2 + 2x - 8
\]

\[
= x^2 + 2x + 1 - 1 - 8
\]

\[
= (x^2 + 2x + 1) - 1 - 8
\]

\[
= (x + 1)^2 - 9
\]

Read off the vertex as \((h, k) = (-1, -9)\). The axis of symmetry is a vertical line through the vertex with equation \( x = -1 \).

To find the \( x \)-intercepts algebraically, set \( y = 0 \) and factor.

\[
0 = x^2 + 2x - 8
\]

By the *zero product property*, either

\[ x + 4 = 0 \quad \text{or} \quad x - 2 = 0. \]

Solve these linear equations independently.

\[ x = -4 \quad \text{or} \quad x = 2 \]
So the $x$-intercepts are $(-4, 0)$ and $(2, 0)$.
Lastly, to find the $y$-intercept, set $x = 0$ in the equation and solve for $y$:

\[
y = x^2 + 2x - 8 \\
y = 0^2 + 2(0) - 8 \\
y = -8
\]

So the $y$-intercept is $(0, -8)$. Finally, put this all together to make the graph.

To find the domain of $f$, mentally project every point of the graph onto the $x$-axis, as shown on the left below. This covers the entire $x$-axis, so the domain $= (-\infty, \infty)$. To find the range, mentally project every point of the graph onto the $y$-axis, as shown on the right below. The shaded interval on the $y$-axis is range $= [-9, \infty)$.

25. First, complete the square:

\[
f(x) = x^2 + 4x - 12 \\
= x^2 + 4x + 4 - 4 - 12 \\
= (x^2 + 4x + 4) - 4 - 12 \\
= (x + 2)^2 - 16
\]
Read off the vertex as \((h,k) = (-2, -16)\). The axis of symmetry is a vertical line through the vertex with equation \(x = -2\).

To find the \(x\)-intercepts algebraically, set \(y = 0\) and factor.

\[
\begin{align*}
0 &= x^2 + 4x - 12 \\
0 &= (x + 6)(x - 2)
\end{align*}
\]

By the zero product property, either

\[x + 6 = 0 \quad \text{or} \quad x - 2 = 0\]

Solve these linear equations independently.

\[x = -6 \quad \text{or} \quad x = 2\]

So the \(x\)-intercepts are \((-6,0)\) and \((2,0)\).

Lastly, to find the \(y\)-intercept, set \(x = 0\) in the equation and solve for \(y\):

\[
\begin{align*}
y &= x^2 + 4x - 12 \\
y &= 0^2 + 4(0) - 12 \\
y &= -12
\end{align*}
\]

So the \(y\)-intercept is \((0,-12)\).

Finally, put this all together to make the graph.

To find the domain of \(f\), mentally project every point of the graph onto the \(x\)-axis, as shown on the left below. This covers the entire \(x\)-axis, so the domain= \((-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \(y\)-axis, as shown on the right below. The shaded interval on the \(y\)-axis is range= \([-16, \infty)\).
Chapter 5  Quadratic Functions

27. First, complete the square:

\[ f(x) = -x^2 - 2x + 8 \]
\[ = -(x^2 + 2x - 8) \]
\[ = -(x^2 + 2x + 1 - 1 - 8) \]
\[ = - ((x^2 + 2x + 1) - 1 - 8) \]
\[ = - (x + 1)^2 - 9 \]
\[ = -(x + 1)^2 + 9 \]

Read off the vertex as \((h, k) = (-1, 9)\). The axis of symmetry is a vertical line through the vertex with equation \(x = -1\).

To find the \(x\)-intercepts algebraically, set \(y = 0\) and factor.

\[
0 = -x^2 - 2x + 8 \\
0 = -(x^2 + 2x - 8) \\
0 = -(x + 4)(x - 2)
\]

By the zero product property, either

\[ x + 4 = 0 \quad \text{or} \quad x - 2 = 0 \]

Solve these linear equations independently.

\[ x = -4 \quad \text{or} \quad x = 2 \]

So the \(x\)-intercepts are \((-4, 0)\) and \((2, 0)\).

Lastly, to find the \(y\)-intercept, set \(x = 0\) in the equation and solve for \(y\):

\[
y = -x^2 - 2x + 8 \\
y = -0^2 - 2(0) + 8 \\
y = 8
\]

Version: Fall 2007
So the $y$-intercept is $(0,8)$.
Finally, put this all together to make the graph.

To find the domain of $f$, mentally project every point of the graph onto the $x$-axis, as shown on the left below. This covers the entire $x$-axis, so the domain $= (-\infty, \infty)$. To find the range, mentally project every point of the graph onto the $y$-axis, as shown on the right below. The shaded interval on the $y$-axis is range $= (-\infty, 9]$.

29. First, complete the square:

$$f(x) = -x^2 - 8x + 48$$
$$= -(x^2 + 8x - 48)$$
$$= -(x^2 + 8x + 16 - 16 - 48)$$
$$= -((x^2 + 8x + 16) - 16 - 48)$$
$$= -((x + 4)^2 - 64)$$
$$= -(x + 4)^2 + 64$$
Read off the vertex as $(h, k) = (-4, 64)$. The axis of symmetry is a vertical line through the vertex with equation $x = -4$.

To find the $x$-intercepts algebraically, set $y = 0$ and factor.

$$0 = -x^2 - 8x + 48$$
$$0 = -(x^2 + 8x - 48)$$
$$0 = -(x + 12)(x - 4)$$

By the zero product property, either

$$x + 12 = 0 \quad \text{or} \quad x - 4 = 0.$$ 

Solve these linear equations independently.

$$x = -12 \quad \text{or} \quad x = 4$$

So the $x$-intercepts are $(-12, 0)$ and $(4, 0)$.

Lastly, to find the $y$-intercept, set $x = 0$ in the equation and solve for $y$:

$$y = -x^2 - 8x + 48$$
$$y = -0^2 - 8(0) + 48$$
$$y = 48$$

So the $y$-intercept is $(0, 48)$.

Finally, put this all together to make the graph.

To find the domain of $f$, mentally project every point of the graph onto the $x$-axis, as shown on the left below. This covers the entire $x$-axis, so the domain $\left( -\infty, \infty \right)$. To find the range, mentally project every point of the graph onto the $y$-axis, as shown on the right below. The shaded interval on the $y$-axis is range $\left( -\infty, 64 \right]$.

Version: Fall 2007
31. Using the $ac$-test, first look for two numbers whose product is $ac = (42)(-2) = -84$ and whose sum is $b = 5$. The solutions are $-7$ and $12$. Then

\[ 42x^2 + 5x - 2 = 42x^2 - 7x + 12x - 2 \]
\[ = 7x(6x - 1) + 2(6x - 1) \]
\[ = (7x + 2)(6x - 1) \]

Now verify the factorization by multiplying $(7x + 2)(6x - 1)$ to obtain $42x^2 + 5x - 2$.

33. Using the $ac$-test, first look for two numbers whose product is $ac = (5)(12) = 60$ and whose sum is $b = -19$. The solutions are $-4$ and $-15$. Then

\[ 5x^2 - 19x + 12 = 5x^2 - 4x - 15x + 12 \]
\[ = x(5x - 4) - 3(5x - 4) \]
\[ = (x - 3)(5x - 4) \]

Now verify the factorization by multiplying $(x - 3)(5x - 4)$ to obtain $5x^2 - 19x + 12$.

35. Using the $ac$-test, first look for two numbers whose product is $ac = (-4)(-5) = 20$ and whose sum is $b = 9$. The solutions are $4$ and $5$. Then

\[ -4x^2 + 9x - 5 = -4x^2 + 4x + 5x - 5 \]
\[ = 4x(-x + 1) - 5(-x + 1) \]
\[ = (4x - 5)(-x + 1) \]

Now verify the factorization by multiplying $(4x - 5)(-x + 1)$ to obtain $-4x^2 + 9x - 5$. 
37. Using the ac-test, first look for two numbers whose product is \( ac = (2)(-35) = -70 \) and whose sum is \( b = -3 \). The solutions are \(-10\) and \(7\). Then
\[
2x^2 - 3x - 35 = 2x^2 - 10x + 7x - 35 \\
= 2x(x - 5) + 7(x - 5) \\
= (2x + 7)(x - 5)
\]
Now verify the factorization by multiplying \((2x + 7)(x - 5)\) to obtain \(2x^2 - 3x - 35\).

39. To find the zeroes, set \( f(x) = 0 \) and factor.
\[
0 = 2x^2 - 3x - 20 \\
0 = (2x + 5)(x - 4)
\]
By the zero product property, either
\[
2x + 5 = 0 \quad \text{or} \quad x - 4 = 0.
\]
Solve these linear equations independently.
\[
x = -5/2 \quad \text{or} \quad x = 4
\]

41. To find the zeroes, set \( f(x) = 0 \) and factor.
\[
0 = -2x^2 + x + 28 \\
0 = -(2x^2 - x - 28) \\
0 = -(2x + 7)(x - 4)
\]
By the zero product property, either
\[
2x + 7 = 0 \quad \text{or} \quad x - 4 = 0.
\]
Solve these linear equations independently.
\[
x = -7/2 \quad \text{or} \quad x = 4
\]

43. To find the zeroes, set \( f(x) = 0 \) and factor.
\[
0 = 3x^2 - 20x + 12 \\
0 = (3x - 2)(x - 6)
\]
By the zero product property, either
\[
3x - 2 = 0 \quad \text{or} \quad x - 6 = 0.
\]
Solve these linear equations independently.
\[
x = 2/3 \quad \text{or} \quad x = 6
\]
45. To find the zeroes, set \( f(x) = 0 \) and factor.

\[
0 = -4x^2 + 4x + 15 \\
0 = -(4x^2 - 4x - 15) \\
0 = -(2x + 3)(2x - 5)
\]

By the zero product property, either

\[
2x + 3 = 0 \quad \text{or} \quad 2x - 5 = 0.
\]

Solve these linear equations independently.

\[
x = -3/2 \quad \text{or} \quad x = 5/2
\]

47. To find the zeroes with your calculator, press 2nd TRACE to access the CALC menu and choose 2:zero.

Repeat this process for the second zero. We get \((3.5, 0)\) and \((-5, 0)\).
Chapter 5  Quadratic Functions

\[ 0 = 2x^2 + 3x - 35 \]
\[ 0 = (2x - 7)(x + 5) \]

By the zero product property, either
\[ 2x - 7 = 0 \quad \text{or} \quad x + 5 = 0. \]

Solve these linear equations independently.
\[ x = \frac{7}{2} \quad \text{or} \quad x = -5 \]

49. To find the zeroes with your calculator, press 2nd TRACE to access the CALC menu and choose 2:zero.

![Calculator images showing zero finding process](image)

Use the left arrow to move the cursor along the curve until it is to the left of the first zero. Hit ENTER.

Use the right arrow to move the cursor until it is to the right of the same zero. Hit ENTER.

Finally hit ENTER near that same zero for the guess, and you get the zero.

Repeat this process for the second zero. We get \((5.5, 0)\) and \((-3, 0)\).

![Graph of quadratic function](image)

To find the zeroes algebraically, set \(y = 0\) and solve for \(x\):
\[ 0 = -2x^2 + 5x + 33 \]
\[ 0 = -(2x^2 - 5x - 33) \]
\[ 0 = -(2x - 11)(x + 3) \]
By the zero product property, either
\[ 2x - 11 = 0 \quad \text{or} \quad x + 3 = 0. \]

Solve these linear equations independently.
\[ x = 11/2 \quad \text{or} \quad x = -3 \]

51. To find the zeroes with your calculator, press 2nd TRACE to access the CALC menu and choose 2:zero.

Repeat this process for the second zero. We get (−0.5, 0) and (6.5, 0).

To find the zeroes, set \( y = 0 \) and solve for \( x \):
\[ 0 = 4x^2 - 24x - 13 \]
\[ 0 = (2x + 1)(2x - 13) \]

By the zero product property, either
\[ 2x + 1 = 0 \quad \text{or} \quad 2x - 13 = 0. \]
Solve these linear equations independently.

\[ x = -\frac{1}{2} \quad \text{or} \quad x = \frac{13}{2} \]

53. First, complete the square:

\[
\begin{align*}
f(x) &= 2x^2 - 8x - 24 \\
&= 2(x^2 - 4x - 12) \\
&= 2 \left( x^2 - 4x + 4 - 4 - 12 \right) \\
&= 2 \left( (x - 2)^2 - 16 \right) \\
&= 2(x - 2)^2 - 32
\end{align*}
\]

Read off the vertex as \((h, k) = (2, -32)\). The axis of symmetry is a vertical line through the vertex with equation \(x = 2\).

To find the \(x\)-intercepts algebraically, set \(y = 0\) and factor.

\[
\begin{align*}
0 &= 2x^2 - 8x - 24 \\
0 &= 2(x^2 - 4x - 12) \\
0 &= 2(x - 6)(x + 2)
\end{align*}
\]

By the zero product property, either

\[ x - 6 = 0 \quad \text{or} \quad x + 2 = 0. \]

Solve these linear equations independently.

\[ x = 6 \quad \text{or} \quad x = -2 \]

So the \(x\)-intercepts are \((6, 0)\) and \((-2, 0)\).

Lastly, to find the \(y\)-intercept, set \(x = 0\) in the equation and solve for \(y\):

\[
\begin{align*}
y &= 2x^2 - 8x - 24 \\
y &= 2(0)^2 - 8(0) - 24 \\
y &= -24
\end{align*}
\]

So the \(y\)-intercept is \((0, -24)\).

