5.1 Exercises

In Exercises 1-6, sketch the image of your calculator screen on your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax. Label each graph with its equation. Remember to use a ruler to draw all lines, including axes.

1. Use your graphing calculator to sketch the graphs of \( f(x) = x^2 \), \( g(x) = 2x^2 \), and \( h(x) = 4x^2 \) on one screen. Write a short sentence explaining what you learned in this exercise.

2. Use your graphing calculator to sketch the graphs of \( f(x) = -x^2 \), \( g(x) = -2x^2 \), and \( h(x) = -4x^2 \) on one screen. Write a short sentence explaining what you learned in this exercise.

3. Use your graphing calculator to sketch the graphs of \( f(x) = x^2 \), \( g(x) = (x - 2)^2 \), and \( h(x) = (x - 4)^2 \) on one screen. Write a short sentence explaining what you learned in this exercise.

4. Use your graphing calculator to sketch the graphs of \( f(x) = x^2 \), \( g(x) = (x + 2)^2 \), and \( h(x) = (x + 4)^2 \) on one screen. Write a short sentence explaining what you learned in this exercise.

5. Use your graphing calculator to sketch the graphs of \( f(x) = x^2 \), \( g(x) = x^2 + 2 \), and \( h(x) = x^2 + 4 \) on one screen. Write a short sentence explaining what you learned in this exercise.

6. Use your graphing calculator to sketch the graphs of \( f(x) = x^2 \), \( g(x) = x^2 - 2 \), and \( h(x) = x^2 - 4 \) on one screen. Write a short sentence explaining what you learned in this exercise.

In Exercises 7-14, write down the given quadratic function on your homework paper, then state the coordinates of the vertex.

7. \( f(x) = -5(x - 4)^2 - 5 \)
8. \( f(x) = 5(x + 3)^2 - 7 \)
9. \( f(x) = 3(x + 1)^2 \)
10. \( f(x) = \frac{7}{5} \left( x + \frac{5}{9} \right)^2 - \frac{3}{4} \)
11. \( f(x) = -7(x - 4)^2 + 6 \)
12. \( f(x) = -\frac{1}{2} \left( x - \frac{8}{9} \right)^2 + \frac{2}{9} \)
13. \( f(x) = \frac{1}{6} \left( x + \frac{7}{3} \right)^2 + \frac{3}{8} \)
14. \( f(x) = -\frac{3}{2} \left( x + \frac{1}{2} \right)^2 - \frac{8}{9} \)

In Exercises 15-22, state the equation of the axis of symmetry of the graph of the given quadratic function.

15. \( f(x) = -7(x - 3)^2 + 1 \)
16. \( f(x) = -6(x + 8)^2 + 1 \)

1 Copyrighted material. See: http://msenux.redwoods.edu/IntAlgText/
Chapter 5  Quadratic Functions

17. \( f(x) = -\frac{7}{8} \left(x + \frac{1}{4}\right)^2 + \frac{2}{3} \)

18. \( f(x) = -\frac{1}{2} \left(x - \frac{3}{8}\right)^2 - \frac{5}{7} \)

19. \( f(x) = -\frac{2}{9} \left(x + \frac{2}{3}\right)^2 - \frac{4}{5} \)

20. \( f(x) = -7(x + 3)^2 + 9 \)

21. \( f(x) = -\frac{8}{7} \left(x + \frac{2}{9}\right)^2 + \frac{6}{5} \)

22. \( f(x) = 3(x + 3)^2 + 6 \)

27. \( f(x) = (x - 3)^2 \)

28. \( f(x) = -(x + 2)^2 \)

29. \( f(x) = -x^2 + 7 \)

30. \( f(x) = -x^2 + 7 \)

31. \( f(x) = 2(x - 1)^2 - 6 \)

32. \( f(x) = -2(x + 1)^2 + 5 \)

33. \( f(x) = -\frac{1}{2}(x + 1)^2 + 5 \)

34. \( f(x) = \frac{1}{2}(x - 3)^2 - 6 \)

35. \( f(x) = 2(x - 5/2)^2 - 15/2 \)

36. \( f(x) = -3(x + 7/2)^2 + 15/4 \)

In Exercises 23-36, perform each of the following tasks for the given quadratic function.

i. Set up a coordinate system on graph paper. Label and scale each axis.

ii. Plot the vertex of the parabola and label it with its coordinates.

iii. Draw the axis of symmetry and label it with its equation.

iv. Set up a table near your coordinate system that contains exact coordinates of two points on either side of the axis of symmetry. Plot them on your coordinate system and their “mirror images” across the axis of symmetry.

v. Sketch the parabola and label it with its equation.

vi. Use interval notation to describe both the domain and range of the quadratic function.

