5.3 Exercises

In Exercises 1-8, factor the given quadratic polynomial.

1. $x^2 + 9x + 14$
2. $x^2 + 6x + 5$
3. $x^2 + 10x + 9$
4. $x^2 + 4x - 21$
5. $x^2 - 4x - 5$
6. $x^2 + 7x - 8$
7. $x^2 - 7x + 12$
8. $x^2 + 5x - 24$

In Exercises 9-16, find the zeros of the given quadratic function.

9. $f(x) = x^2 - 2x - 15$
10. $f(x) = x^2 + 4x - 32$
11. $f(x) = x^2 + 10x - 39$
12. $f(x) = x^2 + 4x - 45$
13. $f(x) = x^2 - 14x + 40$
14. $f(x) = x^2 - 5x - 14$
15. $f(x) = x^2 + 9x - 36$
16. $f(x) = x^2 + 11x - 26$

In Exercises 17-22, perform each of the following tasks for the quadratic functions.

i. Load the function into Y1 of the Y= of your graphing calculator. Adjust the window parameters so that the vertex is visible in the viewing window.

ii. Set up a coordinate system on your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax. Make a reasonable copy of the image in the viewing window of your calculator on this coordinate system and label it with its equation.

iii. Use the zero utility on your graphing calculator to find the zeros of the function. Use these results to plot the x-intercepts on your coordinate system and label them with their coordinates.

iv. Use a strictly algebraic technique (no calculator) to find the zeros of the given quadratic function. Show your work next to your coordinate system. Be stubborn! Work the problem until your algebraic and graphically zeros are a reasonable match.

17. $f(x) = x^2 + 5x - 14$
18. $f(x) = x^2 + x - 20$
19. $f(x) = -x^2 + 3x + 18$
20. $f(x) = -x^2 + 3x + 40$
21. $f(x) = x^2 - 16x - 36$
22. $f(x) = x^2 + 4x - 96$

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1 Copyrighted material. See: http://msenux.redwoods.edu/IntAlgText/
In Exercises 23-30, perform each of the following tasks for the given quadratic function.

i. Set up a coordinate system on graph paper. Label and scale each axis. Remember to draw all lines with a ruler.

ii. Use the technique of completing the square to place the quadratic function in vertex form. Plot the vertex on your coordinate system and label it with its coordinates. Draw the axis of symmetry on your coordinate system and label it with its equation.

iii. Use a strictly algebraic technique (no calculators) to find the x-intercepts of the graph of the given quadratic function. Plot them on your coordinate system and label them with their coordinates.

iv. Find the y-intercept of the graph of the quadratic function. Plot the y-intercept on your coordinate system and its mirror image across the axis of symmetry, then label these points with their coordinates.

v. Using all the information plotted, draw the graph of the quadratic function and label it with the vertex form of its equation. Use interval notation to describe the domain and range of the quadratic function.

23. \( f(x) = x^2 + 2x - 8 \)
24. \( f(x) = x^2 - 6x + 8 \)
25. \( f(x) = x^2 + 4x - 12 \)
26. \( f(x) = x^2 + 8x + 12 \)
27. \( f(x) = -x^2 - 2x + 8 \)
28. \( f(x) = -x^2 - 2x + 24 \)
29. \( f(x) = -x^2 - 8x + 48 \)
30. \( f(x) = -x^2 - 8x + 20 \)

In Exercises 31-38, factor the given quadratic polynomial.

31. \( 42x^2 + 5x - 2 \)
32. \( 3x^2 + 7x - 20 \)
33. \( 5x^2 - 19x + 12 \)
34. \( 54x^2 - 3x - 1 \)
35. \( -4x^2 + 9x - 5 \)
36. \( 3x^2 - 5x - 12 \)
37. \( 2x^2 - 3x - 35 \)
38. \( -6x^2 + 25x + 9 \)

In Exercises 39-46, find the zeros of the given quadratic functions.

