5.4 Exercises

In **Exercises 1-8**, find all real solutions of the given equation. Use a calculator to approximate the answers, correct to the nearest hundredth (two decimal places).

1. \(x^2 = 36\)
2. \(x^2 = 81\)
3. \(x^2 = 17\)
4. \(x^2 = 13\)
5. \(x^2 = 0\)
6. \(x^2 = -18\)
7. \(x^2 = -12\)
8. \(x^2 = 3\)

In **Exercises 9-16**, find all real solutions of the given equation. Use a calculator to approximate your answers to the nearest hundredth.

9. \((x - 1)^2 = 25\)
10. \((x + 3)^2 = 9\)
11. \((x + 2)^2 = 0\)
12. \((x - 3)^2 = -9\)
13. \((x + 6)^2 = -81\)
14. \((x + 7)^2 = 10\)
15. \((x - 8)^2 = 15\)
16. \((x + 10)^2 = 37\)

In **Exercises 17-28**, perform each of the following tasks for the given quadratic function.

i. Set up a coordinate system on a sheet of graph paper. Label and scale each axis. *Remember to draw all lines with a ruler.*

ii. Place the quadratic function in vertex form. Plot the vertex on your coordinate system and label it with its coordinates. Draw the axis of symmetry on your coordinate system and label it with its equation.

iii. Use the quadratic formula to find the \(x\)-intercepts of the parabola. Use a calculator to approximate each intercept, correct to the nearest tenth, and use these approximations to plot the \(x\)-intercepts on your coordinate system. However, label each \(x\)-intercept with its **exact** coordinates.

iv. Plot the \(y\)-intercept on your coordinate system and its mirror image across the axis of symmetry and label each with their coordinates.

v. Using all of the information on your coordinate system, draw the graph of the parabola, then label it with the vertex form of the function. Use interval notation to state the domain and range of the quadratic function.

17. \(f(x) = x^2 - 4x - 8\)
18. \(f(x) = x^2 + 6x - 1\)
19. \(f(x) = x^2 + 6x - 3\)
20. \(f(x) = x^2 - 8x + 1\)
21. \(f(x) = -x^2 + 2x + 10\)

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1 Copyrighted material. See: http://msenux.redwoods.edu/IntAlgText/
22. \( f(x) = -x^2 - 8x - 8 \)

23. \( f(x) = -x^2 - 8x - 9 \)

24. \( f(x) = -x^2 + 10x - 20 \)

25. \( f(x) = 2x^2 - 20x + 40 \)

26. \( f(x) = 2x^2 - 16x + 12 \)

27. \( f(x) = -2x^2 + 16x + 8 \)

28. \( f(x) = -2x^2 - 24x - 52 \)

In Exercises 29-32, perform each of the following tasks for the given quadratic equation.

i. Set up a coordinate system on a sheet of graph paper. Label and scale each axis. Remember to draw all lines with a ruler.

ii. Show that the discriminant is negative.

iii. Use the technique of completing the square to put the quadratic function in vertex form. Plot the vertex on your coordinate system and label it with its coordinates. Draw the axis of symmetry on your coordinate system and label it with its equation.

iv. Plot the \( y \)-intercept and its mirror image across the axis of symmetry and label each with their coordinates.

v. Because the discriminant is negative (did you remember to show that?), there are no \( x \)-intercepts. Use the given equation to calculate one additional point, then plot the point and its mirror image across the axis of symmetry and label each with their coordinates.

vi. Using all of the information on your coordinate system, draw the graph of the parabola, then label it with the vertex form of function. Use interval notation to describe the domain and range of the quadratic function.

29. \( f(x) = x^2 + 4x + 8 \)

30. \( f(x) = x^2 - 4x + 9 \)

31. \( f(x) = -x^2 + 6x - 11 \)

32. \( f(x) = -x^2 - 8x - 20 \)

In Exercises 33-36, perform each of the following tasks for the given quadratic function.

i. Set up a coordinate system on a sheet of graph paper. Label and scale each axis. Remember to draw all lines with a ruler.

ii. Use the discriminant to help determine the value of \( k \) so that the graph of the given quadratic function has exactly one \( x \)-intercept.

iii. Substitute this value of \( k \) back into the given quadratic function, then use the technique of completing the square to put the quadratic function in vertex form. Plot the vertex on your coordinate system and label it with its coordinates. Draw the axis of symmetry on your coordinate system and label it with its equation.

iv. Plot the \( y \)-intercept and its mirror image across the axis of symmetry and label each with their coordinates.

v. Use the equation to calculate an additional point on either side of the axis of symmetry, then plot this point and its mirror image across the axis of symmetry and label each with their coordinates.

vi. Using all of the information on your coordinate system, draw the graph of the parabola, then label it with the vertex form of the function. Use
interval notation to describe the domain and range of the quadratic function.

33. \( f(x) = x^2 - 4x + 4k \)

34. \( f(x) = x^2 + 6x + 3k \)

35. \( f(x) = kx^2 - 16x - 32 \)

36. \( f(x) = kx^2 - 24x + 48 \)

37. Find all values of \( k \) so that the graph of the quadratic function \( f(x) = kx^2 - 3x + 5 \) has exactly two \( x \)-intercepts.