Finally, put this all together to make the graph.
To find the domain of $f$, mentally project every point of the graph onto the $x$-axis, as shown on the left below. This covers the entire $x$-axis, so the domain $= (-\infty, \infty)$. To find the range, mentally project every point of the graph onto the $y$-axis, as shown on the right below. The shaded interval on the $y$-axis is range $= [-32, \infty)$.

55. First, complete the square:

\[
f(x) = -2x^2 - 4x + 16
= -2(x^2 + 2x - 8)
= -2(x^2 + 2x + 1 - 1 - 8)
= -2((x^2 + 2x + 1) - 1 - 8)
= -2((x + 1)^2 - 9)
= -2(x + 1)^2 + 18
\]

Read off the vertex as $(h, k) = (-1, 18)$. The axis of symmetry is a vertical line through the vertex with equation $x = -1$.

To find the $x$-intercepts algebraically, set $y = 0$ and factor.
0 = \(-2x^2 - 4x + 16\)
\[0 = -2(x^2 + 2x - 8)\]
\[0 = -2(x + 4)(x - 2)\]

By the zero product property, either
\[x + 4 = 0\quad \text{or}\quad x - 2 = 0.\]

Solve these linear equations independently.
\[x = -4\quad \text{or}\quad x = 2\]

So the \(x\)-intercepts are \((-4,0)\) and \((2,0)\).

Lastly, to find the \(y\)-intercept, set \(x = 0\) in the equation and solve for \(y\):
\[y = -2x^2 - 4x + 16\]
\[y = -2(0)^2 - 4(0) + 16\]
\[y = 16\]

So the \(y\)-intercept is \((0,16)\).

Finally, put this all together to make the graph.

To find the domain of \(f\), mentally project every point of the graph onto the \(x\)-axis, as shown on the left below. This covers the entire \(x\)-axis, so the domain \(= (-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \(y\)-axis, as shown on the right below. The shaded interval on the \(y\)-axis is range \(= (-\infty, 18]\).
57. First, complete the square:

\[ f(x) = 3x^2 + 18x - 48 \]
\[ = 3(x^2 + 6x - 16) \]
\[ = 3 \left( x^2 + 6x + 9 - 9 - 16 \right) \]
\[ = 3 \left( (x + 3)^2 - 25 \right) \]
\[ = 3(x + 3)^2 - 75 \]

Read off the vertex as \((h, k) = (-3, -75)\). The axis of symmetry is a vertical line through the vertex with equation \(x = -3\).

To find the \(x\)-intercepts algebraically, set \(y = 0\) and factor.

\[ 0 = 3x^2 + 18x - 48 \]
\[ 0 = 3(x^2 + 6x - 16) \]
\[ 0 = 3(x + 8)(x - 2) \]

By the zero product property, either

\[ x + 8 = 0 \quad \text{or} \quad x - 2 = 0. \]

Solve these linear equations independently.

\[ x = -8 \quad \text{or} \quad x = 2 \]

So the \(x\)-intercepts are \((-8, 0)\) and \((2, 0)\).

Lastly, to find the \(y\)-intercept, set \(x = 0\) in the equation and solve for \(y\):

\[ y = 3x^2 + 18x - 48 \]
\[ y = 3(0)^2 + 18(0) - 48 \]
\[ y = -48 \]
So the $y$-intercept is $(0, -48)$.

Finally, put this all together to make the graph.

To find the domain of $f$, mentally project every point of the graph onto the $x$-axis, as shown on the left below. This covers the entire $x$-axis, so the domain $= (-\infty, \infty)$. To find the range, mentally project every point of the graph onto the $y$-axis, as shown on the right below. The shaded interval on the $y$-axis is range $= [-75, \infty)$.
59. First, complete the square:

\[ f(x) = 2x^2 + 10x - 48 \]

\[ = 2(x^2 + 5x - 24) \]

\[ = 2 \left( x^2 + 5x + \frac{25}{4} - \frac{25}{4} - 24 \right) \]

\[ = 2 \left( x^2 + 5x + \frac{25}{4} - \frac{25}{4} - 24 \right) \]

\[ = 2 \left( \left( x + \frac{5}{2} \right)^2 - \frac{25}{4} - 96 \right) \]

\[ = 2 \left( \left( x + \frac{5}{2} \right)^2 - \frac{121}{4} \right) \]

\[ = 2 \left( x + \frac{5}{2} \right)^2 - \frac{121}{2} \]

Read off the vertex as \((h, k) = (-5/2, -121/2)\). The axis of symmetry is a vertical line through the vertex with equation \(x = -5/2\).

To find the \(x\)-intercepts algebraically, set \(y = 0\) and factor.

\[ 0 = 2x^2 + 10x - 48 \]

\[ 0 = 2(x^2 + 5x - 24) \]

\[ 0 = 2(x + 8)(x - 3) \]

By the zero product property, either

\[ x + 8 = 0 \quad \text{or} \quad x - 3 = 0. \]

Solve these linear equations independently.

\[ x = -8 \quad \text{or} \quad x = 3 \]

So the \(x\)-intercepts are \((-8, 0)\) and \((3, 0)\).

Lastly, to find the \(y\)-intercept, set \(x = 0\) in the equation and solve for \(y\):

\[ y = 2x^2 + 10x - 48 \]

\[ y = 2(0)^2 + 10(0) - 48 \]

\[ y = -48 \]

So the \(y\)-intercept is \((0, -48)\).

Finally, put this all together to make the graph.
To find the domain of $f$, mentally project every point of the graph onto the $x$-axis, as shown on the left below. This covers the entire $x$-axis, so the domain = $(-\infty, \infty)$. To find the range, mentally project every point of the graph onto the $y$-axis, as shown on the right below. The shaded interval on the $y$-axis is range = $[-121/2, \infty)$.

61. To find the solutions of $ax^2+bx+c = 0$, note where the graph of $f(x) = ax^2+bx+c$ crosses the $x$-axis (see figure below). Thus, the solutions are $-2$ and $3$. 
63. To find the solutions of $ax^2 + bx + c = 0$, note where the graph of $f(x) = ax^2 + bx + c$ crosses the $x$-axis (see figure below). Thus, the solutions are $-3$ and $0$.

65. To find the solutions of $ax^2 + bx + c = 0$, note where the graph of $f(x) = ax^2 + bx + c$ crosses the $x$-axis (see figure below). Thus, the solutions are $-3$ and $0$. 
5.4 Exercises

In Exercises 1-8, find all real solutions of the given equation. Use a calculator to approximate the answers, correct to the nearest hundredth (two decimal places).

1. \( x^2 = 36 \)
2. \( x^2 = 81 \)
3. \( x^2 = 17 \)
4. \( x^2 = 13 \)
5. \( x^2 = 0 \)
6. \( x^2 = -18 \)
7. \( x^2 = -12 \)
8. \( x^2 = 3 \)

In Exercises 9-16, find all real solutions of the given equation. Use a calculator to approximate your answers to the nearest hundredth.

9. \( (x - 1)^2 = 25 \)
10. \( (x + 3)^2 = 9 \)
11. \( (x + 2)^2 = 0 \)
12. \( (x - 3)^2 = -9 \)
13. \( (x + 6)^2 = -81 \)
14. \( (x + 7)^2 = 10 \)
15. \( (x - 8)^2 = 15 \)
16. \( (x + 10)^2 = 37 \)

In Exercises 17-28, perform each of the following tasks for the given quadratic function.

i. Set up a coordinate system on a sheet of graph paper. Label and scale each axis. Remember to draw all lines with a ruler.

ii. Place the quadratic function in vertex form. Plot the vertex on your coordinate system and label it with its coordinates. Draw the axis of symmetry on your coordinate system and label it with its equation.

iii. Use the quadratic formula to find the \( x \)-intercepts of the parabola. Use a calculator to approximate each intercept, correct to the nearest tenth, and use these approximations to plot the \( x \)-intercepts on your coordinate system. However, label each \( x \)-intercept with its exact coordinates.

iv. Plot the \( y \)-intercept on your coordinate system and its mirror image across the axis of symmetry and label each with their coordinates.

v. Using all of the information on your coordinate system, draw the graph of the parabola, then label it with the vertex form of the function. Use interval notation to state the domain and range of the quadratic function.

17. \( f(x) = x^2 - 4x - 8 \)
18. \( f(x) = x^2 + 6x - 1 \)
19. \( f(x) = x^2 + 6x - 3 \)
20. \( f(x) = x^2 - 8x + 1 \)
21. \( f(x) = -x^2 + 2x + 10 \)

1 Copyrighted material. See: http://msenux.redwoods.edu/IntAlgText/
22. \( f(x) = -x^2 - 8x - 8 \)
23. \( f(x) = -x^2 - 8x - 9 \)
24. \( f(x) = -x^2 + 10x - 20 \)
25. \( f(x) = 2x^2 - 20x + 40 \)
26. \( f(x) = 2x^2 - 16x + 12 \)
27. \( f(x) = -2x^2 + 16x + 8 \)
28. \( f(x) = -2x^2 - 24x - 52 \)

In Exercises 29-32, perform each of the following tasks for the given quadratic equation.

i. Set up a coordinate system on a sheet of graph paper. Label and scale each axis. Remember to draw all lines with a ruler.

ii. Show that the discriminant is negative.

iii. Use the technique of completing the square to put the quadratic function in vertex form. Plot the vertex on your coordinate system and label it with its coordinates. Draw the axis of symmetry on your coordinate system and label it with its equation.

iv. Plot the \( y \)-intercept and its mirror image across the axis of symmetry on your coordinate system and label each with their coordinates.

v. Because the discriminant is negative (did you remember to show that?), there are no \( x \)-intercepts. Use the given equation to calculate one additional point, then plot the point and its mirror image across the axis of symmetry and label each with their coordinates.

vi. Using all of the information on your coordinate system, draw the graph of the parabola, then label it with the vertex form of function. Use interval notation to describe the domain and range of the quadratic function.

29. \( f(x) = x^2 + 4x + 8 \)
30. \( f(x) = x^2 - 4x + 9 \)
31. \( f(x) = -x^2 + 6x - 11 \)
32. \( f(x) = -x^2 - 8x - 20 \)

In Exercises 33-36, perform each of the following tasks for the given quadratic function.

i. Set up a coordinate system on a sheet of graph paper. Label and scale each axis. Remember to draw all lines with a ruler.

ii. Use the discriminant to help determine the value of \( k \) so that the graph of the given quadratic function has exactly one \( x \)-intercept.

iii. Substitute this value of \( k \) back into the given quadratic function, then use the technique of completing the square to put the quadratic function in vertex form. Plot the vertex on your coordinate system and label it with its coordinates. Draw the axis of symmetry on your coordinate system and label it with its equation.

iv. Plot the \( y \)-intercept and its mirror image across the axis of symmetry on your coordinate system and label each with their coordinates.

v. Use the equation to calculate an additional point on either side of the axis of symmetry, then plot this point and its mirror image across the axis of symmetry and label each with their coordinates.

vi. Using all of the information on your coordinate system, draw the graph of the parabola, then label it with the vertex form of the function. Use
interval notation to describe the domain and range of the quadratic function.

33. \( f(x) = x^2 - 4x + 4k \)
34. \( f(x) = x^2 + 6x + 3k \)
35. \( f(x) = kx^2 - 16x - 32 \)
36. \( f(x) = kx^2 - 24x + 48 \)

37. Find all values of \( k \) so that the graph of the quadratic function \( f(x) = kx^2 - 3x + 5 \) has exactly two \( x \)-intercepts.

38. Find all values of \( k \) so that the graph of the quadratic function \( f(x) = 2x^2 + 7x - 4k \) has exactly two \( x \)-intercepts.

39. Find all values of \( k \) so that the graph of the quadratic function \( f(x) = 2x^2 - x + 5k \) has no \( x \)-intercepts.

40. Find all values of \( k \) so that the graph of the quadratic function \( f(x) = kx^2 - 2x - 4 \) has no \( x \)-intercepts.

In Exercises 41-50, find all real solutions, if any, of the equation \( f(x) = b \).

41. \( f(x) = 63x^2 + 74x - 1; b = 8 \)
42. \( f(x) = 64x^2 + 128x + 64; b = 0 \)
43. \( f(x) = x^2 - x - 5; b = 2 \)
44. \( f(x) = 5x^2 - 5x; b = 3 \)
45. \( f(x) = 4x^2 + 4x - 1; b = -2 \)
46. \( f(x) = 2x^2 - 9x - 3; b = -1 \)
47. \( f(x) = 2x^2 + 4x + 6; b = 0 \)
48. \( f(x) = 24x^2 - 54x + 27; b = 0 \)
49. \( f(x) = -3x^2 + 2x - 13; b = -5 \)
50. \( f(x) = x^2 - 5x - 7; b = 0 \)

In Exercises 51-60, find all real solutions, if any, of the quadratic equation.

51. \( -2x^2 + 7 = -3x \)
52. \( -x^2 = -9x + 7 \)
53. \( x^2 - 2 = -3x \)
54. \( 81x^2 = -162x - 81 \)
55. \( 9x^2 + 81 = -54x \)
56. \( -30x^2 - 28 = -62x \)
57. \( -x^2 + 6 = 7x \)
58. \( -8x^2 = 4x + 2 \)
59. \( 4x^2 + 3 = -x \)
60. \( 27x^2 = -66x + 16 \)

In Exercises 61-66, find all of the \( x \)-intercepts, if any, of the given function.

