6.3 Exercises

In Exercises 1–8, perform each of the following tasks for the given polynomial.

i. Without the aid of a calculator, use an algebraic technique to identify the zeros of the given polynomial. Factor if necessary.

ii. On graph paper, set up a coordinate system. Label each axis, but scale only the $x$-axis. Use the zeros and the end-behavior to draw a “rough graph” of the given polynomial without the aid of a calculator.

iii. Classify each local extrema as a relative minimum or relative maximum. Note: It is not necessary to find the coordinates of the relative extrema. Indeed, this would be difficult without a calculator. All that is required is that you label each extrema as a relative maximum or minimum.

1. $p(x) = (x + 6)(x - 1)(x - 5)$
2. $p(x) = (x + 2)(x - 4)(x - 7)$
3. $p(x) = x^3 - 6x^2 - 4x + 24$
4. $p(x) = x^3 + x^2 - 36x - 36$
5. $p(x) = 2x^3 + 5x^2 - 42x$
6. $p(x) = 2x^3 - 3x^2 - 44x$
7. $p(x) = -2x^3 + 4x^2 + 70x$
8. $p(x) = -6x^3 - 21x^2 + 90x$

In Exercises 9–16, perform each of the following tasks for the given polynomial.

i. Use a graphing calculator to draw the graph of the polynomial. Adjust the viewing window so that the extrema or “turning points” of the polynomial are visible in the viewing window. Copy the resulting image onto your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax.

ii. Use the maximum and/or minimum utility in your calculator’s CALC menu to find the coordinates of the extrema. Label each extremum on your homework copy with its coordinates and state whether the extremum is a relative or absolute maximum or minimum.

9. $p(x) = x^3 - 8x^2 - 5x + 84$
10. $p(x) = x^3 + 3x^2 - 33x - 35$
11. $p(x) = -x^3 + 21x - 20$
12. $p(x) = -x^3 + 5x^2 + 12x - 36$
13. $p(x) = x^4 - 50x^2 + 49$
14. $p(x) = x^4 - 29x^2 + 100$
15. $p(x) = x^4 - 2x^3 - 39x^2 + 72x + 108$
16. $p(x) = x^4 - 3x^3 - 31x^2 + 63x + 90$

17. A square piece of cardboard measures 12 inches per side. Cherie cuts four smaller squares from each corner of the cardboard square, tossing the material aside. She then bends up the sides of the remaining cardboard to form an open box with no top. Find the dimensions of the squares cut from each corner of the original piece of cardboard so that
Cherie maximizes the volume of the resulting box. Perform each of the following steps in your analysis.

**a)** Set up an equation that determines the volume of the box as a function of \( x \), the length of the edge of each square cut from the four corners of the cardboard. Include any pictures used to determine this volume function.

**b)** State the empirical domain of the function created in part (a). Use your calculator to sketch the graph of the function over this empirical domain. Adjust the viewing window so that all extrema are visible in the viewing window.

**c)** Copy the image in your viewing window onto your homework paper. Label and scale each axis with \( xmin \), \( xmax \), \( ymin \), and \( ymax \). Use the maximum utility to find the coordinates of the absolute maximum on the function’s empirical domain.

**d)** What are the measures of the four squares cut from each corner of the original cardboard? What is the maximum volume of the box?

18. A rectangular piece of cardboard measures 8 inches by 12 inches. Schuyler cuts four smaller squares from each corner of the cardboard square, tossing the material aside. He then bends up the sides of the remaining cardboard to form an open box with no top. Find the dimensions of the squares cut from each corner of the original piece of cardboard so that Schuyler maximizes the volume of the resulting box. Perform each of the following steps in your analysis.

**a)** Set up an equation that determines the volume of the box as a function of \( x \), the length of the edge of each square cut from the four corners of the cardboard. Include any pictures used to determine this volume function.

**b)** State the empirical domain of the function created in part (a). Use your calculator to sketch the graph of the function over this empirical domain. Adjust the viewing window so that all extrema are visible in the viewing window.

**c)** Copy the image in your viewing window onto your homework paper. Label and scale each axis with \( xmin \), \( xmax \), \( ymin \), and \( ymax \). Use the maximum utility to find the coordinates of the absolute maximum on the function’s empirical domain.

**d)** What are the measures of the four squares cut from each corner of the original cardboard? What is the maximum volume of the box?

19. Restrict the graph of the parabola \( y = 4 - x^2/4 \) to the first quadrant, then inscribe a rectangle inside the parabola, as shown in the figure that follows.

\[ y = 4 - x^2/4 \]

**a)** Express the area of the inscribed rectangle as a function of \( x \).

**b)** State the empirical domain of the
function defined in part (a). Use your calculator to graph the area function over its empirical domain. Adjust the window parameters so that all extrema are visible in the viewing window.

c) Copy the image in your viewing window to your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax. Use the maximum utility to find the coordinates of the absolute maximum on the function’s empirical domain. Label your graph with this result.

d) What are the dimensions of the rectangle of maximum area?

20. Restrict the graph of the parabola \( y = 4 - x^2/4 \) to the first quadrant, then inscribe a triangle inside the parabola, as shown in the figure that follows.

\[ y \]
\[ (x, y) \]
\[ x \]

a) Express the area of the inscribed triangle as a function of \( x \).

b) State the empirical domain of the function defined in part (a). Use your calculator to graph the area function over its empirical domain. Adjust the window parameters so that all extrema are visible in the viewing window.

c) Copy the image in your viewing window to your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax. Use the maximum utility to find the coordinates of the absolute maximum on the function’s empirical domain. Label your graph with this result.

d) What are the length of the base and height of the triangle of maximum area?
6.3 Answers

1. Local Maximum
   \[(x, y) = (-6, 0) \quad (1, 0) \quad (5, 0)\]
   Local Minimum

3. Local Maximum
   \[(-2, 0) \quad (2, 0) \quad (6, 0)\]
   Local Minimum

5. Local Maximum
   \[(-6, 0) \quad (0, 0) \quad (7/2, 0)\]
   Local Minimum

7. Local Maximum
   \[(-5, 0) \quad (0, 0) \quad (7, 0)\]
   Local Minimum

9. Relative max: \((-0.2960664, 84.753138)\)
   Relative min: \((5.6293978, -19.27166)\)
   Answers may differ slightly due to round-off error.

\[p(x) = x^3 - 8x^2 - 5x + 84\]
11. Relative min: \((-2.645751, -57.04052)\)
Relative max: \((2.6457518, 17.040518)\)
Answers may differ slightly due to round-off error.

13. Absolute min: \((-5, -576)\)
Relative max: \((0, 49)\)
Absolute min: \((5, -576)\)
Answers may differ slightly due to round-off error.

15. Absolute min: \((-4.189858, -423.0327)\)
Relative max: \((0.89817915, 140.40823)\)
Relative min: \((4.7876796, -135.313)\)
Answers may differ slightly due to round-off error.

17. 
   a) \(V = x(12 - 2x)^2\)
   b) \([0, 6]\)
   c) Absolute max: \((2, 128)\)
   d) Cut square 2 inches on a side to produce a box having value 128 in\(^3\).
19.

a) \( A = x(4 - x^2/4) \)

b) \([0, 4]\)

c) Absolute max: \((2.3094011, 6.1584029)\)

d) \( x = 2.3094011, y = 2.6666666 \)