23. \( f(x) = (x + 2)^2 - 3 \)

24. \( f(x) = (x - 3)^2 - 4 \)

25. \( f(x) = -(x - 2)^2 + 5 \)

26. \( f(x) = -(x + 4)^2 + 4 \)

In Exercises 37-44, write the given quadratic function on your homework paper, then use set-builder and interval notation to describe the domain and the range of the function.

37. \( f(x) = 7(x + 6)^2 - 6 \)

38. \( f(x) = 8(x + 1)^2 + 7 \)

39. \( f(x) = -3(x + 4)^2 - 7 \)

40. \( f(x) = -6(x - 7)^2 + 9 \)

41. \( f(x) = -7(x + 5)^2 - 7 \)

42. \( f(x) = 8(x - 4)^2 + 3 \)

43. \( f(x) = -4(x - 1)^2 + 2 \)

44. \( f(x) = 7(x - 2)^2 - 3 \)
In **Exercises 45-52**, using the given value of $a$, find the specific quadratic function of the form $f(x) = a(x - h)^2 + k$ that has the graph shown. Note: $h$ and $k$ are integers. Check your solution with your graphing calculator.

45. $a = -2$

46. $a = 0.5$

47. $a = 2$

48. $a = 0.5$

49. $a = 2$

50. $a = -0.5$
51. \( a = 2 \)

52. \( a = 0.5 \)

In Exercises 53-54, use the graph to determine the range of the function \( f(x) = ax^2 + bx + c \). The arrows on the graph are meant to indicate that the graph continues indefinitely in the continuing pattern and direction of each arrow. Describe your solution using interval notation.

53.

54.

In Exercises 55-56, use the graph to determine the domain of the function \( f(x) = ax^2 + bx + c \). The arrows on the graph are meant to indicate that the graph continues indefinitely in the continuing pattern and direction of each arrow. Use interval notation to describe your solution.

55.

56.
5.1 Solutions

1. First, enter the functions into the Y= menu. Then press GRAPH to view a comparison of the three graphs.

Note how the graph of $y = 2x^2$ is narrower and taller than the graph of $y = x^2$, and the graph of $y = 4x^2$ is narrower and taller still. We see therefore that multiplying $x^2$ by a positive number such as 2 or 4 stretches or scales the graph vertically, making it taller (and narrower).

3. First, enter the functions into the Y= menu. Then press GRAPH to view a comparison of the three graphs.

Note how the graph of $g(x) = (x-2)^2$ has the same shape as the graph of $f(x) = x^2$ but it is shifted 2 units to the right; and the graph of $h(x) = (x-4)^2$ also has the same shape, but is shifted 4 units to the right. It thus appears that $y = (x-c)^2$, for positive $c$, has the same shape as $f(x) = x^2$ but is shifted $c$ units to the right.
5. First, enter the functions into the Y= menu. Then press GRAPH to view a comparison of the three graphs.

Note how the graph of \( g(x) = x^2 + 2 \) has the same shape as the graph of \( f(x) = x^2 \) but it is shifted 2 units up; and the graph of \( h(x) = x^2 + 4 \) also has the same shape, but is shifted 4 units up.

It thus appears that \( y = x^2 + k \), for a positive \( k \), has the same shape as \( f(x) = x^2 \) but is shifted \( k \) units up.

7. The function \( f(x) = -5(x - 4)^2 - 5 \) is given in vertex form \( f(x) = a(x - h)^2 + k \), where \( a = -5 \), \( h = 4 \), and \( k = -5 \). The vertex is \((h,k) = (4,-5)\).

9. The function \( f(x) = 3(x+1)^2 \) is given in vertex form \( f(x) = a(x - h)^2 + k \), where \( a = 3 \), \( h = -1 \), and \( k = 0 \). The vertex is \((h,k) = (-1,0)\).

11. The function \( f(x) = -7(x - 4)^2 + 6 \) is given in vertex form \( f(x) = a(x - h)^2 + k \), where \( a = -7 \), \( h = 4 \), and \( k = 6 \). The vertex is \((h,k) = (4,6)\).

13. The function \( f(x) = \frac{1}{6}(x + \frac{7}{3})^2 + \frac{3}{4} \) is given in vertex form \( f(x) = a(x - h)^2 + k \), where \( a = 1/6 \), \( h = -7/3 \), and \( k = 3/8 \). The vertex is \((h,k) = (-\frac{7}{3},\frac{3}{8})\).

15. The function \( f(x) = -7(x - 3)^2 + 1 \) is given in vertex form \( f(x) = a(x - h)^2 + k \), where \( a = -7 \), \( h = 3 \), and \( k = 1 \). The axis of symmetry is the vertical line through the vertex. \( h = 3 \), so the axis of symmetry is \( x = 3 \).