61. \( f(x) = -4x^2 - 4x - 5 \)
62. \( f(x) = 49x^2 - 28x + 4 \)
63. \( f(x) = -56x^2 + 47x + 18 \)
64. \( f(x) = 24x^2 + 34x + 12 \)
65. \( f(x) = 36x^2 + 96x + 64 \)
66. \( f(x) = 5x^2 + 2x + 3 \)

In Exercises 67-74, determine the number of real solutions of the equation.

67. \( 9x^2 + 6x + 1 = 0 \)
68. \(7x^2 - 12x + 7 = 0\)

69. \(-6x^2 + 4x - 7 = 0\)

70. \(-8x^2 + 11x - 4 = 0\)

71. \(-5x^2 - 10x - 5 = 0\)

72. \(6x^2 + 11x + 2 = 0\)

73. \(-7x^2 - 4x + 5 = 0\)

74. \(6x^2 + 10x + 4 = 0\)
5.4 Solutions

1. Because $6^2 = 36$ and $(-6)^2 = 36$, the equation $x^2 = 36$ has two solutions, $x = \pm 6$. Take these square roots on your calculator as shown in (a) and (b).

3. The equation $x^2 = 17$ has two solutions, $x = \pm \sqrt{17}$, since $(\sqrt{17})^2 = 17$ and $(-\sqrt{17})^2 = 17$. Approximate these square roots on your calculator as shown in (a) and (b).

5. The solutions of the equation $x^2 = 0$ are $x = \pm \sqrt{0}$, but $\sqrt{0} = 0$, so we have $x = \pm 0$. However, 0 is the same as $-0$, so really there is only one solution, $x = 0$. This single solution is also supported by the calculator screens (a) and (b).
7. The equation $x^2 = -12$ has no real solutions. It is not possible to square a real number and get $-12$. The fact that $\sqrt{-12}$ is not a real number is supported by the following calculator results (provided you have REAL selected in the MODE menu).

![Calculator Display](a) ![Calculator Display](b)

In similar fashion, $-\sqrt{-12}$ is not a real number.

![Calculator Display](c) ![Calculator Display](d)

9. If $(x - 1)^2 = 25$, then there are two possibilities for $x - 1$, namely

$$x - 1 = \pm \sqrt{25}.$$  

To solve for $x$, add 1 to both sides of this last equation.

$$x = 1 \pm \sqrt{25}$$

$$x = 1 \pm 5.$$  

So the solutions are $1 + 5 = 6$ and $1 - 5 = -4$. These can also be computed on your calculator.

![Calculator Display](a) ![Calculator Display](b)
11. If \( (x + 2)^2 = 0 \), then there is only one possibility for \( x + 2 \), namely
\[
x + 2 = 0.
\]
To solve for \( x \), subtract 2 from both sides of this last equation.
\[
x = -2.
\]
So the solution is \( x = -2 \).

13. If \( x \) is a real number, then so is \( x + 6 \). It’s not possible to square the real number \( x + 6 \) and get a negative number like -81. Hence, the equation \( (x + 6)^2 = -81 \) has no real solutions.

15. If \( (x - 8)^2 = 15 \), then there are two possibilities for \( x - 8 \), namely
\[
x - 8 = \pm \sqrt{15}.
\]
To solve for \( x \), add 8 to both sides of this last equation.
\[
x = 8 \pm \sqrt{15}.
\]
Our TI83 gives the following approximations.

17. First, complete the square:

\[
f(x) = x^2 - 4x - 8
= x^2 - 4x + 4 - 4 - 8
= (x^2 - 4x + 4) - 4 - 8
= (x - 2)^2 - 12
\]

Read off the vertex as \((h, k) = (2, -12)\). The axis of symmetry is a vertical line through the vertex with equation \( x = 2 \).

To find the \( x \)-intercepts algebraically, set \( x^2 - 4x - 8 = 0 \) and use the quadratic formula with \( a = 1 \), \( b = -4 \) and \( c = -8 \):
Chapter 5  Quadratic Functions

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ = 4 \pm \sqrt{(-4)^2 - 4(1)(-8)} \]

\[ = \frac{4 \pm \sqrt{16 + 32}}{2} \]

\[ = \frac{4 \pm \sqrt{48}}{2} \]

So the x-intercepts are \((4 - \sqrt{48})/2, 0\) and \((4 + \sqrt{48})/2, 0\). These are approximated on your calculator in (a) and (b).

Lastly, to find the y-intercept, set \(x = 0\) in the equation and solve for \(y\):

\[ f(x) = x^2 - 4x - 8 \]

\[ f(0) = 0^2 - 4(0) - 8 \]

\[ f(0) = -8 \]

So the y-intercept is \((0, -8)\). Finally, put this all together to make the graph.

To find the domain of \(f\), mentally project every point of the graph onto the \(x\)-axis, as shown on the left below. This covers the entire \(x\)-axis, so the domain= \((-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \(y\)-axis, as shown on the right below. The shaded interval on the \(y\)-axis is range= \([-12, \infty)\).
19. First, complete the square:

\[ f(x) = x^2 + 6x - 3 \]
\[ = x^2 + 6x + 9 - 9 - 3 \]
\[ = (x^2 + 6x + 9) - 9 - 3 \]
\[ = (x + 3)^2 - 12 \]

Read off the vertex as \((h, k) = (-3, -12)\). The axis of symmetry is a vertical line through the vertex with equation \(x = -3\).

To find the \(x\)-intercepts algebraically, set \(x^2 + 6x - 3 = 0\) and use the quadratic formula with \(a = 1\), \(b = 6\) and \(c = -3\):

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(-3)}}{2(1)} \]
\[ = \frac{-6 \pm \sqrt{36 + 12}}{2} \]
\[ = \frac{-6 \pm \sqrt{48}}{2} \]

So the \(x\)-intercepts are \((-6 - \sqrt{48})/2, 0\) and \((-6 + \sqrt{48})/2, 0\). These are approximated on your calculator in (a) and (b).
Lastly, to find the $y$-intercept, set $x = 0$ in the equation and solve for $y$

\[
f(x) = x^2 + 6x - 3 \\
f(0) = 0^2 + 6(0) - 3 \\
f(0) = -3
\]

So the $y$-intercept is $(0, -3)$. Finally, put this all together to make the graph.

To find the domain of $f$, mentally project every point of the graph onto the $x$-axis, as shown on the left below. This covers the entire $x$-axis, so the domain= $(-\infty, \infty)$. To find the range, mentally project every point of the graph onto the $y$-axis, as shown on the right below. The shaded interval on the $y$-axis is range= $[-12, \infty)$.

21. First, complete the square:
\[ f(x) = -x^2 + 2x + 10 \]
\[ = -(x^2 - 2x - 10) \]
\[ = -(x^2 - 2x + 1 - 10) \]
\[ = -(x^2 - 2x + 1) - 10 \]
\[ = -(x - 1)^2 - 11 \]
\[ = -(x - 1)^2 + 11 \]

Read off the vertex as \((h, k) = (1, 11)\). The axis of symmetry is a vertical line through the vertex with equation \(x = 1\).

To find the \(x\)-intercepts algebraically, set \(-x^2 + 2x + 10 = 0\) and use the quadratic formula with \(a = -1\), \(b = 2\) and \(c = 10\):
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{(2)^2 - 4(-1)(10)}}{2(-1)} = \frac{-2 \pm \sqrt{4 + 40}}{-2} = \frac{-2 \pm \sqrt{44}}{-2}
\]
So the \(x\)-intercepts are \((-2 + \sqrt{44})/(-2), 0\) and \((-2 - \sqrt{44})/(-2), 0\). These are approximated on your calculator in (a) and (b).

\[ \text{(a)} \quad \text{(b)} \]

Lastly, to find the \(y\)-intercept, set \(x = 0\) in the equation and solve for \(y\):
\[ f(x) = -x^2 + 2x + 10 \]
\[ f(0) = -(0)^2 + 2(0) + 10 \]
\[ f(0) = 10 \]
So the \(y\)-intercept is \((0, 10)\). Finally, put this all together to make the graph.
23. First, complete the square:

\[ f(x) = -x^2 - 8x - 9 \]
\[ = -(x^2 + 8x + 9) \]
\[ = - (x^2 + 8x + 16 - 16 + 9) \]
\[ = - ((x^2 + 8x + 16) - 16 + 9) \]
\[ = - ((x + 4)^2 - 7) \]
\[ = - (x + 4)^2 + 7 \]

Read off the vertex as \((h, k) = (-4, 7)\). The axis of symmetry is a vertical line through the vertex with equation \(x = -4\).
To find the \( x \)-intercepts algebraically, set \(-x^2 - 8x - 9 = 0\) and use the quadratic formula with \( a = -1 \), \( b = -8 \) and \( c = -9 \):

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
= \frac{8 \pm \sqrt{(-8)^2 - 4(-1)(-9)}}{2(-1)}
\]

\[
= \frac{8 \pm \sqrt{64 - 36}}{-2}
\]

\[
= \frac{8 \pm \sqrt{28}}{-2}
\]

So the \( x \)-intercepts are \(((8 - \sqrt{28})/(-2), 0)\) and \(((8 + \sqrt{28})/(-2), 0)\). These are approximated on your calculator in (a) and (b).

Lastly, to find the \( y \)-intercept, set \( x = 0 \) in the equation and solve for \( y \):

\[
f(x) = -x^2 - 8x - 9
\]

\[
f(0) = -(0)^2 - 8(0) - 9
\]

\[
f(0) = -9
\]

So the \( y \)-intercept is \((0, -9)\).

Finally, put this all together to make the graph.
To find the domain of $f$, mentally project every point of the graph onto the $x$-axis, as shown on the left below. This covers the entire $x$-axis, so the domain = $(-\infty, \infty)$. To find the range, mentally project every point of the graph onto the $y$-axis, as shown on the right below. The shaded interval on the $y$-axis is range = $(-\infty, 7]$.

25. First, complete the square:

\[
f(x) = 2x^2 - 20x + 40
\]
\[
= 2(x^2 - 10x + 20)
\]
\[
= 2 \left( x^2 - 10x + 25 - 25 + 20 \right)
\]
\[
= 2 \left( (x - 5)^2 - 5 \right)
\]
\[
= 2 (x - 5)^2 - 10
\]

Read off the vertex as $(h, k) = (5, -10)$. The axis of symmetry is a vertical line through the vertex with equation $x = 5$.

To find the $x$-intercepts algebraically, set $2x^2 - 20x + 40 = 0$ and use the quadratic formula with $a = 2$, $b = -20$ and $c = 40$:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
= \frac{20 \pm \sqrt{(-20)^2 - 4(2)(40)}}{2(2)}
\]
\[
= \frac{20 \pm \sqrt{400 - 320}}{4}
\]
\[
= \frac{20 \pm \sqrt{80}}{4}
\]
So the x-intercepts are \((20-\sqrt{80})/4, 0\) and \((20+\sqrt{80})/4, 0\). These are approximated on your calculator in (a) and (b).

Lastly, to find the y-intercept, set \(x = 0\) in the equation and solve for \(y\):

\[
f(x) = 2x^2 - 20x + 40
\]
\[
f(0) = 2(0)^2 - 20(0) + 40
\]
\[
f(0) = 40
\]

So the y-intercept is \((0, 40)\).

Finally, put this all together to make the graph.

To find the domain of \(f\), mentally project every point of the graph onto the \(x\)-axis, as shown on the left below. This covers the entire \(x\)-axis, so the domain= \((-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \(y\)-axis, as shown on the right below. The shaded interval on the \(y\)-axis is range= \([-10, \infty)\).
27. First, complete the square:

\[ f(x) = -2x^2 + 16x + 8 \]
\[ = -2(x^2 - 8x - 4) \]
\[ = -2 \left( x^2 - 8x + 16 - 16 - 4 \right) \]
\[ = -2 \left( (x - 4)^2 - 20 \right) \]
\[ = -2(x - 4)^2 + 40 \]

Read off the vertex as \((h, k) = (4, 40)\). The axis of symmetry is a vertical line through the vertex with equation \(x = 4\).

To find the \(x\)-intercepts algebraically, set \(-2x^2 + 16x + 8 = 0\) and use the quadratic formula with \(a = -2\), \(b = 16\) and \(c = 8\):

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
= \frac{-16 \pm \sqrt{(16)^2 - 4(-2)(8)}}{2(-2)}
\]
\[
= \frac{-16 \pm \sqrt{256 + 64}}{-4}
\]
\[
= \frac{-16 \pm \sqrt{320}}{-4}
\]

So the \(x\)-intercepts are \(((−16 − \sqrt{320})/(-4), 0)\) and \(((−16 + \sqrt{320})/(-4), 0)\). These are approximated on your calculator in \((a)\) and \((b)\).
Lastly, to find the $y$-intercept, set $x = 0$ in the equation and solve for $y$:

$$f(x) = -2x^2 + 16x + 8$$
$$f(0) = -2(0)^2 + 16(0) + 8$$
$$f(0) = 8$$

So the $y$-intercept is $(0, 8)$.

Finally, put this all together to make the graph.

To find the domain of $f$, mentally project every point of the graph onto the $x$-axis, as shown on the left below. This covers the entire $x$-axis, so the domain $= (-\infty, \infty)$. To find the range, mentally project every point of the graph onto the $y$-axis, as shown on the right below. The shaded interval on the $y$-axis is range $= (-\infty, 40]$. 
29. For the quadratic function \( f(x) = x^2 + 4x + 8 \), \( a = 1 \), \( b = 4 \) and \( c = 8 \), so the discriminant is \( D = b^2 - 4ac = (4)^2 - 4(1)(8) = 16 - 32 = -16 < 0 \). A negative discriminant tells us that this function has no \( x \)-intercepts (zeroes).

