7.4 Exercises

In Exercises 1-10, reduce the product to a single fraction in lowest terms.

1. \( \dfrac{108}{14} \cdot \dfrac{6}{100} = \dfrac{9}{11} \cdot \dfrac{3}{50} = \dfrac{27}{55} \)

2. \( \dfrac{75}{63} \cdot \dfrac{18}{45} = \dfrac{5}{7} \cdot \dfrac{2}{5} = \dfrac{2}{7} \)

3. \( \dfrac{189}{56} \cdot \dfrac{12}{27} = \dfrac{63}{16} \cdot \dfrac{4}{9} = \dfrac{7}{4} \cdot \dfrac{1}{3} = \dfrac{7}{12} \)

4. \( \dfrac{45}{72} \cdot \dfrac{63}{64} = \dfrac{5}{8} \cdot \dfrac{7}{8} = \dfrac{35}{64} \)

5. \( \dfrac{15}{36} \cdot \dfrac{28}{100} = \dfrac{5}{12} \cdot \dfrac{7}{25} = \dfrac{35}{300} = \dfrac{7}{60} \)

6. \( \dfrac{189}{49} \cdot \dfrac{32}{25} = \dfrac{9}{7} \cdot \dfrac{256}{25} = \dfrac{384}{7} \cdot \dfrac{1}{25} = \dfrac{384}{175} \)

7. \( \dfrac{21}{100} \cdot \dfrac{125}{16} = \dfrac{3}{25} \cdot \dfrac{125}{16} = \dfrac{3}{2} \cdot \dfrac{1}{16} = \dfrac{3}{32} \)

8. \( \dfrac{21}{35} \cdot \dfrac{49}{45} = \dfrac{3}{5} \cdot \dfrac{7}{5} = \dfrac{21}{25} \)

9. \( \dfrac{56}{20} \cdot \dfrac{98}{32} = \dfrac{7}{5} \cdot \dfrac{49}{32} = \dfrac{49}{40} \)

10. \( \dfrac{27}{125} \cdot \dfrac{4}{12} = \dfrac{3}{12} \cdot \dfrac{4}{12} = \dfrac{1}{4} \cdot \dfrac{1}{3} = \dfrac{1}{12} \)

In Exercises 11-34, multiply and simplify. State all restrictions.

11. \( \dfrac{x + 6}{x^2 + 16x + 63} \cdot \dfrac{x^2 + 7x}{x + 4} = \dfrac{x(x + 6)(x + 7)}{(x + 7)(x + 9)} = \dfrac{x}{x + 9} \)

12. \( \dfrac{x^2 + 9x}{x^2 - 25} \cdot \dfrac{x^2 - x - 20}{-18 - 11x - x^2} = \dfrac{(x + 4)(x - 5)}{(x + 5)(x - 5)} = \dfrac{x + 4}{x + 5} \)

13. \( \dfrac{x^2 + 7x + 10}{x^2 - 1} \cdot \dfrac{-9 + 10x - x^2}{x^2 + 9x + 20} = \dfrac{(x + 2)(x + 5)}{(x + 1)(x - 2)} \cdot \dfrac{x(x - 2)}{(x + 4)(x + 5)} = \dfrac{x}{x + 4} \)

14. \( \dfrac{x^2 + 5x}{x - 4} \cdot \dfrac{x - 2}{x^2 + 6x + 5} = \dfrac{x(x + 5)}{x - 4} \cdot \dfrac{x - 2}{(x + 2)(x + 5)} = \dfrac{x(x - 2)}{x - 4} = \dfrac{x^2 - 2x}{x - 4} \)

15. \( \dfrac{x^2 - 5x}{x^2 + 2x - 48} \cdot \dfrac{x^2 + 11x + 24}{x^2 - x} = \dfrac{x(x - 5)}{(x - 6)(x + 8)} \cdot \dfrac{(x + 3)(x + 8)}{x(x - 1)} = \dfrac{x(x + 3)}{x - 1} \)

16. \( \dfrac{x^2 - 6x - 27}{x^2 + 10x + 24} \cdot \dfrac{x^2 + 13x + 42}{x^2 - 11x + 18} = \dfrac{(x - 9)(x + 3)}{(x + 4)(x + 6)} \cdot \dfrac{(x + 6)(x + 7)}{(x - 3)(x - 6)} = \dfrac{(x - 9)(x + 7)}{x - 3} \)