17. The function \( f(x) = -\frac{7}{8}(x + \frac{1}{3})^2 + \frac{2}{3} \) is given in vertex form \( f(x) = a(x - h)^2 + k \), where \( a = -7/8 \), \( h = -1/4 \), and \( k = 2/3 \). The axis of symmetry is the vertical line through the vertex. \( h = -\frac{1}{4} \), so the axis of symmetry is \( x = -\frac{1}{4} \).

19. The function \( f(x) = -\frac{2}{5}(x + \frac{3}{5})^2 - \frac{4}{5} \) is given in vertex form \( f(x) = a(x - h)^2 + k \), where \( a = -2/9 \), \( h = -2/3 \), and \( k = -4/5 \). The axis of symmetry is the vertical line through the vertex. \( h = -\frac{2}{3} \), so the axis of symmetry is \( x = -\frac{2}{3} \).

21. The function \( f(x) = -\frac{8}{7}(x + \frac{3}{7})^2 + \frac{6}{5} \) is given in vertex form \( f(x) = a(x - h)^2 + k \), where \( a = -8/7 \), \( h = -2/9 \), and \( k = 6/5 \). The axis of symmetry is the vertical line through the vertex. \( h = -\frac{2}{9} \), so the axis of symmetry is \( x = -\frac{2}{9} \).
23. First, sketch your coordinate system. Compare the quadratic function \( f(x) = (x + 2)^2 - 3 \) with \( f(x) = a(x - h)^2 + k \) and note that \( h = -2 \) and \( k = -3 \). Hence, the vertex is located at \((h, k) = (-2, -3)\). The axis of symmetry is a vertical line through the vertex with equation \( x = -2 \). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \( f(x) = (x + 2)^2 - 3 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = (x + 2)^2 - 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain= \((-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range= \([-3, \infty)\).
25. First, sketch your coordinate system. Compare the quadratic function \( f(x) = -(x - 2)^2 + 5 \) with \( f(x) = a(x - h)^2 + k \) and note that \( h = 2 \) and \( k = 5 \). Hence, the vertex is located at \((h, k) = (2, 5)\). The axis of symmetry is a vertical line through the vertex with equation \( x = 2 \). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \( f(x) = -(x - 2)^2 + 5 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = -(x - 2)^2 + 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain = \((-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range = \((-\infty, 5]\).
27. First, sketch your coordinate system. Compare the quadratic function \( f(x) = (x - 3)^2 \) with \( f(x) = a(x - h)^2 + k \) and note that \( h = 3 \) and \( k = 0 \). Hence, the vertex is located at \((h, k) = (3, 0)\). The axis of symmetry is a vertical line through the vertex with equation \( x = 3 \). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \( f(x) = (x - 3)^2 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = (x - 3)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain = \(( -\infty, \infty )\). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range \( = [0, \infty) \).
29. First, sketch your coordinate system. Compare the quadratic function \( f(x) = -x^2 + 7 \) with \( f(x) = a(x - h)^2 + k \) and note that \( h = 0 \) and \( k = 7 \). Hence, the vertex is located at \( (h, k) = (0, 7) \). The axis of symmetry is a vertical line through the vertex with equation \( x = 0 \). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \( f(x) = -x^2 + 7 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = -x^2 + 7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain = \(( -\infty, \infty )\). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range = \(( -\infty, 7] \).
31. First, sketch your coordinate system. Compare the quadratic function $f(x) = 2(x - 1)^2 - 6$ with $f(x) = a(x - h)^2 + k$ and note that $h = 1$ and $k = -6$. Hence, the vertex is located at $(h, k) = (1, -6)$. The axis of symmetry is a vertical line through the vertex with equation $x = 1$. Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of $f(x) = 2(x - 1)^2 - 6$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = 2(x - 1)^2 - 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-4</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

To find the domain of $f$, mentally project every point of the graph onto the $x$-axis, as shown on the left below. This covers the entire $x$-axis, so the domain = $(-\infty, \infty)$. To find the range, mentally project every point of the graph onto the $y$-axis, as shown on the right below. The shaded interval on the $y$-axis is range = $[-6, \infty)$. 
33. First, sketch your coordinate system. Compare the quadratic function $f(x) = -\frac{1}{2}(x+1)^2 + 5$ with $f(x) = a(x-h)^2 + k$ and note that $h = -1$ and $k = 5$. Hence, the vertex is located at $(h, k) = (-1, 5)$. The axis of symmetry is a vertical line through the vertex with equation $x = -1$. Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of $f(x) = -\frac{1}{2}(x+1)^2 + 5$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = -\frac{1}{2}(x+1)^2 + 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9/2</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