Complete the square to find the vertex.

\[
\begin{align*}
  f(x) &= x^2 + 4x + 8 \\
  &= x^2 + 4x + 4 - 4 + 8 \\
  &= (x^2 + 4x + 4) - 4 + 8 \\
  &= (x + 2)^2 - 4 + 8 \\
  &= (x + 2)^2 + 4
\end{align*}
\]

Read off the vertex as \((h, k) = (-2, 4)\). The axis of symmetry is a vertical line through the vertex with equation \( x = -2 \).

To find the \( y \)-intercept, set \( x = 0 \) in the equation and solve for \( y \):

\[
\begin{align*}
  f(x) &= x^2 + 4x + 8 \\
  f(0) &= 0^2 + 4(0) + 8 \\
  f(0) &= 8
\end{align*}
\]

So the \( y \)-intercept is \((0, 8)\).

Now calculate an additional point and mirror it over, and then complete the graph.
To find the domain of $f$, mentally project every point of the graph onto the $x$-axis, as shown on the left below. This covers the entire $x$-axis, so the domain = $(-\infty, \infty)$. To find the range, mentally project every point of the graph onto the $y$-axis, as shown on the right below. The shaded interval on the $y$-axis is range $= [4, \infty)$.

31. For the quadratic function $f(x) = -x^2 + 6x - 11$, $a = -1$, $b = 6$ and $c = -11$, so the discriminant is $D = b^2 - 4ac = (6)^2 - 4(-1)(-11) = 36 - 44 = -8 < 0$. A negative discriminant tells us that this function has no $x$-intercepts (zeroes).

Complete the square to find the vertex.

$$f(x) = -x^2 + 6x - 11$$
$$= -(x^2 - 6x + 11)$$
$$= -(x^2 - 6x + 9 - 9 + 11)$$
$$= -((x^2 - 6x + 9) - 9 + 11)$$
$$= -((x - 3)^2 + 2)$$
$$= -(x - 3)^2 - 2$$
Read off the vertex as \((h, k) = (3, -2)\). The axis of symmetry is a vertical line through the vertex with equation \(x = 3\).

To find the \(y\)-intercept, set \(x = 0\) in the equation and solve for \(y\):

\[
\begin{align*}
f(x) &= -x^2 + 6x - 11 \\
f(0) &= -(0)^2 + 6(0) - 11 \\
f(0) &= -11
\end{align*}
\]

So the \(y\)-intercept is \((0, -11)\).

Now calculate an additional point and mirror it over, and then complete the graph.

\[
\begin{array}{c|c}
 x & y = - (x - 3)^2 - 2 \\
 4 & -3
\end{array}
\]

To find the domain of \(f\), mentally project every point of the graph onto the \(x\)-axis, as shown on the left below. This covers the entire \(x\)-axis, so the domain = \((−\infty, \infty)\). To find the range, mentally project every point of the graph onto the \(y\)-axis, as shown on the right below. The shaded interval on the \(y\)-axis is range = \((−\infty, -2]\).

33. The graph of a quadratic function has exactly one \(x\)-intercept when the discriminant is zero. For this function \(f(x) = x^2 - 4x + 4k\), \(a = 1\), \(b = -4\) and \(c = 4k\), so the discriminant is \(D = b^2 - 4ac = (-4)^2 - 4(1)(4k) = 16 - 16k\). In order for there to be
only one $x$-intercept, this must be zero. So, set it equal to zero and solve for the value of $k$.

\[
0 = D \\
0 = 16 - 16k
\]

Subtracting 16 from each side,

\[-16 = -16k
\]

Now divide both sides by -16:

\[1 = k
\]

So, if $k = 1$, then $f(x)$ will have one $x$-intercept. So, replacing $k$ with 1, we get that $f(x) = x^2 - 4x + 4$.

Complete the square to find the vertex. $f(x)$ already has the constant term we need, so just factor to complete the square.

\[
f(x) = x^2 - 4x + 4 \\
= (x - 2)^2
\]

Read off the vertex as $(h, k) = (2, 0)$. The axis of symmetry is a vertical line through the vertex with equation $x = 2$.

To find the $y$-intercept, set $x = 0$ in the equation and solve for $y$:

\[
f(x) = x^2 - 4x + 4 \\
f(0) = (0)^2 - 4(0) + 4 \\
f(0) = 4
\]

So the $y$-intercept is $(0, 4)$.

Now calculate an additional point and mirror it over, and then complete the graph.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = (x - 2)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>16</td>
</tr>
</tbody>
</table>
To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain = \(( -\infty, \infty )\). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range = \([0, \infty)\).

35. The graph of a quadratic function has exactly one \( x \)-intercept when the discriminant is zero. For this function \( f(x) = kx^2 - 16x - 32 \), \( a = k \), \( b = -16 \) and \( c = -32 \), so the discriminant is
\[
D = b^2 - 4ac = (-16)^2 - 4(k)(-32) = 256 + 128k.
\]
In order for there to be only one \( x \)-intercept, this must be zero. So, set it equal to zero and solve for the value of \( k \):
\[
0 = D \\
0 = 256 + 128k
\]
Subtracting 256 from each side,
\[
-256 = 128k
\]
Now divide both sides by 128:
\[
-2 = k
\]
So, if \( k = -2 \), then \( f(x) \) will have one \( x \)-intercept. So, replacing \( k \) with \(-2 \), we get that
\[
f(x) = -2x^2 - 16x - 32
\]
Complete the square to find the vertex.
\[
f(x) = -2x^2 - 16x - 32 \\
= -2(x^2 + 8x + 16) \\
= -2(x + 4)^2
\]
Read off the vertex as \((h, k) = (-4, 0)\). The axis of symmetry is a vertical line through the vertex with equation \( x = -4 \).
To find the \( y \)-intercept, set \( x = 0 \) in the equation and solve for \( y \):
Section 5.4 The Quadratic Formula

\[ f(x) = -2x^2 - 16x - 32 \]
\[ f(0) = -2(0)^2 - 16(0) - 32 \]
\[ f(0) = -32 \]

So the \( y \)-intercept is \((0, -32)\).

Now calculate an additional point and mirror it over, and then complete the graph.

\[
\begin{array}{c|c}
  x & y = -2(x + 4)^2 \\
  -2 & -8 \\
\end{array}
\]

To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain = \((-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range = \((-\infty, 0]\).

37. The graph of a quadratic function has exactly two \( x \)-intercepts when the discriminant is positive. For this function \( f(x) = kx^2 - 3x + 5 \), \( a = k \), \( b = -3 \) and \( c = 5 \), so the discriminant is \( D = b^2 - 4ac = (-3)^2 - 4(k)(5) = 9 - 20k \). In order for there to be two \( x \)-intercepts, this must be positive. So, set it greater than zero and solve for the value of \( k \).
0 < D
0 < 9 − 20k

Subtracting 9 from each side,

−9 < 20k

Now divide both sides by -20 (reverse the direction of the inequality because we’re dividing by a negative number):

9/20 > k

So, any k less than 9/20 will work. Our solution set is therefore \( \{ k : k < 9/20 \} \)

39. The graph of a quadratic function has no x-intercepts when the discriminant is negative. For this function \( f(x) = 2x^2 - x + 5k \), \( a = 2 \), \( b = -1 \) and \( c = 5k \), so the discriminant is \( D = b^2 - 4ac = (-1)^2 - 4(2)(5k) = 1 - 40k \). In order for there to be no x-intercepts, this must be negative. So, set it less than zero and solve for the value of k.

0 > D
0 > 1 − 40k

Subtracting 1 from each side,

−1 > −40k

Now divide both sides by -40 (reverse the direction of the inequality because we’re dividing by a negative number):

1/40 < k

So, any k greater than 1/40 will work. Our solution set is therefore \( \{ k : k > 1/40 \} \)

41. Use factoring and the principle of zero products:

\[ 63x^2 + 74x - 1 = 8 \]
\[ \Rightarrow 63x^2 + 74x - 9 = 0 \]
\[ \Rightarrow (7x + 9)(9x - 1) = 0 \]
\[ \Rightarrow x = -\frac{9}{7}, \frac{1}{9} \]

Alternatively, use the quadratic formula \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \).

43. \( x^2 - x - 5 = 2 \Rightarrow x^2 - x - 7 = 0 \). The polynomial on the left side does not factor, so use the quadratic formula:
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-7)}}{2(1)} \]

\[ = \frac{1 \pm \sqrt{29}}{2} \]

\[ = \frac{1 + \sqrt{29}}{2}, \frac{1 - \sqrt{29}}{2} \]

45. Use factoring and the principle of zero products:

\[ 4x^2 + 4x - 1 = -2 \]

\[ \implies 4x^2 + 4x + 1 = 0 \]

\[ \implies (2x + 1)^2 = 0 \]

\[ \implies x = -\frac{1}{2} \]

Alternatively, use the quadratic formula \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \).

47. The polynomial \( 2x^2 + 4x + 6 \) does not factor, so use the quadratic formula:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ = \frac{-4 \pm \sqrt{4^2 - 4(2)(6)}}{2(2)} \]

\[ = \frac{-4 \pm \sqrt{-32}}{4} \]

Since \( \sqrt{-32} \) is not a real number, there are no real solutions.

49. \( -3x^2 + 2x - 13 = -5 \implies -3x^2 + 2x - 8 = 0 \). The polynomial on the left side does not factor, so use the quadratic formula:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ = \frac{-2 \pm \sqrt{2^2 - 4(-3)(-8)}}{2(-3)} \]

\[ = \frac{-2 \pm \sqrt{-92}}{-6} \]

Since \( \sqrt{-92} \) is not a real number, there are no real solutions.
51. \(-2x^2 + 7 = -3x \implies -2x^2 + 3x + 7 = 0\). The polynomial on the left side does not factor, so use the quadratic formula:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
= \frac{-3 \pm \sqrt{3^2 - 4(-2)(7)}}{2(-2)}
\]

\[
= \frac{-3 \pm \sqrt{65}}{-4}
\]

\[
= \frac{3 - \sqrt{65}}{4}, \frac{3 + \sqrt{65}}{4}
\]

53. \(x^2 - 2 = -3x \implies x^2 + 3x - 2 = 0\). The polynomial on the left side does not factor, so use the quadratic formula:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
= \frac{-3 \pm \sqrt{3^2 - 4(1)(-2)}}{2(1)}
\]

\[
= \frac{-3 \pm \sqrt{17}}{2}
\]

\[
= -\frac{3 - \sqrt{17}}{2}, -\frac{3 + \sqrt{17}}{2}
\]

55. Make one side zero, then note that each term is divisible by 9.

\[
9x^2 + 81 = -54x
\]

\(\implies 9x^2 + 54x + 81 = 0\)

\(\implies x^2 + 6x + 9 = 0\)

Factor.

\[(x + 3)^2 = 0\]

Hence, \(x = -3\) is the only solution. Alternatively, use the quadratic formula \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\).
57. \(-x^2 + 6 = 7x \implies -x^2 - 7x + 6 = 0\). The polynomial on the left side does not factor, so use the quadratic formula:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
= \frac{7 \pm \sqrt{(-7)^2 - 4(-1)(6)}}{2(-1)}
\]

\[
= \frac{7 \pm \sqrt{73}}{-2}
\]

\[
= \frac{7 + \sqrt{73}}{2}, \quad \frac{7 - \sqrt{73}}{2}
\]

59. \(4x^2 + 3 = -x \implies 4x^2 + x + 3 = 0\). The polynomial on the left side does not factor, so use the quadratic formula:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
= \frac{-1 \pm \sqrt{1^2 - 4(4)(3)}}{2(4)}
\]

\[
= \frac{-1 \pm \sqrt{-47}}{8}
\]

Since \(\sqrt{-47}\) is not a real number, there are no real solutions.

61. To find the \(x\)-intercepts, solve the equation \(f(x) = 0\). In other words, solve the equation

\(-4x^2 - 4x - 5 = 0\)

The polynomial on the left side does not factor, so use the quadratic formula:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
= \frac{4 \pm \sqrt{(-4)^2 - 4(-4)(-5)}}{2(-4)}
\]

\[
= \frac{4 \pm \sqrt{-64}}{-8}
\]

Since \(\sqrt{-64}\) is not a real number, there are no real solutions, and therefore no \(x\)-intercepts.
63. To find the x-intercepts, solve the equation $f(x) = 0$:

$$-56x^2 + 47x + 18 = 0$$

$$\Rightarrow (-8x + 9)(7x + 2) = 0$$

$$\Rightarrow x = \frac{9}{8}, -\frac{2}{7}$$

Alternatively, use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

65. To find the x-intercepts, solve the equation $f(x) = 0$. Note that each term is divisible by 4, then factor the resulting perfect square trinomial.

$$36x^2 + 96x + 64 = 0$$

$$\Rightarrow 9x^2 + 24x + 16 = 0$$

$$\Rightarrow (3x + 4)^2 = 0$$

$$\Rightarrow x = -\frac{4}{3}$$

Alternatively, use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

67. Compute the discriminant $b^2 - 4ac$ to determine the number of real solutions.

- If $b^2 - 4ac > 0$, then the equation has two real solutions.
- If $b^2 - 4ac = 0$, then the equation has one real solution.
- If $b^2 - 4ac < 0$, then the equation has no real solutions.