38. Find all values of \( k \) so that the graph of the quadratic function \( f(x) = 2x^2 + 7x - 4k \) has exactly two \( x \)-intercepts.

39. Find all values of \( k \) so that the graph of the quadratic function \( f(x) = 2x^2 - x + 5k \) has no \( x \)-intercepts.

40. Find all values of \( k \) so that the graph of the quadratic function \( f(x) = kx^2 - 2x - 4 \) has no \( x \)-intercepts.

In Exercises 41-50, find all real solutions, if any, of the equation \( f(x) = b \).

41. \( f(x) = 63x^2 + 74x - 1; b = 8 \)

42. \( f(x) = 64x^2 + 128x + 64; b = 0 \)

43. \( f(x) = x^2 - x - 5; b = 2 \)

44. \( f(x) = 5x^2 - 5x; b = 3 \)

45. \( f(x) = 4x^2 + 4x - 1; b = -2 \)

46. \( f(x) = 2x^2 - 9x - 3; b = -1 \)

47. \( f(x) = 2x^2 + 4x + 6; b = 0 \)

48. \( f(x) = 24x^2 - 54x + 27; b = 0 \)

49. \( f(x) = -3x^2 + 2x - 13; b = -5 \)

50. \( f(x) = x^2 - 5x - 7; b = 0 \)

In Exercises 51-60, find all real solutions, if any, of the quadratic equation.

51. \(-2x^2 + 7 = -3x\)

52. \(-x^2 = -9x + 7\)

53. \(x^2 - 2 = -3x\)

54. \(81x^2 = -162x - 81\)

55. \(9x^2 + 81 = -54x\)

56. \(-30x^2 - 28 = -62x\)

57. \(-x^2 + 6 = 7x\)

58. \(-8x^2 = 4x + 2\)

59. \(4x^2 + 3 = -x\)

60. \(27x^2 = -66x + 16\)

In Exercises 61-66, find all of the \( x \)-intercepts, if any, of the given function.

61. \( f(x) = -4x^2 - 4x - 5 \)

62. \( f(x) = 49x^2 - 28x + 4 \)

63. \( f(x) = -56x^2 + 47x + 18 \)

64. \( f(x) = 24x^2 + 34x + 12 \)

65. \( f(x) = 36x^2 + 96x + 64 \)

66. \( f(x) = 5x^2 + 2x + 3 \)

In Exercises 67-74, determine the number of real solutions of the equation.

67. \( 9x^2 + 6x + 1 = 0 \)
68. \(7x^2 - 12x + 7 = 0\)
69. \(-6x^2 + 4x - 7 = 0\)
70. \(-8x^2 + 11x - 4 = 0\)
71. \(-5x^2 - 10x - 5 = 0\)
72. \(6x^2 + 11x + 2 = 0\)
73. \(-7x^2 - 4x + 5 = 0\)
74. \(6x^2 + 10x + 4 = 0\)
5.4 Answers

1. \( x = \pm 6 \)
2. \( x = \pm 6 \)
3. \( x = \pm \sqrt{17} = \pm 4.12 \)
4. \( x = 0 \)
5. No real solutions.
6. \( x = -4 \) or \( x = 6 \)
7. \( x = -2 \)
8. No real solutions.
9. \( x = 8 \pm \sqrt{15} \approx 4.13, 11.87 \)
10. Domain = \((-\infty, \infty)\), Range = \([-12, \infty)\)
11. Domain = \((-\infty, \infty)\), Range = \([-12, \infty)\)
12. Domain = \((-\infty, \infty)\), Range = \([-\infty, 11\) \]
13. Domain = \((-\infty, \infty)\), Range = \((-\infty, 11)\)
23. Domain = \((-\infty, \infty)\),  
Range = \((-\infty, 7]\)

25. Domain = \((-\infty, \infty)\),  
Range = \([-10, \infty)\)

27. Domain = \((-\infty, \infty)\),  
Range = \((-\infty, 40]\)

29. Domain = \((-\infty, \infty)\),  
Range = \([4, \infty)\)
31. Domain = \((-\infty, \infty),\)  
Range = \((-\infty, -2]\)

33. \(k = 1,\) Domain = \((-\infty, \infty),\)  
Range = \([0, \infty)\)

35. \(k = -2,\) Domain = \((-\infty, \infty),\)  
Range = \((-\infty, 0]\)

37. \(\{k : k < 9/20\}\)

39. \(\{k : k > 1/40\}\)

41. \(-9/7, 1/9\)

43. \(1+\sqrt{29}/2, 1-\sqrt{29}/2\)

45. \(-1/2\)

47. no real solutions

49. no real solutions

51. \(3-\sqrt{65}/4, 3+\sqrt{65}/4\)

53. \(-3-\sqrt{17}/2, -3+\sqrt{17}/2\)

55. \(-3\)

57. \(-7+\sqrt{73}/2, -7-\sqrt{73}/2\)

59. no real solutions

61. no x-intercepts

63. \((9/8, 0), (-2/7, 0)\)

65. \((-4/3, 0)\)
67. 1
69. 0
71. 1
73. 2