39. \( f(x) = 2x^2 - 3x - 20 \)
40. \( f(x) = 2x^2 - 7x - 30 \)
41. \( f(x) = -2x^2 + x + 28 \)
42. \( f(x) = -2x^2 + 15x - 22 \)
43. \( f(x) = 3x^2 - 20x + 12 \)
44. \( f(x) = 4x^2 + 11x - 20 \)
45. \( f(x) = -4x^2 + 4x + 15 \)
46. \( f(x) = -6x^2 - x + 12 \)
In Exercises 47-52, perform each of the following tasks for the given quadratic functions.

i. Load the function into Y1 of the Y= of your graphing calculator. Adjust the window parameters so that the vertex is visible in the viewing window.

ii. Set up a coordinate system on your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax. Make a reasonable copy of the image in the viewing window of your calculator on this coordinate system and label it with its equation.

iii. Use the zero utility on your graphing calculator to find the zeros of the function. Use these results to plot the x-intercepts on your coordinate system and label them with their coordinates.

iv. Use a strictly algebraic technique (no calculator) to find the zeros of the given quadratic function. Show your work next to your coordinate system. Be stubborn! Work the problem until your algebraic and graphically zeros are a reasonable match.

47. \( f(x) = 2x^2 + 3x - 35 \)
48. \( f(x) = 2x^2 - 5x - 42 \)
49. \( f(x) = -2x^2 + 5x + 33 \)
50. \( f(x) = -2x^2 - 5x + 52 \)
51. \( f(x) = 4x^2 - 24x - 13 \)
52. \( f(x) = 4x^2 + 24x - 45 \)

In Exercises 53-60, perform each of the following tasks for the given quadratic functions.

i. Set up a coordinate system on graph paper. Label and scale each axis. Remember to draw all lines with a ruler.

ii. Use the technique of completing the square to place the quadratic function in vertex form. Plot the vertex on your coordinate system and label it with its coordinates. Draw the axis of symmetry on your coordinate system and label it with its equation.

iii. Use a strictly algebraic method (no calculators) to find the x-intercepts of the graph of the quadratic function. Plot them on your coordinate system and label them with their coordinates.

iv. Find the y-intercept of the graph of the quadratic function. Plot the y-intercept on your coordinate system and its mirror image across the axis of symmetry, then label these points with their coordinates.

v. Using all the information plotted, draw the graph of the quadratic function and label it with the vertex form of its equation. Use interval notation to describe the domain and range of the quadratic function.

53. \( f(x) = 2x^2 - 8x - 24 \)
54. \( f(x) = 2x^2 - 4x - 6 \)
55. \( f(x) = -2x^2 - 4x + 16 \)
56. \( f(x) = -2x^2 - 16x + 40 \)
57. \( f(x) = 3x^2 + 18x - 48 \)
58. \( f(x) = 3x^2 + 18x - 216 \)
59. \( f(x) = 2x^2 + 10x - 48 \)
60. \( f(x) = 2x^2 - 10x - 100 \)
In Exercises 61-66, Use the graph of \( f(x) = ax^2 + bx + c \) shown to find all solutions of the equation \( f(x) = 0 \). (Note: Every solution is an integer.)

61.

62.

63.

64.

65.

66.
5.3 Solutions

1. Look for \( p \) and \( q \) such that \((p+q)(x+q) = x^2 + 9x + 14\). It follows that \( p+q = 9 \) and \( pq = 14 \), so \( p = 2 \) and \( q = 7 \). Now verify the factorization by multiplying \((x+2)(x+7)\) to obtain \( x^2 + 9x + 14 \).

3. Look for \( p \) and \( q \) such that \((p+q)(x+q) = x^2 + 10x + 9\). It follows that \( p+q = 10 \) and \( pq = 9 \), so \( p = 9 \) and \( q = 1 \). Now verify the factorization by multiplying \((x+9)(x+1)\) to obtain \( x^2 + 10x + 9 \).

5. Look for \( p \) and \( q \) such that \((p+q)(x+q) = x^2 - 4x - 5\). It follows that \( p+q = -4 \) and \( pq = -5 \), so \( p = -5 \) and \( q = 1 \). Now verify the factorization by multiplying \((x-5)(x+1)\) to obtain \( x^2 - 4x - 5 \).