In this case, $b^2 - 4ac = 6^2 - 4(9)(1) = 0$, so the equation has one real solution.

69. Compute the discriminant $b^2 - 4ac$ to determine the number of real solutions.

- If $b^2 - 4ac > 0$, then the equation has two real solutions.
- If $b^2 - 4ac = 0$, then the equation has one real solution.
- If $b^2 - 4ac < 0$, then the equation has no real solutions.

In this case, $b^2 - 4ac = 4^2 - 4(-6)(-7) = -152 < 0$, so the equation has no real solutions.

71. Compute the discriminant $b^2 - 4ac$ to determine the number of real solutions.

- If $b^2 - 4ac > 0$, then the equation has two real solutions.
- If $b^2 - 4ac = 0$, then the equation has one real solution.
- If $b^2 - 4ac < 0$, then the equation has no real solutions.

In this case, $b^2 - 4ac = (-10)^2 - 4(-5)(-5) = 0$, so the equation has one real solution.
73. Compute the discriminant \( b^2 - 4ac \) to determine the number of real solutions. 

If \( b^2 - 4ac > 0 \), then the equation has two real solutions.

If \( b^2 - 4ac = 0 \), then the equation has one real solution.

If \( b^2 - 4ac < 0 \), then the equation has no real solutions.

In this case, \( b^2 - 4ac = (-4)^2 - 4(-7)(5) = 156 > 0 \), so the equation has two real solutions.
5.5 Exercises

In Exercises 1-12, write down the formula $d = vt$ and solve for the unknown quantity in the problem. Once that is completed, substitute the known quantities in the result and simplify. Make sure to check that your units cancel and provide the appropriate units for your solution.

1. If Martha maintains a constant speed of 30 miles per hour, how far will she travel in 5 hours?

2. If Jamal maintains a constant speed of 25 miles per hour, how far will he travel in 5 hours?

3. If Arturo maintains a constant speed of 30 miles per hour, how long will it take him to travel 120 miles?

4. If Mei maintains a constant speed of 25 miles per hour, how long will it take her to travel 150 miles?

5. If Allen maintains a constant speed and travels 250 miles in 5 hours, what is his constant speed?

6. If Jane maintains a constant speed and travels 300 miles in 6 hours, what is her constant speed?

7. If Jose maintains a constant speed of 15 feet per second, how far will he travel in 5 minutes?

8. If Tami maintains a constant speed of 1.5 feet per second, how far will she travel in 4 minutes?

9. If Carmen maintains a constant speed of 80 meters per minute, how far will she travel in 600 seconds?

10. If Alphonso maintains a constant speed of 15 feet per second, how long will it take him to travel 1 mile? Note: 1 mile equals 5280 feet.

11. If Hoshi maintains a constant speed of 200 centimeters per second, how long will it take her to travel 20 meters? Note: 100 centimeters equals 1 meter.

12. If Maeko maintains a constant speed and travels 5 miles in 12 minutes, what is her speed in miles per hour?

In Exercises 13-18, a plot of speed $v$ versus time $t$ is presented.

i. Make an accurate duplication of the plot on graph paper. Label and scale each axis. Mark the units on each axis.

ii. Use the graph to determine the distance traveled over the time period $[0, 5]$, using the time units given on the graph.

13. [Graph showing a plot of speed $v$ versus time $t$.]

\[ v \text{ (ft/s)} \]

\[ 0 \quad 5 \quad t \text{ (s)} \]

\[ 0 \quad 30 \quad 50 \]

\[ v \]

---

1 Copyrighted material. See: http://msenux.redwoods.edu/IntAlgText/

Version: Fall 2007
14. You’re told that a car moves with a constant acceleration of 7.5 ft/s². In your own words, explain what this means.

15. You’re told that an object will fall on a distant planet with constant acceleration 6.5 m/s². In your own words, explain what this means.

16. You’re told that the acceleration of a car is −18 ft/s². In your own words, explain what this means.

Version: Fall 2007
22. An observer on a distant planet throws an object into the air and as it moves upward he reports that the object has a constant acceleration of \(-4.5 \text{ m/s}^2\). In your own words, explain what this means.

In Exercises 23-28, perform each of the following tasks.

i. Solve the equation \(v = v_0 + at\) for the unknown quantity.

ii. Substitute the known quantities (with units) into your result, then simplify. Make sure the units cancel and provide appropriate units for your solution.

23. A rocket accelerates from rest with constant acceleration 15.8 m/s². What will be the speed of the rocket after 3 minutes?

24. A stone is dropped from rest on a distant planet and it accelerates towards the ground with constant acceleration 3.8 ft/s². What will be the speed of the stone after 2 minutes?

25. A stone is thrown downward on a distant planet with an initial speed of 20 ft/s. If the stone experiences constant acceleration of 32 ft/s², what will be the speed of the stone after 1 minute?

26. A ball is hurled upward with an initial speed of 80 m/s. If the ball experiences a constant acceleration of \(-9.8 \text{ m/s}^2\), what will be the speed of the ball at the end of 5 seconds?

27. An object is shot into the air with an initial speed of 100 m/s. If the object experiences constant deceleration of 9.8 m/s², how long will it take the ball to reach its maximum height?

28. An object is released from rest on a distant planet and after 5 seconds, its speed is 98 m/s. If the object falls with constant acceleration, determine the acceleration of the object.

In Exercises 29-42, use the appropriate equation of motion, either \(v = v_0 + at\) or \(x = x_0 + v_0t + (1/2)at^2\) or both, to solve the question posed in the exercise.

i. Select the appropriate equation of motion and solve for the unknown quantity.

ii. Substitute the known quantities (with their units) into your result and simplify. Check that cancellation of units provide units appropriate for your solution.

iii. Find a decimal approximation for your answer.

29. A rocket with initial velocity 30 m/s moves along a straight line with constant acceleration 2.5 m/s². Find the velocity and the distance traveled by the rocket at the end of 10 seconds.

30. A car is traveling at 88 ft/s when it applies the brakes and begins to slow with constant deceleration of 5 ft/s². What is its speed and how far has it traveled at the end of 5 seconds?

31. A car is traveling at 88 ft/s when it applies the brakes and slows to 58 ft/s in 10 seconds. Assuming constant deceleration, find the deceleration and the distance traveled by the car in the 10 second time interval. Hint: Compute the deceleration first.
32. A stone is hurled downward from above the surface of a distant planet with initial speed 45 m/s. At then end of 10 seconds, the velocity of the stone is 145 m/s. Assuming constant acceleration, find the acceleration of the stone and the distance traveled in the 10 second time period.

33. An object is shot into the air from the surface of the earth with an initial velocity of 180 ft/s. Find the maximum height of the object and the time it takes the object to reach that maximum height. 
*Hint: The acceleration due to gravity near the surface of the earth is well known.*

34. An object is shot into the air from the surface of a distant planet with an initial velocity of 180 m/s. Find the maximum height of the object and the time it takes the object to reach that maximum height. Assume that the acceleration due to gravity on this distant planet is 5.8 m/s². 
*Hint: Calculate the time to the maximum height first.*

35. A car is traveling down the highway at 55 mi/h when the driver spots a slide of rocks covering the road ahead and hits the brakes, providing a constant deceleration of 12 ft/s². How long does it take the car to come to a halt and how far does it travel during this time period?

36. A car is traveling down the highway in Germany at 81 km/h when the driver spots that traffic is stopped in the road ahead and hits the brakes, providing a constant deceleration of 2.3 m/s². How long does it take the car to come to a halt and how far does it travel during this time period? 
*Note: 1 kilometer equals 1000 meters.*

37. An object is released from rest at some distance over the surface of the earth. How far (in meters) will the object fall in 5 seconds and what will be its velocity at the end of this 5 second time period? 
*Hint: You should know the acceleration due to gravity near the surface of the earth.*

38. An object is released from rest at some distance over the surface of a distant planet. How far (in meters) will the object fall in 5 seconds and what will be its velocity at the end of this 5 second time period? Assume the acceleration due to gravity on the distant planet is 13.5 m/s².

39. An object is released from rest at a distance of 352 feet over the surface of the earth. How long will it take the object to impact the ground?

40. An object is released from rest at a distance of 400 meters over the surface of a distant planet. How long will it take the object to impact the ground? Assume that the acceleration due to gravity on the distant planet equals 5.3 m/s².

41. On earth, a ball is thrown upward from an initial height of 5 meters with an initial velocity of 100 m/s. How long will it take the ball to return to the ground?

42. On earth, a ball is thrown upward from an initial height of 5 feet with an initial velocity of 100 ft/s. How long will it take the ball to return to the ground?
A ball is thrown into the air near the surface of the earth. In Exercises 43-46, the initial height of the ball and the initial velocity of the ball are given. Complete the following tasks.

i. Use \( y = y_0 + v_0 t + (1/2)at^2 \) to set up a formula for the height \( y \) of the ball as a function of time \( t \). Use the appropriate constant for the acceleration due to gravity near the surface of the earth.

ii. Load the equation from the previous part into Y1 in your graphing calculator. Adjust your viewing window so that both the vertex and the time when the ball returns to the ground are visible. Copy the image onto your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax.

iii. Use the zero utility in the CALC menu of your graphing calculator to determine the time when the ball returns to the ground. Record this answer in the appropriate location on your graph.

iv. Use the quadratic formula to determine the time the ball returns to the ground. Use your calculator to find a decimal approximation of your solution. It should agree with that found using the zero utility on your graphing calculator. Be stubborn! Check your work until the answers agree.

43. \( y_0 = 50 \text{ ft}, \ v_0 = 120 \text{ ft/s} \).

44. \( y_0 = 30 \text{ m}, \ v_0 = 100 \text{ m/s} \).

45. \( y_0 = 20 \text{ m}, \ v_0 = 110 \text{ m/s} \).

46. \( y_0 = 100 \text{ ft}, \ v_0 = 200 \text{ ft/s} \).

47. A rock is thrown upward at an initial speed of 64 ft/s. How many seconds will it take the rock to rise 61 feet? Round your answer to the nearest hundredth of a second.

48. A penny is thrown downward from the top of a tree at an initial speed of 28 ft/s. How many seconds will it take the penny to fall 289 feet? Round your answer to the nearest hundredth of a second.

49. A water balloon is thrown downward from the roof of a building at an initial speed of 24 ft/s. The building is 169 feet tall. How many seconds will it take the water balloon to hit the ground? Round your answer to the nearest hundredth of a second.

50. A rock is thrown upward at an initial speed of 60 ft/s. How many seconds will it take the rock to rise 51 feet? Round your answer to the nearest hundredth of a second.

51. A ball is thrown upward from a height of 42 feet at an initial speed of 63 ft/s. How many seconds will it take the ball to hit the ground? Round your answer to the nearest hundredth of a second.

52. A rock is thrown upward from a height of 32 feet at an initial speed of 25 ft/s. How many seconds will it take the rock to hit the ground? Round your answer to the nearest hundredth of a second.
53. A penny is thrown downward from the top of a tree at an initial speed of 16 ft/s. The tree is 68 feet tall. How many seconds will it take the penny to hit the ground? Round your answer to the nearest hundredth of a second.

54. A penny is thrown downward off of the edge of a cliff at an initial speed of 32 ft/s. How many seconds will it take the penny to fall 210 feet? Round your answer to the nearest hundredth of a second.
5.5 Solutions

1. We are trying to solve for d, so \( d = vt \) is the formula we want to use.

\[
d = vt
\]
\[
= 30 \frac{\text{mi}}{\text{hr}} \times 5 \text{ hr}
\]
\[
= 150 \text{ mi}
\]

3. We are trying to solve for t, so divide \( d = vt \) by v on each side first.

\[
t = \frac{d}{v}
\]
\[
= \frac{120 \text{ mi}}{30 \text{ mi/hr}}
\]
\[
= 120 \frac{\text{mi}}{30 \text{ mi}} \times \frac{1 \text{ hr}}{30 \text{ mi}}
\]
\[
= 4 \text{ hrs}
\]

5. We are trying to solve for v, so divide \( d = vt \) by t on each side first.

\[
v = \frac{d}{t}
\]
\[
= \frac{250 \text{ mi}}{5 \text{ hrs}}
\]
\[
= 50 \text{ miles per hour}
\]

7. We are trying to solve for d, so \( d = vt \) is the formula we want to use. But notice that the rate \( v \) is given in ft/s, while time \( t \) is given as 5 minutes. We must first make the units match, so convert

\[
t = 5 \text{ min}
\]
\[
= 5 \text{ min} \times \frac{60 \text{ s}}{1 \text{ min}}
\]
\[
= 300 \text{ s}
\]

Now plug into the formula.

\[
d = vt
\]
\[
= 15 \frac{\text{ft}}{s} \times 300 \text{ s}
\]
\[
= 4500 \text{ ft}
\]
9. We are trying to solve for $d$, so $d = vt$ is the formula we want to use. But notice that the rate $v$ is given in m/min, while time $t$ is given as 600 seconds. We must first make the units match, so convert

\[
t = 600 \text{ s} \\
= 600 \text{ s} \times \frac{1 \text{ min}}{60 \text{ s}} \\
= 10 \text{ min}
\]

Now plug into the formula.

\[
d = vt \\
= 80 \text{ m/min} \times 10 \text{ min} \\
= 800 \text{ m}
\]

11. We are trying to solve for $t$, so divide $d = vt$ by $v$ on each side first to get $t = d/v$. But notice that $v$ is given in cm/s, while distance $d$ is given in meters. We must first make the units match.

\[
d = 20 \text{ m} \\
= 20 \text{ m} \times \frac{100 \text{ cm}}{1 \text{ m}} \\
= 2000 \text{ cm}
\]

Now plug in to the formula

\[
t = \frac{d}{v} \\
= \frac{2000 \text{ cm}}{200 \text{ cm/s}} \\
= \frac{2000 \text{ cm} \times 1 \text{ s}}{200 \text{ cm}} \\
= 10 \text{ s}
\]
The distance traveled is the area under the curve, which is a rectangle, so the area is length times width, or \(d = 30 \text{ ft/s} \times 5 \text{ s} = 150 \text{ ft}\).