17. \( \dfrac{-x - x^2}{x^2 - 9x + 8} \cdot \dfrac{x^2 - 4x + 3}{x^2 + 4x + 3} = \dfrac{-x(1 + x)}{(x - 1)(x - 8)} \cdot \dfrac{(x - 1)(x - 3)}{(x + 1)(x + 3)} = \dfrac{-x(x - 3)}{x + 3} \)

18. \( \dfrac{x^2 - 12x + 35}{x^2 + 2x - 15} \cdot \dfrac{45 + 4x - x^2}{x^2 + x} = \dfrac{(x - 5)(x - 7)}{(x - 3)(x + 5)} \cdot \dfrac{(x + 9)(x - 5)}{(x + 1)(x - 1)} = \dfrac{(x - 7)(x + 9)}{(x - 3)(x + 1)} \)

19. \( \dfrac{x + 2}{7 - x} \cdot \dfrac{x^2 + x - 56}{x^2 + 7x + 6} = \dfrac{x + 2}{7 - x} \cdot \dfrac{(x + 7)(x - 8)}{(x + 7)(x + 1)} = \dfrac{x + 2}{x + 1} \)

20. \( \dfrac{x^2 - 2x - 15}{x^2 + x} \cdot \dfrac{x^2 + 7x}{x^2 + 12x + 27} = \dfrac{(x - 5)(x + 3)}{(x + 1)(x - 1)} \cdot \dfrac{x(x + 7)}{(x + 9)(x + 3)} = \dfrac{x(x + 7)}{x + 9} \)

21. \( \dfrac{x^2 - 9}{x^2 - 4x - 45} \cdot \dfrac{x - 6}{-3 - x} = \dfrac{(x - 3)(x + 3)}{(x - 9)(x + 5)} \cdot \dfrac{x - 6}{-1 - (x + 3)} = \dfrac{(x - 3)(x - 6)}{-x - 9} \)

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22. \[
\frac{x^2 - 12x + 27}{x - 4} \cdot \frac{x - 5}{x^2 - 18x + 81}
\]
32. \[
\frac{x^2 + 9x}{x^2 + 7x + 12} \cdot \frac{27 + 6x - x^2}{x^2 - 5x}
\]
33. \[
\frac{5 - x}{x + 3} \cdot \frac{x^2 + 3x - 18}{2x^2 - 7x - 15}
\]
34. \[
\frac{4x^2 + 21x + 5}{18 - 7x - x^2} \cdot \frac{x^2 + 11x + 18}{x^2 - 25}
\]

In Exercises 35-58, divide and simplify. State all restrictions.

35. \[
\frac{x^2 - 14x + 48}{x^2 + 10x + 16} \div \frac{-24 + 11x - x^2}{x^2 - x - 72}
\]
36. \[
\frac{x - 1}{x^2 - 14x + 48} \div \frac{x + 5}{x^2 - 3x - 18}
\]
37. \[
\frac{x^2 - 1}{x^2 - 7x + 12} \div \frac{x^2 + 6x + 5}{-24 + 10x - x^2}
\]
38. \[
\frac{x^2 - 13x + 42}{x^2 - 2x - 63} \div \frac{x^2 - x - 42}{x^2 + 8x + 7}
\]
39. \[
\frac{x^2 - 25}{x + 1} \div \frac{5x^2 + 23x - 10}{x - 3}
\]
40. \[ \frac{x^2 - 3x}{x^2 - 7x + 6} \div \frac{x^2 - 4x}{3x^2 - 11x - 42} \]

41. \[ \frac{x^2 + 10x + 21}{x - 4} \div \frac{x^2 + 3x}{x + 8} \]

42. \[ \frac{x^2 + 8x + 15}{x^2 - 14x + 45} \div \frac{x^2 + 11x + 30}{-30 + 11x - x^2} \]

43. \[ \frac{x^2 - 6x - 16}{x^2 + x - 42} \div \frac{x^2 - 64}{x^2 + 12x + 35} \]