7. Look for \( p \) and \( q \) such that \((p+q)(x+q) = x^2 - 7x + 12\). It follows that \( p+q = -7 \) and \( pq = 12 \), so \( p = -4 \) and \( q = -3 \). Now verify the factorization by multiplying \((x-4)(x-3)\) to obtain \( x^2 - 7x + 12 \).

9. To find the zeroes, set \( f(x) = 0 \) and factor.

\[
0 = x^2 - 2x - 15 \\
0 = (x - 5)(x + 3)
\]

By the zero product property, either

\[
x - 5 = 0 \quad \text{or} \quad x + 3 = 0.
\]

Solve these linear equations independently.

\[
x = 5 \quad \text{or} \quad x = -3
\]

11. To find the zeroes, set \( f(x) = 0 \) and factor.

\[
0 = x^2 + 10x - 39 \\
0 = (x + 13)(x - 3)
\]

By the zero product property, either

\[
x + 13 = 0 \quad \text{or} \quad x - 3 = 0.
\]

Solve these linear equations independently.

\[
x = -13 \quad \text{or} \quad x = 3
\]
13. To find the zeroes, set \( f(x) = 0 \) and factor.

\[
0 = x^2 - 14x + 40 \\
0 = (x - 4)(x - 10)
\]

By the zero product property, either

\[
x - 4 = 0 \quad \text{or} \quad x - 10 = 0.
\]

Solve these linear equations independently.

\[
x = 4 \quad \text{or} \quad x = 10
\]

15. To find the zeroes, set \( f(x) = 0 \) and factor.

\[
0 = x^2 + 9x - 36 \\
0 = (x - 3)(x + 12)
\]

By the zero product property, either

\[
x - 3 = 0 \quad \text{or} \quad x + 12 = 0.
\]

Solve these linear equations independently.

\[
x = 3 \quad \text{or} \quad x = -12
\]

17. To find the zeroes with your calculator, press 2nd TRACE to access the CALC menu and choose 2:zero.

Use the left arrow to move the cursor along the curve until it is to the left of the first zero. Hit ENTER.

Use the right arrow to move the cursor until it is to the right of the same zero. Hit ENTER.

Finally hit ENTER near that same zero for the guess, and you get the zero.

Repeat this process for the second zero. We get \((-7, 0)\) and \((2, 0)\).
To find the zeroes algebraically, set $f(x) = 0$ and factor.

$$0 = x^2 + 5x - 14$$

By the zero product property, either

$$x + 7 = 0 \quad \text{or} \quad x - 2 = 0.$$ 

Solve these linear equations independently.

$$x = -7 \quad \text{or} \quad x = 2$$

So the zeroes are $(-7, 0)$ and $(2, 0)$.

19. To find the zeroes with your calculator, press 2nd TRACE to access the CALC menu and choose 2:zero.

Use the left arrow to move the cursor along the curve until it is to the left of the first zero. Hit ENTER.

Use the right arrow to move the cursor until it is to the right of the same zero. Hit ENTER.

Finally hit ENTER near that same zero for the guess, and you get the zero.

Repeat this process for the second zero. We get $(6, 0)$ and $(-3, 0)$. 
To find the zeroes algebraically, set \( f(x) = 0 \) and factor.

\[
0 = -x^2 + 3x + 18 \\
0 = -(x^2 - 3x - 18) \\
0 = -(x - 6)(x + 3)
\]

By the zero product property, either

\[
x - 6 = 0 \quad \text{or} \quad x + 3 = 0.
\]

Solve these linear equations independently.

\[
x = 6 \quad \text{or} \quad x = -3
\]

21. To find the zeroes with your calculator, press 2nd TRACE to access the CALC menu and choose 2:zero.

Repeat this process for the second zero. We get \((-2,0)\) and \((18,0)\).
To find the zeroes algebraically, set \( f(x) = 0 \) and factor.

\[
0 = x^2 - 16x - 36
\]

\[
0 = (x + 2)(x - 18)
\]

By the zero product property, either

\[
x + 2 = 0 \quad \text{or} \quad x - 18 = 0.
\]

Solve these linear equations independently.

\[
x = -2 \quad \text{or} \quad x = 18
\]

23. First, complete the square:

\[
f(x) = x^2 + 2x - 8
\]

\[
= x^2 + 2x + 1 - 1 - 8
\]

\[
= (x^2 + 2x + 1) - 9
\]

Read off the vertex as \((h, k) = (-1, -9)\). The axis of symmetry is a vertical line through the vertex with equation \(x = -1\).