The distance traveled is the area under the curve, which is a triangle. The formula for area of a triangle is \(\frac{1}{2} \times \text{base} \times \text{height}\), so

\[
d = \frac{1}{2} \times \text{base} \times \text{height}
\]

\[
= \frac{1}{2} \times 5 \text{ s} \times 40 \text{ m/s}
\]

\[
= 100 \text{ m}
\]
The distance traveled is the area under the curve. This can be divided into two simple geometric shapes—a triangle and a rectangle. The area is

\[
d = \text{area of triangle} + \text{area of rectangle} = \left(\frac{1}{2} \times \text{base} \times \text{height}\right) + (\text{length} \times \text{width})
\]

\[
= \left(\frac{1}{2} \times 5 \text{mi} \times 30 \text{mi/hr}\right) + \left(5 \text{mi/hr} \times 20 \text{mi/hr}\right)
\]

\[
= (75 \text{ mi}) + (100 \text{ mi})
\]

\[
= 175 \text{ mi}
\]

19. It means that the velocity of the car increases at a rate of 7.5 feet per second every second.

21. It means that the velocity of the car is decreasing at a rate of 18 feet per second every second.

23. We are trying to solve for \(v\), so \(v = v_0 + at\) is the formula we want to use. But notice that the acceleration \(a\) is given in \(m/s^2\), while time \(t\) is given as 3 min. We must first make the units match, so convert

\[
t = 3 \text{ min} = 3 \text{ min} \times \frac{60 \text{ s}}{1 \text{ min}} = 180 \text{ s}
\]

Now plug into the formula. Note that the rocket “accelerates from rest,” which means that the initial velocity is \(v_0 = 0 \text{ m/s}\).
Section 5.5 Motion

\[ v = v_0 + at \]
\[ = 0 \frac{m}{s} + \left( 15.8 \frac{m/s}{s} \times 180 \right) \]
\[ = 0 \frac{m}{s} + 2844 \frac{m}{s} \]
\[ = 2844 \text{ m/s} \]

25. We are trying to solve for \( v \), so \( v = v_0 + at \) is the formula we want to use. But notice that the acceleration \( a \) and initial speed \( v_0 \) are given in units involving seconds, while time \( t \) is given as 1 minute. We must first make the units match, so convert \( t = 1 \text{ min} = 60 \text{s} \).

The initial speed is \( v_0 = 20 \text{ ft/s} \) and the acceleration is \( a = 32 \text{ ft/s}^2 \). Now plug into the formula.

\[ v = v_0 + at \]
\[ = 20 \frac{\text{ft}}{\text{s}} + \left( 32 \frac{\text{ft/s}}{s} \times 60 \frac{s}{s} \right) \]
\[ = 20 \frac{\text{ft}}{\text{s}} + \left( 1920 \frac{\text{ft}}{s} \right) \]
\[ = 1940 \text{ ft/s} \]

27. We are given \( v_0 = 100 \text{ m/s} \), \( a = -9.8 \text{ m/s}^2 \) and are asked to find \( t \) when the ball has reached maximum height. The ball will reach its maximum when \( v = 0 \text{ m/s} \). So, we are really given \( v, v_0, \) and \( a \) and asked to find \( t \). First, solve the formula for \( t \).

\[ v = v_0 + at \]
\[ v - v_0 = at \]
\[ at = v - v_0 \]
\[ t = \frac{v - v_0}{a} \]

And now plug in what we know.

\[ t = \frac{v - v_0}{a} \]
\[ = \frac{0 \text{ m/s} - 100 \text{ m/s}}{-9.8 \text{ (m/s)/s}} \]
\[ \approx 10.2 \text{ m/s} \times \frac{s}{\text{m/s}} \]
\[ = 10.2 \text{ s} \]
29. We’re given \( v_0 = 30 \text{ m/s}, a = 2.5 \text{ m/s}^2 \), and \( t = 10 \text{ s} \), and asked to find \( v \) and \( x \). We will thus need to use both formulas. For the velocity,

\[
v = v_0 + at
\]

\[
= 30 \text{ m/s} + \left( 2.5 \frac{\text{m}}{\text{s}} \times 10 \text{ s} \right)
\]

\[
= 30 \text{ m/s} + 25 \text{ m/s}
\]

\[
= 55 \text{ m/s}
\]

For the distance traveled, \( x_0 = 0 \text{ m} \) since the rocket hasn’t traveled any distance at time \( t = 0 \text{ s} \).

\[
x = x_0 + v_0t + \frac{1}{2}at^2
\]

\[
= 0 \text{ m} + (30 \frac{\text{m}}{\text{s}} \times 10 \text{ s}) + \left( \frac{1}{2} \times 2.5 \frac{\text{m}}{\text{s}^2} \times (10 \text{ s})^2 \right)
\]

\[
= 0 \text{ m} + (300 \text{ m}) + \left( \frac{1}{2} \times 2.5 \text{ m/s}^2 \times 100 \text{ s}^2 \right)
\]

\[
= 0 \text{ m} + 300 \text{ m} + 125 \text{ m}
\]

\[
= 425 \text{ m}
\]

31. We’re given \( v_0 = 88 \text{ ft/s}, v = 58 \text{ ft/s}, \) and \( t = 10 \text{ s} \), and asked to find \( a \) and \( x \). We will thus need to use both formulas. First, since we know \( v, v_0, \) and \( t \), compute \( a \). Solve the velocity formula for \( a \):

\[
v = v_0 + at
\]

\[
v - v_0 = at
\]

\[
at = v - v_0
\]

\[
a = \frac{v - v_0}{t}
\]

And plug in the known values:

\[
a = \frac{v - v_0}{t}
\]

\[
= \frac{58 \text{ ft/s} - 88 \text{ ft/s}}{10 \text{ s}}
\]

\[
= -30 \text{ ft/s}
\]

\[
= \frac{-30 \text{ ft/s}}{10 \text{ s}}
\]

\[
= -3 \frac{\text{ft}}{\text{s}^2}
\]
For the distance traveled, \(x_0 = 0\) ft since the car hasn’t traveled any distance at time \(t = 0\) s.

\[
x = x_0 + v_0 t + \frac{1}{2} a t^2
\]

\[
= 0 \text{ ft} + (88 \frac{\text{ft}}{s} \times 10 \text{ s}) + \left(\frac{1}{2} \times -3 \frac{\text{ft}}{s^2} \times (10 \text{ s})^2\right)
\]

\[
= 0 \text{ ft} + (88 \frac{\text{ft}}{s} \times 10 \text{ s}) + \left(\frac{1}{2} \times -3 \frac{\text{ft}}{s^2} \times 100 \text{ s}^2\right)
\]

\[
= 0 \text{ ft} + 880 \text{ ft} - 150 \text{ ft}
\]

\[
= 730 \text{ ft}
\]

33. We are given the initial velocity \(v_0 = 180\) ft/s and the initial position—the object is shot from the surface of earth, so its initial distance is \(x_0 = 0\) ft. It is well known that the acceleration due to gravity is \(a = -32\) \(\text{ft/s}^2\) when we measure in feet (and it is negative because it is pulling down on the object). The maximum height of the object occurs when the velocity is \(v = 0\) ft/s. So our task is to find \(x\). First, find \(t\) by using,

\[
v = v_0 + at
\]

\[
v - v_0 = at
\]

\[
at = v - v_0
\]

\[
t = \frac{v - v_0}{a}
\]

and plugging in the known values:

\[
t = \frac{v - v_0}{a}
\]

\[
= \frac{0 \text{ ft/s} - 180 \text{ ft/s}}{-32(\text{ft/s})/\text{s}}
\]

\[
= -180 \text{ ft/s} \times -\frac{1 \text{ s}}{-32 \text{ ft/s}^2}
\]

\[
= 5.625 \text{ s}
\]

And now use the distance formula to find \(x\).

\[
x = x_0 + v_0 t + \frac{1}{2} a t^2
\]

\[
= 0 \text{ ft} + (180 \frac{\text{ft}}{s} \times 5.625 \text{ s}) + \left(\frac{1}{2} \times -32 \frac{\text{ft}}{s^2} \times (5.625 \text{ s})^2\right)
\]

\[
= 0 \text{ ft} + (180 \frac{\text{ft}}{s} \times 5.625 \text{ s}) + \left(\frac{1}{2} \times -32 \frac{\text{ft}}{s^2} \times 31.640625 \text{ s}^2\right)
\]

\[
= 0 \text{ ft} + (180 \frac{\text{ft}}{s} \times 5.625 \text{ s}) + \left(\frac{1}{2} \times -32 \frac{\text{ft}}{s^2} \times 31.640625 \text{ s}^2\right)
\]
So, at time \( t = 5.625 \) s, the object reaches its maximum height of \( x = 506.25 \) ft.

35. We are given that the car’s initial speed is \( v_0 = 55 \) mi/h and its acceleration (deceleration actually, so negative in sign) is \( a = -12 \) ft/s\(^2\). We must find the time \( t \) and the distance traveled \( x \) when the car has come to a stop; that is, when \( v = 0 \) ft/s.

First, note that the velocity is given in units involving miles and hours, while the acceleration is given in units involving feet and seconds. We must first make the units match. Convert the initial velocity to ft/s as follows:

\[
v_0 = 55 \frac{\text{mi}}{\text{h}}
\]

\[
v_0 = 55 \frac{\text{mi}}{\text{h}} \times \frac{1 \text{K}}{3600\text{ s}} \times \frac{5280 \text{ ft}}{1 \text{ mi}}
\]

\[
\approx 80.667 \text{ ft/s}
\]

Now find \( t \) by using,

\[
v = v_0 + at
\]

\[
v - v_0 = at
\]

\[
at = v - v_0
\]

\[
t = \frac{v - v_0}{a}
\]

and plugging in the known values:

\[
t = \frac{v - v_0}{a}
\]

\[
= \frac{0 \text{ ft/s} - 80.667 \text{ ft/s}}{-12(\text{ft/s}/\text{s})/\text{s}}
\]

\[
= -80.667\text{ft/s} \times \frac{1\text{s}}{-120\text{s}}
\]

\[
\approx 6.72 \text{ s}
\]

Finally, use the distance formula to find \( x \). The initial distance is \( x_0 = 0 \) ft since the car has not traveled any distance when \( t = 0 \).

\[
x = x_0 + v_0t + \frac{1}{2}at^2
\]

\[
= 0 \text{ ft} + (80.667\frac{\text{ft}}{\text{s}} \times 6.72 \text{ s}) + \left(\frac{1}{2} \times -12\frac{\text{ft}}{\text{s}^2} \times (6.72 \text{ s})^2\right)
\]

\[
= 0 \text{ ft} + (80.667\frac{\text{ft}}{\text{s}} \times 6.72 \text{ s}) + \left(\frac{1}{2} \times -12\frac{\text{ft}}{\text{s}^2} \times 45.158 \text{ s}^2\right)
\]

\[
= 0 \text{ ft} + (80.667\frac{\text{ft}}{\text{s}} \times 6.72 \text{ s}) + \left(\frac{1}{2} \times -12\frac{\text{ft}}{\text{s}^2} \times 45.158 \text{ s}^2\right)
\]
\[ x = x_0 + v_0 t + \frac{1}{2} a t^2 \]
\[ = 0 \text{ m} + \left( 0 \frac{\text{m}}{\text{s}} \times 5 \text{ s} \right) + \left( \frac{1}{2} \times -9.8 \frac{\text{m}}{\text{s}^2} \times (5 \text{ s})^2 \right) \]
\[ = 0 \text{ m} + \left( 0 \frac{\text{m}}{\text{s}} \times 5 \text{ s} \right) + \left( \frac{1}{2} \times -9.8 \frac{\text{m}}{\text{s}^2} \times 25 \text{ s}^2 \right) \]
\[ = 0 \text{ m} + (0 \frac{\text{m}}{\text{s}} \times 5 \text{ s}) + \left( \frac{1}{2} \times -9.8 \frac{\text{m}}{\text{s}^2} \times 25 \text{ s}^2 \right) \]
\[ = 0 \text{ m} + 0 \text{ m} - 122.5 \text{ m} \]
\[ = -122.5 \text{ m} \]

The negative value indicates that the height of the object decreased (it fell).

So, the object drops with a velocity of \(-49 \text{ m/s}\), falling 122.5 m.

39. The object is released from rest, so \( v_0 = 0 \) ft/s; its initial height is \( x_0 = 352 \) ft. Its only acceleration is due to gravity, which is well known to be \( a = -32 \text{ ft/s}^2 \) when using ft (negative because it is pulling down, decreasing the height of the object). To find how long it will take to hit the ground, we must find the time \( t \) when \( x = 0 \) (the height is zero, or the object is on the ground).