44. \[ \frac{x^2 + 3x + 2}{x^2 - 9x + 18} \div \frac{x^2 + 7x + 6}{x^2 - 6x} \]

45. \[ \frac{x^2 + 12x + 35}{x + 4} \div \frac{x^2 + 10x + 25}{x + 9} \]

46. \[ \frac{x^2 - 8x + 7}{x^2 + 3x - 18} \div \frac{x^2 - 7x}{x^2 + 6x - 27} \]

47. \[ \frac{x^2 + x - 30}{x^2 + 5x - 36} \div -6 - x \div x + 8 \]

48. \[ \frac{2x - x^2}{x^2 - 15x + 54} \div \frac{x^2 + x}{x^2 - 11x + 30} \]

49. \[ \frac{x^2 - 9x + 8}{x^2 - 9} \div \frac{x^2 - 8x}{-15 - 8x - x^2} \]

50. \[ \frac{x + 5}{x^2 + 2x + 1} \div \frac{x - 2}{x^2 + 10x + 9} \]

51. \[ \frac{x^2 - 4}{x + 8} \div \frac{x^2 - 10x + 16}{x + 3} \]

52. \[ \frac{27 - 6x - x^2}{x^2 + 9x + 20} \div \frac{x^2 - 12x + 27}{x^2 + 5x} \]

53. \[ \frac{x^2 + 5x + 6}{x^2 - 36} \div \frac{x - 7}{-6 - x} \]

54. \[ \frac{2 - x}{x - 5} \div \frac{x^2 + 3x - 10}{x^2 - 14x + 48} \]
55. \[
\frac{x + 3}{x^2 + 4x - 12} \div \frac{x - 4}{x^2 - 36}
\]

56. \[
\frac{x + 3}{x^2 - x - 2} \div \frac{x}{x^2 - 3x - 4}
\]

57. \[
\frac{x^2 - 11x + 28}{x^2 + 5x + 6} \div \frac{7x^2 - 30x + 8}{x^2 - x - 6}
\]

58. \[
\frac{x - 7}{3 - x} \div \frac{2x^2 + 3x - 5}{x^2 - 12x + 27}
\]

59. Let \[
f(x) = \frac{x^2 - 7x + 10}{x^2 + 4x - 21}
\]
and \[
g(x) = \frac{5x - x^2}{x^2 + 15x + 56}
\]
Compute \(f(x)/g(x)\) and simplify your answer.

60. Let \[
f(x) = \frac{x^2 + 15x + 56}{x^2 - x - 20}
\]
and \[
g(x) = \frac{-7 - x}{x + 1}
\]
Compute \(f(x)/g(x)\) and simplify your answer.

61. Let \[
f(x) = \frac{x^2 + 12x + 35}{x^2 + 4x - 32}
\]
and \[
g(x) = \frac{x^2 - 2x - 35}{x^2 + 8x}
\]
Compute \(f(x)/g(x)\) and simplify your answer.

62. Let \[
f(x) = \frac{x^2 + 4x + 3}{x - 1}
\]
and \[
g(x) = \frac{x^2 - 4x - 21}{x + 5}
\]
Compute \(f(x)/g(x)\) and simplify your answer.

63. Let \[
f(x) = \frac{x^2 + x - 20}{x}
\]
and \[
g(x) = \frac{x - 1}{x^2 - 2x - 35}
\]
Compute \(f(x)g(x)\) and simplify your answer.

64. Let \[
f(x) = \frac{x^2 + 10x + 24}{x^2 - 13x + 42}
\]
and \[
g(x) = \frac{x^2 - 6x - 7}{x^2 + 8x + 12}
\]
Compute \(f(x)g(x)\) and simplify your answer.
65. Let 
\[ f(x) = \frac{x + 5}{-6 - x} \]
and 
\[ g(x) = \frac{x^2 + 8x + 12}{x^2 - 49} \]
Compute \( f(x)g(x) \) and simplify your answer.