To find the \(x\)-intercepts algebraically, set \(y = 0\) and factor.

\[
0 = x^2 + 2x - 8
\]

\[
0 = (x + 4)(x - 2)
\]

By the zero product property, either

\[
x + 4 = 0 \quad \text{or} \quad x - 2 = 0.
\]

Solve these linear equations independently.

\[
x = -4 \quad \text{or} \quad x = 2
\]
So the $x$-intercepts are $(-4, 0)$ and $(2, 0)$.

Lastly, to find the $y$-intercept, set $x = 0$ in the equation and solve for $y$:

\[
y = x^2 + 2x - 8
\]

\[
y = 0^2 + 2(0) - 8
\]

\[
y = -8
\]

So the $y$-intercept is $(0, -8)$. Finally, put this all together to make the graph.

To find the domain of $f$, mentally project every point of the graph onto the $x$-axis, as shown on the left below. This covers the entire $x$-axis, so the domain $= (-\infty, \infty)$. To find the range, mentally project every point of the graph onto the $y$-axis, as shown on the right below. The shaded interval on the $y$-axis is range $= [-9, \infty)$.

25. First, complete the square:

\[
f(x) = x^2 + 4x - 12
\]

\[
= x^2 + 4x + 4 - 4 - 12
\]

\[
= (x^2 + 4x + 4) - 16
\]

\[
= (x + 2)^2 - 16
\]
Section 5.3  Zeros of the Quadratic

Read off the vertex as \((h, k) = (-2, -16)\). The axis of symmetry is a vertical line through the vertex with equation \(x = -2\).

To find the \(x\)-intercepts algebraically, set \(y = 0\) and factor.

\[
0 = x^2 + 4x - 12 \\
0 = (x + 6)(x - 2)
\]

By the zero product property, either

\[
x + 6 = 0 \quad \text{or} \quad x - 2 = 0.
\]

Solve these linear equations independently.

\[
x = -6 \quad \text{or} \quad x = 2
\]

So the \(x\)-intercepts are \((-6, 0)\) and \((2, 0)\).

Lastly, to find the \(y\)-intercept, set \(x = 0\) in the equation and solve for \(y\):

\[
y = x^2 + 4x - 12
\]

\[
y = 0^2 + 4(0) - 12
\]

\[
y = -12
\]

So the \(y\)-intercept is \((0, -12)\).

Finally, put this all together to make the graph.

To find the domain of \(f\), mentally project every point of the graph onto the \(x\)-axis, as shown on the left below. This covers the entire \(x\)-axis, so the domain= \((-∞, ∞)\). To find the range, mentally project every point of the graph onto the \(y\)-axis, as shown on the right below. The shaded interval on the \(y\)-axis is range= \([-16, \infty)\).
27. First, complete the square:

\[
f(x) = -x^2 - 2x + 8
\]

\[
= -(x^2 + 2x - 8)
\]

\[
= -(x^2 + 2x + 1 - 1 - 8)
\]

\[
= - \left( (x^2 + 2x + 1) - 1 - 8 \right)
\]

\[
= - \left( (x + 1)^2 - 9 \right)
\]

\[
= - (x + 1)^2 + 9
\]

Read off the vertex as \((h, k) = (-1, 9)\). The axis of symmetry is a vertical line through the vertex with equation \(x = -1\).