We want to find \( t \), so solve the formula for \( t \).
This is a quadratic equation in \( t \) and we want to solve for \( t \), so use the quadratic formula with \( a = \frac{1}{2} \), \( b = v_0 \) and \( c = x_0 - x \).

\[
t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(\frac{1}{2}a\right)(x_0 - x)}}{2\left(\frac{1}{2}a\right)}
\]
\[
t = \frac{-v_0 \pm \sqrt{v_0^2 - 2a(x_0 - x)}}{a}
\]

Now substitute in the known values.

\[
t = \frac{-v_0 \pm \sqrt{v_0^2 - 2a(x_0 - x)}}{a}
\]
\[
t = \frac{-0 \text{ ft/s} \pm \sqrt{(0\text{ ft/s})^2 - 2(-32 \text{ ft/s}^2)(352 \text{ ft} - 0 \text{ ft})}}{-32 \text{ ft/s}^2}
\]
\[
t = \frac{\pm \sqrt{64 \text{ ft/s}^2(352 \text{ ft})}}{-32 \text{ ft/s}^2}
\]
\[
t = \frac{\pm \sqrt{22528 \text{ ft}^2/s^2}}{-32 \text{ ft/s}^2}
\]
\[
t = \frac{\pm 150.09 \text{ ft/s}}{-32 \text{ ft/s}^2}
\]
\[
t \approx \frac{\pm 150.09 \text{ ft/s}}{-32 \text{ ft/s}^2} \times \frac{1 \text{ s}}{-32 \text{ ft/s}}
\]
\[
t = \pm 4.69 \text{ s} \approx t
\]

We throw out the negative result because time must be positive. Thus, the object will hit the surface of the earth in \( t = 4.69 \text{ s} \).

41. The initial height of the ball is \( x_0 = 5 \text{ m} \) and its initial velocity is \( v = 100 \text{ m/s} \). Because we are using the metric system, we need to use the well-known value of \( a = -9.8 \text{ m/s}^2 \) for the acceleration due to gravity. We must find the time \( t \) at which the height is \( x = 0 \text{ m} \).
\[ x = x_0 + v_0 t + \frac{1}{2} at^2 \]

\[ 0 = 5 \text{ m} + (100 \text{ m/s})t + \left(\frac{1}{2}\right)(-9.8 \text{ m/s}^2)t^2 \]

\[ 0 = 5 \text{ m} + (100 \text{ m/s})t + (-4.9 \text{ m/s}^2)t^2 \]

\[ 0 = (-4.9 \text{ m/s}^2)t^2 + (100 \text{ m/s})t + (5 \text{ m}) \]

This is a quadratic equation with \( a = -4.9 \text{ m/s}^2 \), \( b = 100 \text{ m/s} \) and \( c = 5 \text{ m} \), so use the quadratic formula to solve.

\[ t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ t = \frac{-100 \text{ m/s} \pm \sqrt{(100 \text{ m/s})^2 - 4(-4.9 \text{ m/s}^2)(5 \text{ m})}}{2(-4.9 \text{ m/s}^2)} \]

\[ t = \frac{-100 \text{ m/s} \pm \sqrt{10000 \text{ m}^2/\text{s}^2 + 98 \text{ m}^2/\text{s}^2}}{-9.8 \text{ m/s}^2} \]

\[ t = \frac{-100 \text{ m/s} \pm \sqrt{10098 \text{ m}^2/\text{s}^2}}{-9.8 \text{ m/s}^2} \]

\[ t \approx \frac{-100 \text{ m/s} \pm 100.49 \text{ m/s}}{-9.8 \text{ m/s}^2} \]

\[ t \approx \frac{(-100 \pm 100.49)\text{m/s}}{-9.8 \text{ m/s}^2} \times \frac{\text{m/s}}{\text{m/s}^2} \]

\[ t \approx \frac{-100 \pm 100.49}{-9.8} \times \frac{\text{m}}{\text{s}} \times \frac{\text{s}^2}{\text{m}} \]

\[ t \approx \frac{-100 \pm 100.49}{-9.8} \times \text{ft/} \times \text{s} \times \text{ft} \]

\[ t \approx 20.46 \text{ s}, -0.50 \text{ s} \]

We throw out the negative value for time. Thus the ball will hit the ground in approximately \( t = 20.46 \text{ s} \).

43. We are given that \( y_0 = 50 \text{ ft} \) and \( v_0 = 120 \text{ ft/s} \), and the well-known acceleration due to gravity is \( a = -32 \text{ ft/s}^2 \) when using ft. Plug these into the height formula:

\[ y = y_0 + v_0 t + \frac{1}{2}at^2 \]

\[ y = 50 + 120t - 16t^2 \]

\[ y = -16t^2 + 120t + 50 \]
This is a quadratic function and we can graph it. We enter it into our calculator using \( x \)'s instead of \( t \)'s. A good window to use is \( X_{\text{min}} = 0, X_{\text{max}} = 10, Y_{\text{min}} = -100 \) and \( Y_{\text{max}} = 400 \). We only need to look at time greater than or equal to zero.

To find when the ball hits the ground (the zero of the function) with your calculator, press \texttt{2nd TRACE} to access the \texttt{CALC} menu and choose \texttt{2:zero}.

We get \((7.895781, 0)\).

Now sketch the graph on your paper.

Finally, we verify the time it takes the ball to hit the ground with the quadratic formula. The ball is on the ground when its height is \( y = 0 \) ft. So, set \( y = 0 \) and solve the equation.

\[
y = -16t^2 + 120t + 50
\]

\[
0 = -16t^2 + 120t + 50
\]

This is a quadratic equation with \( a = -16, b = 120 \) and \( c = 50 \). So, the solutions are,
Section 5.5  Motion

\[ t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ t = \frac{-120 \pm \sqrt{(120)^2 - 4(-16)(50)}}{2(-16)} \]
\[ t = \frac{-120 \pm \sqrt{14400 + 3200}}{-32} \]
\[ t = \frac{-120 \pm \sqrt{17600}}{-32} \]
\[ t \approx -0.396, 7.896 \]

As usual, we throw out the negative time, so \( t \approx 7.896 \text{ s} \). This agrees with the zero that we found using the calculator.

45. We are given that \( y_0 = 20 \text{ m} \) and \( v_0 = 110 \text{ m/s} \), and the well-known acceleration due to gravity is \( a = -9.8 \text{ m/s}^2 \) when using the metric system. Plug these into the height formula:

\[ y = y_0 + v_0 t + \frac{1}{2} a t^2 \]
\[ y = 20 + 110 t - 4.9t^2 \]
\[ y = -4.9t^2 + 110t + 20 \]

This is a quadratic function and we can graph it. We enter it into our calculator using \( x \)'s instead of \( t \)'s. A good window to use is \( X_{\min} = 0, X_{\max} = 30, Y_{\min} = -200 \) and \( Y_{\max} = 1000 \). We only need to look at time greater than or equal to zero.

To find when the ball hits the ground (the zero of the function) with your calculator, press 2nd TRACE to access the CALC menu and choose 2:zero.

We get \((22.629349, 0)\).

Now sketch the graph on your paper.
Finally, we verify the time it takes the ball to hit the ground with the quadratic formula. The ball is on the ground when its height is \( y = 0 \) ft. So, set \( y = 0 \) and solve the equation.

\[
y = -4.9t^2 + 110t + 20
\]

\[
0 = -4.9t^2 + 110t + 20
\]

This is a quadratic equation with \( a = -4.9 \), \( b = 110 \) and \( c = 20 \). So, the solutions are,

\[
t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
t = \frac{-110 \pm \sqrt{(110)^2 - 4(-4.9)(20)}}{2(-4.9)}
\]

\[
t = \frac{-110 \pm \sqrt{12100 + 392}}{-9.8}
\]

\[
t = \frac{-110 \pm \sqrt{12492}}{-9.8}
\]

\[
t \approx -0.180, 22.629
\]

As usual, we throw out the negative time, so \( t \approx 22.629 \) s. This agrees with the zero that we found using the calculator.

47. If the \( y \)-axis is oriented with the positive direction upward, and the 0 mark is set at the initial position of the rock, then the height (in feet) of the rock at \( t \) seconds is given by the function

\[
s(t) = -16t^2 + 64t
\]

Then

\[
\text{height} = 61 \implies s(t) = 61
\]

\[
\implies -16t^2 + 64t = 61
\]

\[
\implies -16t^2 + 64t - 61 = 0
\]
Section 5.5 Motion

Now use either the quadratic formula or the ZERO routine on your calculator to find the approximate solutions of this equation:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ = \frac{-64 \pm \sqrt{64^2 - 4(-16)(-61)}}{2(-16)} \]

\[ = \frac{-64 \pm \sqrt{192}}{-32} \]

\[ \approx 1.567, 2.433 \]

Thus, it takes approximately 1.567 seconds for the rock to rise to a height of 61 feet. It also attains that height a second time on the way back down at approximately 2.433 seconds. Rounded to the nearest hundredth of a second, the answer is 1.57.

49. If the y-axis is oriented with the positive direction upward, and the 0 mark is set at ground level, then the height (in feet) of the water balloon after \( t \) seconds is given by the function

\[ h(t) = -16t^2 - 24t + 169 \]

Note that

\[ \text{height} = 0 \implies h(t) = 0 \implies -16t^2 - 24t + 169 = 0 \]

Now use either the quadratic formula or the ZERO routine on your calculator to find the approximate solutions of this equation:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ = \frac{24 \pm \sqrt{(-24)^2 - 4(-16)(169)}}{2(-16)} \]

\[ = \frac{24 \pm \sqrt{11392}}{-32} \]

\[ \approx -4.085, 2.585 \]

Since \(-4.085\) is meaningless in the context of the question, the approximate answer is 2.585. Rounded to the nearest hundredth of a second, the answer is 2.59.

51. If the y-axis is oriented with the positive direction upward, and the 0 mark is set at ground level, then the height (in feet) of the ball after \( t \) seconds is given by the function

\[ h(t) = -16t^2 + 63t + 42 \]

Note that
height = 0  \implies h(t) = 0  \implies -16t^2 + 63t + 42 = 0

Now use either the quadratic formula or the ZERO routine on your calculator to find the approximate solutions of this equation:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ = \frac{-63 \pm \sqrt{63^2 - 4(-16)(42)}}{2(-16)} \]

\[ = \frac{-63 \pm \sqrt{6657}}{-32} \]

\[ \approx -0.581, 4.518 \]

Since −0.581 is meaningless in the context of the question, the approximate answer is 4.518. Rounded to the nearest hundredth of a second, the answer is 4.52.

53. If the y-axis is oriented with the positive direction upward, and the 0 mark is set at ground level, then the height (in feet) of the penny after t seconds is given by the function

\[ h(t) = -16t^2 - 16t + 68 \]

Note that

height = 0  \implies h(t) = 0  \implies -16t^2 - 16t + 68 = 0

Now use either the quadratic formula or the ZERO routine on your calculator to find the approximate solutions of this equation:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ = \frac{16 \pm \sqrt{(-16)^2 - 4(-16)(68)}}{2(-16)} \]

\[ = \frac{16 \pm \sqrt{4608}}{-32} \]

\[ \approx -2.621, 1.621 \]

Since −2.621 is meaningless in the context of the question, the approximate answer is 1.621. Rounded to the nearest hundredth of a second, the answer is 1.62.
5.6 Exercises

1. Find the exact maximum value of the function \( f(x) = -x^2 - 3x \).

2. Find the exact maximum value of the function \( f(x) = -x^2 - 5x - 2 \).

3. Find the vertex of the graph of the function \( f(x) = -3x^2 - x - 6 \).

4. Find the range of the function \( f(x) = -2x^2 - 9x + 2 \).

5. Find the exact maximum value of the function \( f(x) = -3x^2 - 9x - 4 \).

6. Find the equation of the axis of symmetry of the graph of the function \( f(x) = -x^2 - 5x - 9 \).

7. Find the vertex of the graph of the function \( f(x) = 3x^2 + 3x + 9 \).

8. Find the exact minimum value of the function \( f(x) = x^2 + x + 1 \).

9. Find the exact minimum value of the function \( f(x) = x^2 + 9x \).

10. Find the range of the function \( f(x) = 5x^2 - 3x - 4 \).

11. Find the range of the function \( f(x) = -3x^2 + 8x - 2 \).

12. Find the exact minimum value of the function \( f(x) = 2x^2 + 5x - 6 \).

13. Find the range of the function \( f(x) = 4x^2 + 9x - 8 \).

14. Find the exact maximum value of the function \( f(x) = -3x^2 - 8x - 1 \).

15. Find the equation of the axis of symmetry of the graph of the function \( f(x) = -4x^2 - 2x + 9 \).

16. Find the exact minimum value of the function \( f(x) = 5x^2 + 2x - 3 \).

17. A ball is thrown upward at a speed of 8 ft/s from the top of a 182 foot high building. How many seconds does it take for the ball to reach its maximum height? Round your answer to the nearest hundredth of a second.

18. A ball is thrown upward at a speed of 9 ft/s from the top of a 143 foot high building. How many seconds does it take for the ball to reach its maximum height? Round your answer to the nearest hundredth of a second.

19. A ball is thrown upward at a speed of 52 ft/s from the top of a 293 foot high building. What is the maximum height of the ball? Round your answer to the nearest hundredth of a foot.

20. A ball is thrown upward at a speed of 23 ft/s from the top of a 71 foot high building. What is the maximum height of the ball? Round your answer to the nearest hundredth of a foot.

21. Find two numbers whose sum is 20 and whose product is a maximum.

22. Find two numbers whose sum is 36 and whose product is a maximum.

23. Find two numbers whose difference is 12 and whose product is a minimum.

\(^1\) Copyrighted material. See: http://msenux.redwoods.edu/IntAlgText/

Version: Fall 2007
24. Find two numbers whose difference is 24 and whose product is a minimum.