To find the domain of $f$, mentally project every point of the graph onto the $x$-axis, as shown on the left below. This covers the entire $x$-axis, so the domain = $(-\infty, \infty)$. To find the range, mentally project every point of the graph onto the $y$-axis, as shown on the right below. The shaded interval on the $y$-axis is range = $(-\infty, 5]$. 
35. First, sketch your coordinate system. Compare the quadratic function \( f(x) = 2(x - 5/2)^2 - 15/2 \) with \( f(x) = a(x - h)^2 + k \) and note that \( h = 5/2 \) and \( k = -15/2 \). Hence, the vertex is located at \((h, k) = (5/2, -15/2)\). The axis of symmetry is a vertical line through the vertex with equation \( x = 5/2 \). Make a table to find two points on either side of the axis of symmetry. Plot them and mirror them across the axis of symmetry. Use all of this information to complete the graph of \( f(x) = 2(x - 5/2)^2 - 15/2 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 2(x - 5/2)^2 - 15/2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>-7</td>
</tr>
</tbody>
</table>

To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain = \((−∞, ∞)\). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range = \([-15/2, ∞)\).
37. The graph opens upward since $a = 7 > 0$, and the vertex is at $(h, k) = (-6, -6)$. Thus the domain is $[-\infty, \infty)$ and the range is $\{y : y \geq -6\} = [-6, \infty)$.

39. The graph opens downward since $a = -3 < 0$, and the vertex is at $(h, k) = (-4, -7)$. Thus the domain is $[-\infty, \infty)$ and the range is $\{y : y \leq -7\} = [-7, \infty)$.

41. The graph opens downward since $a = -7 < 0$, and the vertex is at $(h, k) = (-5, -7)$. Thus the domain is $[-\infty, \infty)$ and the range is $\{y : y \leq -7\} = [-7, \infty)$.

43. The graph opens downward since $a = -4 < 0$, and the vertex is at $(h, k) = (1, 2)$. Thus the domain is $[-\infty, \infty)$ and the range is $\{y : y \leq 2\} = [2, \infty)$.

45. Note that the parabola opens downward (see figure below). Hence, let’s start with the form $f(x) = -2x^2$, which is a parabola that opens downward, with vertex at the origin. Next, the parabola in the image has been shifted 3 units to the right, so we must replace $x$ with $x - 3$ in $f(x) = -2x^2$, arriving at $f(x) = -2(x - 3)^2$. Finally, we see that the graph has been shifted 1 unit up, so we add 1 to our last form to arrive at the final answer, $f(x) = -2(x - 3)^2 + 1$.

47. Note that the parabola opens upward (see figure below). Hence, let’s start with the form $f(x) = 2x^2$, which is a parabola that opens upward, with vertex at the origin. Next, the parabola in the image has been shifted 1 unit to the left, so we must replace $x$ with $x + 1$ in $f(x) = 2x^2$, arriving at $f(x) = 2(x + 1)^2$. Finally, we see that the graph has been shifted 1 unit down, so we add $-1$ to our last form to arrive at the final answer, $f(x) = 2(x + 1)^2 - 1$. 

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49. Note that the parabola opens upward (see figure below). Hence, let’s start with the form \( f(x) = 2x^2 \), which is a parabola that opens upward, with vertex at the origin. Next, the parabola in the image has been shifted 2 units to the left, so we must replace \( x \) with \( x + 2 \) in \( f(x) = 2x^2 \), arriving at \( f(x) = 2(x + 2)^2 \). Finally, we see that the graph has been shifted 1 unit up, so we add 1 to our last form to arrive at the final answer, \( f(x) = 2(x + 2)^2 + 1 \).

![Figure 1](image1)

51. Note that the parabola opens upward (see figure below). Hence, let’s start with the form \( f(x) = 2x^2 \), which is a parabola that opens upward, with vertex at the origin. Next, the parabola in the image has been shifted 3 units to the right, so we must replace \( x \) with \( x - 3 \) in \( f(x) = 2x^2 \), arriving at \( f(x) = 2(x - 3)^2 \). Finally, we see that the graph has been shifted 1 unit down, so we add \(-1\) to our last form to arrive at the final answer, \( f(x) = 2(x - 3)^2 - 1 \).

![Figure 2](image2)
53. To find the range of \( f(x) = ax^2 + bx + c \), examine the graph and mentally project each point of the graph onto the \( y \)-axis (see figure below). Note that the arrows on the ends of the blue graph imply that the blue graph opens downward and to the left and right indefinitely. Thus, the range is all real numbers less than or equal to \(-2\), or in interval notation, \((−\infty, −2]\).

55. To find the domain of \( f(x) = ax^2 + bx + c \), examine the graph and mentally project each point of the graph onto the \( x \)-axis (see figure below). Note that the arrows on the ends of the blue graph imply that the blue graph opens downward and to the left and right indefinitely. Thus, the domain is all real numbers, or in interval notation, \((−\infty, \infty)\).