To find the \(x\)-intercepts algebraically, set \(y = 0\) and factor.

\[
0 = -x^2 - 2x + 8
\]

\[
0 = -(x^2 + 2x - 8)
\]

\[
0 = -(x + 4)(x - 2)
\]

By the zero product property, either

\[
x + 4 = 0 \quad \text{or} \quad x - 2 = 0.
\]

Solve these linear equations independently.

\[
x = -4 \quad \text{or} \quad x = 2
\]

So the \(x\)-intercepts are \((-4, 0)\) and \((2, 0)\).

Lastly, to find the \(y\)-intercept, set \(x = 0\) in the equation and solve for \(y\):

\[
y = -x^2 - 2x + 8
\]

\[
y = -0^2 - 2(0) + 8
\]

\[
y = 8
\]
So the $y$-intercept is $(0, 8)$.

Finally, put this all together to make the graph.

To find the domain of $f$, mentally project every point of the graph onto the $x$-axis, as shown on the left below. This covers the entire $x$-axis, so the domain $= (-\infty, \infty)$. To find the range, mentally project every point of the graph onto the $y$-axis, as shown on the right below. The shaded interval on the $y$-axis is range $= (-\infty, 9]$.

29. First, complete the square:

\[
f(x) = -x^2 - 8x + 48
= -(x^2 + 8x - 48)
= -(x^2 + 8x + 16 - 16 - 48)
= -((x^2 + 8x + 16) - 16 - 48)
= -((x + 4)^2 - 64)
= -(x + 4)^2 + 64
\]
Read off the vertex as \((h, k) = (-4, 64)\). The axis of symmetry is a vertical line through the vertex with equation \(x = -4\).

To find the \(x\)-intercepts algebraically, set \(y = 0\) and factor.

\[
0 = -x^2 - 8x + 48 \\
0 = -(x^2 + 8x - 48) \\
0 = -(x + 12)(x - 4)
\]

By the zero product property, either

\[
x + 12 = 0 \quad \text{or} \quad x - 4 = 0.
\]

Solve these linear equations independently.

\[
x = -12 \quad \text{or} \quad x = 4
\]

So the \(x\)-intercepts are \((-12, 0)\) and \((4, 0)\).

Lastly, to find the \(y\)-intercept, set \(x = 0\) in the equation and solve for \(y\):

\[
y = -x^2 - 8x + 48 \\
y = -0^2 - 8(0) + 48 \\
y = 48
\]

So the \(y\)-intercept is \((0, 48)\).

Finally, put this all together to make the graph.

To find the domain of \(f\), mentally project every point of the graph onto the \(x\)-axis, as shown on the left below. This covers the entire \(x\)-axis, so the domain\(= (-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \(y\)-axis, as shown on the right below. The shaded interval on the \(y\)-axis is range\(= (-\infty, 64]\).
31. Using the *ac*-test, first look for two numbers whose product is \(ac = (42)(-2) = -84\) and whose sum is \(b = 5\). The solutions are \(-7\) and \(12\). Then
\[
42x^2 + 5x - 2 = 42x^2 - 7x + 12x - 2
\]
\[
= 7x(6x - 1) + 2(6x - 1)
\]
\[
= (7x + 2)(6x - 1)
\]
Now verify the factorization by multiplying \((7x + 2)(6x - 1)\) to obtain \(42x^2 + 5x - 2\).

33. Using the *ac*-test, first look for two numbers whose product is \(ac = (5)(12) = 60\) and whose sum is \(b = -19\). The solutions are \(-4\) and \(-15\). Then
\[
5x^2 - 19x + 12 = 5x^2 - 4x - 15x + 12
\]
\[
= x(5x - 4) - 3(5x - 4)
\]
\[
= (x - 3)(5x - 4)
\]
Now verify the factorization by multiplying \((x - 3)(5x - 4)\) to obtain \(5x^2 - 19x + 12\).