25. One number is 3 larger than twice a second number. Find two such numbers so that their product is a minimum.

26. One number is 2 larger than 5 times a second number. Find two such numbers so that their product is a minimum.

27. Among all pairs of numbers whose sum is −10, find the pair such that the sum of their squares is the smallest possible.

28. Among all pairs of numbers whose sum is −24, find the pair such that the sum of their squares is the smallest possible.

29. Among all pairs of numbers whose sum is 14, find the pair such that the sum of their squares is the smallest possible.

30. Among all pairs of numbers whose sum is 12, find the pair such that the sum of their squares is the smallest possible.

31. Among all rectangles having perimeter 40 feet, find the dimensions (length and width) of the one with the greatest area.

32. Among all rectangles having perimeter 100 feet, find the dimensions (length and width) of the one with the greatest area.

33. A farmer with 1700 meters of fencing wants to enclose a rectangular plot that borders on a river. If no fence is required along the river, and the side parallel to the river is x meters long, find the value of x which will give the largest area of the rectangle.

35. A park ranger with 400 meters of fencing wants to enclose a rectangular plot that borders on a river. If no fence is required along the river, and the side parallel to the river is x meters long, find the value of x which will give the largest area of the rectangle.

36. A rancher with 1000 meters of fencing wants to enclose a rectangular plot that borders on a river. If no fence is required along the river, what is the largest area that can be enclosed?

37. Let x represent the demand (the number the public will buy) for an object and let p represent the object’s unit price (in dollars). Suppose that the unit price and the demand are linearly related by the equation \( p = (-1/3)x + 40 \).

a) Express the revenue \( R \) (the amount earned by selling the objects) as a function of the demand \( x \).

b) Find the demand that will maximize the revenue.

c) Find the unit price that will maximize the revenue.

d) What is the maximum revenue?

38. Let \( x \) represent the demand (the number the public will buy) for an object and let \( p \) represent the object’s unit price (in dollars). Suppose that the unit price and the demand are linearly related by the equation \( p = (-1/5)x + 200 \).

a) Express the revenue \( R \) (the amount earned by selling the objects) as a function of the demand \( x \).

b) Find the demand that will maximize the revenue.

c) Find the unit price that will maximize the revenue.

d) What is the maximum revenue?
earned by selling the objects) as a function of the demand $x$.

b) Find the demand that will maximize the revenue.

c) Find the unit price that will maximize the revenue.

d) What is the maximum revenue?

39. A point from the first quadrant is selected on the line $y = mx + b$. Lines are drawn from this point parallel to the axes to form a rectangle under the line in the first quadrant. Among all such rectangles, find the dimensions of the rectangle with maximum area. What is the maximum area? Assume $m < 0$.

![Graph showing a line $y = mx + b$ and a rectangle formed by lines parallel to the axes.]

40. A rancher wishes to fence a rectangular area. The east-west sides of the rectangle will require stronger support due to prevailing east-west storm winds. Consequently, the cost of fencing for the east-west sides of the rectangular area is $18$ per foot. The cost for fencing the north-south sides of the rectangular area is $12$ per foot. Find the dimension of the largest possible rectangular area that can be fenced for $7200$. 

Version: Fall 2007
5.6 Solutions

1. The graph opens downward since $a = -1 > 0$, and the vertex is at $(h, k)$, where

$$h = -\frac{b}{2a} = -\frac{3}{2}$$

and

$$k = f(h) = f \left( -\frac{3}{2} \right) = \frac{9}{4}.$$ 

Thus, the maximum value of the function is $\frac{9}{4}$.

3. The vertex is $(h, k)$, where $h = -\frac{b}{2a} = -\frac{1}{6}$ and $k = f(h) = f \left( -\frac{1}{6} \right) = -\frac{71}{12}$.

5. The graph opens downward since $a = -3 > 0$, and the vertex is at $(h, k)$, where

$$h = -\frac{b}{2a} = -\frac{3}{2}$$

and

$$k = f(h) = f \left( -\frac{3}{2} \right) = \frac{11}{4}.$$ 

Thus, the maximum value of the function is $\frac{11}{4}$.

7. The vertex is $(h, k)$, where $h = -\frac{b}{2a} = -\frac{1}{2}$ and $k = f(h) = f \left( -\frac{1}{2} \right) = \frac{33}{4}$.

9. The graph opens upward since $a = 1 > 0$, and the vertex is at $(h, k)$, where

$$h = -\frac{b}{2a} = -\frac{9}{2}$$

and

$$k = f(h) = f \left( -\frac{9}{2} \right) = -\frac{81}{4}.$$ 

Thus, the minimum value of the function is $-\frac{81}{4}$.

11. The graph opens downward since $a = -3 < 0$, and the vertex is at $(h, k)$, where

$$h = -\frac{b}{2a} = \frac{4}{3}$$

and

$$k = f(h) = f \left( \frac{4}{3} \right) = \frac{10}{3}.$$ 

Thus, the range is $(-\infty, k] = \left(-\infty, \frac{10}{3}\right]$.

13. The graph opens upward since $a = 4 > 0$, and the vertex is at $(h, k)$, where

$$h = -\frac{b}{2a} = -\frac{9}{8}$$

and

$$k = f(h) = f \left( -\frac{9}{8} \right) = -\frac{209}{16}.$$ 

Thus, the range is $[k, \infty) = \left[ -\frac{209}{16}, \infty \right)$.

15. The axis of symmetry is $x = h$, where $h = -\frac{b}{2a} = -\frac{1}{4}$.

17. If the $y$-axis is oriented with the positive direction upward, and the 0 mark is set at ground level, then the height (in feet) of the ball after $t$ seconds is given by the function

$$h(t) = -16t^2 + 8t + 182.$$
Use either the vertex formula or the MAXIMUM routine on your calculator to find the approximate vertex of the graph:

\[ t = -\frac{b}{2a} = \frac{1}{4} \approx 0.25 \quad \text{and} \quad h(t) \approx 183 \]

Thus, the maximum height is reached in \( \approx 0.25 \) seconds (rounded to the nearest hundredth).

19. If the y-axis is oriented with the positive direction upward, and the 0 mark is set at ground level, then the height (in feet) of the ball after \( t \) seconds is given by the function

\[ h(t) = -16t^2 + 52t + 293 \]

Use either the vertex formula or the MAXIMUM routine on your calculator to find the approximate vertex of the graph:

\[ t = -\frac{b}{2a} = \frac{13}{8} \approx 1.625 \quad \text{and} \quad h(t) \approx 335.25 \]

Thus, the maximum height is \( \approx 335.25 \) feet (rounded to the nearest hundredth).

21. Let \( x \) and \( y \) be the two numbers, so \( x + y = 20 \). The goal is to maximize the product \( P = xy \), but we cannot maximize a function of more than one variable. We must substitute one of the variables out. Solve \( x + y = 20 \) for \( y \) to get \( y = 20 - x \), and substitute into \( P \) to get...

\[ P(x) = x(20 - x) \]

Simplifying yields

\[ P(x) = -x^2 + 20x \]

It is a downward parabola, so the maximum occurs at

\[ x = -\frac{b}{2a} = -\frac{20}{2(-1)} = 10, \]

It follows that \( y = 20 - x = 20 - 10 = 10 \). So, the two numbers are 10 and 10.

23. Let \( x \) and \( y \) be the two numbers, so \( x - y = 12 \). The goal is to minimize the product \( P = xy \), but we cannot minimize a function of more than one variable. We must substitute one of the variables out. Since \( x = 12 + y \), \( P \) can be rewritten as a function of \( y \):

\[ P(y) = (12 + y)y \]

Simplifying yields

\[ P(y) = y^2 + 12y \]

This is an upward parabola, so the minimum occurs at
and \( x = 12 + y = 12 + (-6) = 6 \). So the two numbers are 6 and -6.

25. Let \( x \) and \( y \) be the two numbers, so \( y = 2x + 3 \). The goal is to minimize the product \( P = xy \), but we cannot minimize a function of more than one variable. We must substitute one of the variables out. Substitute \( y = 2x + 3 \) into \( P \) to get...

\[ P(x) = x(2x + 3) \]

Simplifying yields

\[ P(x) = 2x^2 + 3x \]

It is an upward parabola, so the minimum occurs at

\[ x = -\frac{b}{2a} = -\frac{3}{2(2)} = -\frac{3}{4}. \]

It follows that \( y = 2x + 3 = 2(-3/4) + 3 = -3/2 + 3 = 3/2 \). So, the two numbers are -3/4 and 3/2.

27. Let \( x \) and \( y \) be the two numbers, so \( x + y = -10 \). The goal is to minimize the sum of squares \( S = x^2 + y^2 \). Since \( y = -10 - x \), \( S \) can be rewritten as a function of \( x \):

\[ S(x) = x^2 + (-10 - x)^2 \]

Simplifying yields

\[ S(x) = 2x^2 + 20x + 100 \]

The minimum occurs at

\[ x = -\frac{b}{2a} = -5, \]

and \( y = -10 - 5 = -5 \).

29. Let \( x \) and \( y \) be the two numbers, so \( x + y = 14 \). The goal is to minimize the sum of squares \( S = x^2 + y^2 \). Since \( y = 14 - x \), \( S \) can be rewritten as a function of \( x \):

\[ S(x) = x^2 + (14 - x)^2 \]

Simplifying yields

\[ S(x) = 2x^2 - 28x + 196 \]

The minimum occurs at

\[ x = -\frac{b}{2a} = 7, \]
and \( y = 14 - 7 = 7 \).  

**31.** Let the sides of the rectangle be \( x \) and \( y \). Then the perimeter is \( 2x + 2y = 40 \) (two lengths plus two widths make up the perimeter). The goal is to maximize the area, which is \( A = xy \), but we cannot maximize a function of more than one variable. We must substitute one of the variables out. 

Solve \( 2x + 2y = 40 \) for \( y \) to get \( y = (40 - 2x)/2 = 20 - x \). Substitute into \( A = xy \): 

\[
A(x) = x(20 - x)
\]

Simplifying yields 

\[
A(x) = -x^2 + 20x
\]

This is a downward parabola, so the maximum occurs at 

\[
x = -\frac{b}{2a} = -\frac{20}{2(-1)} = 10,
\]

and \( y = 20 - x = 20 - 10 = 10 \). So the dimensions are 10 by 10.  

**33.** Let \( x \) be the length of the portion of the fence parallel to the river. Then the other two sides each have length \((1700 - x)/2\), and the total area is 

\[
A(x) = x \left(\frac{1700 - x}{2}\right) = -\frac{1}{2} x^2 + 850x
\]

The maximum occurs at the vertex of the graph, where \( x = -b/(2a) = 850 \) meters, and \( A(850) = 361250 \), so the maximum area is 361250 square meters.  

**35.** As indicated in the question, let \( x \) be the length of the portion of the fence parallel to the river. Then the other two sides each have length \((400 - x)/2\), and the total area is 

\[
A(x) = x \left(\frac{400 - x}{2}\right) = -\frac{1}{2} x^2 + 200x
\]

The maximum occurs at the vertex of the graph, where \( x = -b/(2a) = 200 \) meters.  

**37.**

a) Since \( p \) represents the unit price and \( x \) represents the number of objects, the revenue from sales is the unit price times the number of units, or \( px \). Thus \( R(x) = px = ((-1/3)x + 40)x = (-1/3)x^2 + 40x \).  

b) In the part (a), you found the revenue \( R \) is given by the quadratic function \( R(x) = (-1/3)x^2 + 40x \). Because \( a = -1/3 \), this is a downward parabola, and so its maximum occurs at the vertex. The \( x \)-coordinate of the vertex is \( x = -b/(2a) = -40/(2(-1/3)) = -40/(-2/3) = -40(-3/2) = 60 \). The maximum revenue occurs when the demand is \( x = 60 \) objects.
c) We know that the demand is $x = 60$ when the revenue is maximum. Our task is to find what $p$ is. Use $p = (-1/3)x + 40$ and plug in 60 for $x$ to get $p = (-1/3)60 + 40 = -20 + 40 = 20$. The unit price should be $20$ dollars to yield maximum revenue.

d) The maximum revenue itself is the R-coordinate of the vertex. We already have $x = 60$. Plug this into the equation for $R$ to get $R(60) = (-1/3)60^2 + 40(60) = 1200$.

39. The dimensions of the rectangle are $x$ by $y$, so its area is $A = xy$. We do not know how to find maximums of equations with more than one variable, so we need to get this down to an equation for $A$ in terms of a single variable. Luckily, we are given that $y = mx + b$, so replace $y$ with $mx + b$ to get $A = x(mx + b)$. Multiply this out to get $A = mx^2 + bx$, which is a quadratic equation. We are given that $m < 0$, so the graph of the area function $A$ is a downward parabola, meaning its maximum occurs at the vertex. Use $x = -b/(2a) = -b/(2m)$ to get one dimension. Now plug in to get

$$y = mx + b = m\left(\frac{-b}{2m}\right) + b = -\frac{b}{2} + b = -\frac{b}{2} + \frac{2b}{2} = \frac{b}{2},$$

and

$$A = xy = \left(\frac{-b}{2m}\right)\left(\frac{b}{2}\right) = -\frac{b^2}{4m}.$$

So, the maximum area of $-b^2/(4m)$ occurs when the dimensions are $-b/(2m)$ by $b/2$. 

Version: Fall 2007