35. Using the *ac*-test, first look for two numbers whose product is \(ac = (-4)(-5) = 20\) and whose sum is \(b = 9\). The solutions are \(4\) and \(5\). Then
\[
-4x^2 + 9x - 5 = -4x^2 + 4x + 5x - 5
\]
\[
= 4x(-x + 1) - 5(-x + 1)
\]
\[
= (4x - 5)(-x + 1)
\]
Now verify the factorization by multiplying \((4x - 5)(-x + 1)\) to obtain \(-4x^2 + 9x - 5\).
Chapter 5  Quadratic Functions

37. Using the ac-test, first look for two numbers whose product is \( ac = (2)(-35) = -70 \) and whose sum is \( b = -3 \). The solutions are -10 and 7. Then

\[
2x^2 - 3x - 35 = 2x^2 - 10x + 7x - 35
= 2x(x - 5) + 7(x - 5)
= (2x + 7)(x - 5)
\]

Now verify the factorization by multiplying \((2x + 7)(x - 5)\) to obtain \(2x^2 - 3x - 35\).

39. To find the zeroes, set \( f(x) = 0 \) and factor.

\[
0 = 2x^2 - 3x - 20
0 = (2x + 5)(x - 4)
\]

By the zero product property, either

\[
2x + 5 = 0 \quad \text{or} \quad x - 4 = 0.
\]

Solve these linear equations independently.

\[
x = -5/2 \quad \text{or} \quad x = 4
\]

41. To find the zeroes, set \( f(x) = 0 \) and factor.

\[
0 = -2x^2 + x + 28
0 = -(2x^2 - x - 28)
0 = -(2x + 7)(x - 4)
\]

By the zero product property, either

\[
2x + 7 = 0 \quad \text{or} \quad x - 4 = 0.
\]

Solve these linear equations independently.

\[
x = -7/2 \quad \text{or} \quad x = 4
\]

43. To find the zeroes, set \( f(x) = 0 \) and factor.

\[
0 = 3x^2 - 20x + 12
0 = (3x - 2)(x - 6)
\]

By the zero product property, either

\[
3x - 2 = 0 \quad \text{or} \quad x - 6 = 0.
\]

Solve these linear equations independently.

\[
x = 2/3 \quad \text{or} \quad x = 6
\]

Version: Fall 2007
45. To find the zeroes, set \( f(x) = 0 \) and factor.

\[
0 = -4x^2 + 4x + 15 \\
0 = -(4x^2 - 4x - 15) \\
0 = -(2x + 3)(2x - 5)
\]

By the zero product property, either

\[
2x + 3 = 0 \quad \text{or} \quad 2x - 5 = 0.
\]

Solve these linear equations independently.

\[
x = -\frac{3}{2} \quad \text{or} \quad x = \frac{5}{2}
\]

47. To find the zeroes with your calculator, press 2nd TRACE to access the CALC menu and choose 2:zero.

Use the left arrow to move the cursor along the curve until it is to the left of the first zero. Hit ENTER.

Use the right arrow to move the cursor until it is to the right of the same zero. Hit ENTER.

Finally hit ENTER near that same zero for the guess, and you get the zero.

Repeat this process for the second zero. We get \((3.5,0)\) and \((-5,0)\).

To find the zeroes algebraically, set \( y = 0 \) and solve for \( x \):
Chapter 5  Quadratic Functions

\[ 0 = 2x^2 + 3x - 35 \]
\[ 0 = (2x - 7)(x + 5) \]

By the zero product property, either

\[ 2x - 7 = 0 \quad \text{or} \quad x + 5 = 0. \]

Solve these linear equations independently.

\[ x = 7/2 \quad \text{or} \quad x = -5 \]

49. To find the zeroes with your calculator, press 2nd TRACE to access the CALC menu and choose 2:zero.

Repeat this process for the second zero. We get (5.5, 0) and (-3, 0).

To find the zeroes algebraically, set \( y = 0 \) and solve for \( x \):

\[ 0 = -2x^2 + 5x + 33 \]
\[ 0 = -(2x^2 - 5x - 33) \]
\[ 0 = -(2x - 11)(x + 3) \]
By the zero product property, either

\[ 2x - 11 = 0 \quad \text{or} \quad x + 3 = 0. \]

Solve these linear equations independently.

\[ x = \frac{11}{2} \quad \text{or} \quad x = -3 \]

51. To find the zeroes with your calculator, press 2nd TRACE to access the CALC menu and choose 2:zero.

Use the left arrow to move the cursor along the curve until it is to the left of the first zero. Hit ENTER. Use the right arrow to move the cursor until it is to the right of the same zero. Hit ENTER. Finally hit ENTER near that same zero for the guess, and you get the zero.

Repeat this process for the second zero. We get \((-0.5, 0)\) and \((6.5, 0)\).

To find the zeroes, set \(y = 0\) and solve for \(x\):

\[ 0 = 4x^2 - 24x - 13 \]

\[ 0 = (2x + 1)(2x - 13) \]

By the zero product property, either

\[ 2x + 1 = 0 \quad \text{or} \quad 2x - 13 = 0. \]
Solve these linear equations independently.

\[ x = -\frac{1}{2} \quad \text{or} \quad x = \frac{13}{2} \]

53. First, complete the square:

\[
\begin{align*}
f(x) &= 2x^2 - 8x - 24 \\
     &= 2(x^2 - 4x - 12) \\
     &= 2 \left( x^2 - 4x + 4 - 4 - 12 \right) \\
     &= 2 \left( (x - 2)^2 - 16 \right) \\
     &= 2 (x - 2)^2 - 32
\end{align*}
\]

Read off the vertex as \((h, k) = (2, -32)\). The axis of symmetry is a vertical line through the vertex with equation \(x = 2\).

To find the \(x\)-intercepts algebraically, set \(y = 0\) and factor.

\[
0 = 2x^2 - 8x - 24 \\
0 = 2(x^2 - 4x - 12) \\
0 = 2(x - 6)(x + 2)
\]

By the zero product property, either

\[ x - 6 = 0 \quad \text{or} \quad x + 2 = 0. \]

Solve these linear equations independently.

\[ x = 6 \quad \text{or} \quad x = -2 \]

So the \(x\)-intercepts are \((6, 0)\) and \((-2, 0)\).

Lastly, to find the \(y\)-intercept, set \(x = 0\) in the equation and solve for \(y\):

\[
\begin{align*}
y &= 2x^2 - 8x - 24 \\
y &= 2(0)^2 - 8(0) - 24 \\
y &= -24
\end{align*}
\]

So the \(y\)-intercept is \((0, -24)\).

Finally, put this all together to make the graph.
To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain is \( (-\infty, \infty) \). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range \( [-32, \infty) \).

55. First, complete the square:

\[
f(x) = -2x^2 - 4x + 16
\]

\[
= -2(x^2 + 2x - 8)
\]

\[
= -2 \left( x^2 + 2x + 1 - 1 - 8 \right)
\]

\[
= -2 \left( (x + 1)^2 - 9 \right)
\]

\[
= -2 (x + 1)^2 + 18
\]

Read off the vertex as \((h, k) = (-1, 18)\). The axis of symmetry is a vertical line through the vertex with equation \( x = -1 \).

To find the \( x \)-intercepts algebraically, set \( y = 0 \) and factor.
Chapter 5  Quadratic Functions

\[ 0 = -2x^2 - 4x + 16 \]
\[ 0 = -2(x^2 + 2x - 8) \]
\[ 0 = -2(x + 4)(x - 2) \]

By the zero product property, either
\[ x + 4 = 0 \quad \text{or} \quad x - 2 = 0. \]

Solve these linear equations independently.
\[ x = -4 \quad \text{or} \quad x = 2 \]

So the \( x \)-intercepts are \((-4, 0)\) and \((2, 0)\).

Lastly, to find the \( y \)-intercept, set \( x = 0 \) in the equation and solve for \( y \):
\[ y = -2x^2 - 4x + 16 \]
\[ y = -2(0)^2 - 4(0) + 16 \]
\[ y = 16 \]

So the \( y \)-intercept is \((0, 16)\).

Finally, put this all together to make the graph.

To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain= \((-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range= \((-\infty, 18]\).
57. First, complete the square:

\[ f(x) = 3x^2 + 18x - 48 \]
\[ = 3(x^2 + 6x - 16) \]
\[ = 3 \left( x^2 + 6x + 9 - 9 - 16 \right) \]
\[ = 3 \left( (x + 3)^2 - 25 \right) \]
\[ = 3(x + 3)^2 - 75 \]

Read off the vertex as \((h, k) = (-3, -75)\). The axis of symmetry is a vertical line through the vertex with equation \(x = -3\).

To find the \(x\)-intercepts algebraically, set \(y = 0\) and factor.

\[ 0 = 3x^2 + 18x - 48 \]
\[ 0 = 3(x^2 + 6x - 16) \]
\[ 0 = 3(x + 8)(x - 2) \]

By the zero product property, either

\[ x + 8 = 0 \quad \text{or} \quad x - 2 = 0. \]

Solve these linear equations independently.

\[ x = -8 \quad \text{or} \quad x = 2 \]

So the \(x\)-intercepts are \((-8, 0)\) and \((2, 0)\).

Lastly, to find the \(y\)-intercept, set \(x = 0\) in the equation and solve for \(y\):

\[ y = 3x^2 + 18x - 48 \]
\[ y = 3(0)^2 + 18(0) - 48 \]
\[ y = -48 \]
So the $y$-intercept is $(0, -48)$.

Finally, put this all together to make the graph.

To find the domain of $f$, mentally project every point of the graph onto the $x$-axis, as shown on the left below. This covers the entire $x$-axis, so the domain $= (-\infty, \infty)$. To find the range, mentally project every point of the graph onto the $y$-axis, as shown on the right below. The shaded interval on the $y$-axis is range $= [-75, \infty)$. 
59. First, complete the square:

\[
f(x) = 2x^2 + 10x - 48
= 2(x^2 + 5x - 24)
= 2 \left( x^2 + 5x + \frac{25}{4} - \frac{25}{4} - 24 \right)
= 2 \left( \left( x + \frac{5}{2} \right)^2 - \frac{25}{4} - 96 \right)
= 2 \left( \left( x + \frac{5}{2} \right)^2 - \frac{121}{4} \right)
= 2 \left( x + \frac{5}{2} \right)^2 - \frac{121}{2}
\]

Read off the vertex as \((h, k) = (-5/2, -121/2)\). The axis of symmetry is a vertical line through the vertex with equation \(x = -5/2\).

To find the \(x\)-intercepts algebraically, set \(y = 0\) and factor.

\[
0 = 2x^2 + 10x - 48
= 2(x^2 + 5x - 24)
= 2(x + 8)(x - 3)
\]

By the zero product property, either

\[
x + 8 = 0 \quad \text{or} \quad x - 3 = 0.
\]

Solve these linear equations independently.

\[
x = -8 \quad \text{or} \quad x = 3
\]

So the \(x\)-intercepts are \((-8, 0)\) and \((3, 0)\).

Lastly, to find the \(y\)-intercept, set \(x = 0\) in the equation and solve for \(y\):

\[
y = 2x^2 + 10x - 48
= 2(0)^2 + 10(0) - 48
= 0
\]

So the \(y\)-intercept is \((0, -48)\).

Finally, put this all together to make the graph.
To find the domain of \( f \), mentally project every point of the graph onto the \( x \)-axis, as shown on the left below. This covers the entire \( x \)-axis, so the domain = \((-\infty, \infty)\). To find the range, mentally project every point of the graph onto the \( y \)-axis, as shown on the right below. The shaded interval on the \( y \)-axis is range = \([-121/2, \infty)\).

61. To find the solutions of \( ax^2 + bx + c = 0 \), note where the graph of \( f(x) = ax^2 + bx + c \) crosses the \( x \)-axis (see figure below). Thus, the solutions are \(-2\) and \(3\).
63. To find the solutions of $ax^2 + bx + c = 0$, note where the graph of $f(x) = ax^2 + bx + c$ crosses the $x$-axis (see figure below). Thus, the solutions are $-3$ and $0$.

65. To find the solutions of $ax^2 + bx + c = 0$, note where the graph of $f(x) = ax^2 + bx + c$ crosses the $x$-axis (see figure below). Thus, the solutions are $-3$ and $0$. 