66. Let 
\[ f(x) = \frac{8 - 7x - x^2}{x^2 - 8x - 9} \]
and 
\[ g(x) = \frac{x^2 - 6x - 7}{x^2 - 6x + 5} \]
Compute \( f(x)g(x) \) and simplify your answer.
7.4 Solutions

1. \[
\frac{108}{14} \cdot \frac{6}{100} = \frac{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3}{2 \cdot 7} \cdot \frac{2 \cdot 3}{2 \cdot 2 \cdot 5 \cdot 5} = \frac{3 \cdot 3 \cdot 3}{7 \cdot 5 \cdot 5} = \frac{81}{175}
\]

3. \[
\frac{189}{56} \cdot \frac{12}{27} = \frac{3 \cdot 3 \cdot 3 \cdot 7 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 7 \cdot 3 \cdot 3} = \frac{1}{2}
\]

5. \[
\frac{15}{36} \cdot \frac{28}{100} = \frac{3 \cdot 5}{2 \cdot 2 \cdot 3 \cdot 3} \cdot \frac{2 \cdot 2 \cdot 7}{2 \cdot 2 \cdot 5 \cdot 5} = \frac{7}{3 \cdot 2 \cdot 5} = \frac{7}{60}
\]

7. \[
\frac{21}{100} \cdot \frac{125}{16} = \frac{3 \cdot 7}{2 \cdot 2 \cdot 5 \cdot 5} \cdot \frac{5 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 2} = \frac{3 \cdot 7 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = \frac{105}{64}
\]

9. \[
\frac{56}{20} \cdot \frac{98}{32} = \frac{2 \cdot 2 \cdot 2 \cdot 7}{2 \cdot 2 \cdot 5} \cdot \frac{2 \cdot 7 \cdot 7}{2 \cdot 2 \cdot 2 \cdot 2} = \frac{7 \cdot 7 \cdot 7}{5 \cdot 2 \cdot 2 \cdot 2} = \frac{343}{40}
\]

11. First factor the numerators and the denominators:
\[
\frac{x + 6}{(x + 9)(x + 7)} \cdot \frac{x(x + 7)}{x + 4}
\]
Then cancel all common factors and multiply the two remaining fractions:
\[
\frac{x(x + 6)}{(x + 9)(x + 4)}
\]
Restricted values are \(-9, -7, \) and \(-4\).

13. First multiply \(-9 + 10x - x^2\) by \(-1\) and also negate the fraction:
\[
-\frac{x^2 + 7x + 10}{x^2 - 1} \cdot \frac{x^2 - 10x + 9}{x^2 + 9x + 20}
\]
Then factor the numerators and the denominators:
\[
-\frac{(x + 2)(x + 5)}{(x - 1)(x + 1)} \cdot \frac{(x - 1)(x - 9)}{(x + 1)(x + 5)}
\]
Finally, cancel all common factors and multiply the two remaining fractions:

\[
\frac{(x + 2)(x - 9)}{(x + 1)(x + 4)}
\]

Restricted values are 1, -1, -4, and -5.

15. First factor the numerators and the denominators:

\[
\frac{x(x - 5)}{(x + 8)(x - 6)} \cdot \frac{(x + 8)(x + 3)}{x(x - 1)}
\]

Then cancel all common factors and multiply the two remaining fractions:

\[
\frac{(x - 5)(x + 3)}{(x - 6)(x - 1)}
\]

Restricted values are -8, 6, 1, and 0.

17. First multiply \(-x - x^2\) by -1 and also negate the fraction:

\[
\frac{-x^2 + x}{x^2 - 9x + 8} \cdot \frac{x^2 - 4x + 3}{x^2 + 4x + 3}
\]

Then factor the numerators and the denominators:

\[
\frac{-x(x + 1)}{(x - 1)(x - 8)} \cdot \frac{(x - 1)(x - 3)}{(x + 3)(x + 1)}
\]

Finally, cancel all common factors and multiply the two remaining fractions:

\[
\frac{-x(x - 3)}{(x - 8)(x + 3)}
\]

Restricted values are 1, 8, -3, and -1.

19. First multiply \(7 - x\) by -1 and also negate the fraction:

\[
\frac{-x + 2}{x - 7} \cdot \frac{x^2 + x - 56}{x^2 + 7x + 6}
\]

Then factor the numerators and the denominators:

\[
\frac{-x + 2}{x - 7} \cdot \frac{(x - 7)(x + 8)}{(x + 1)(x + 6)}
\]

Finally, cancel all common factors and multiply the two remaining fractions:

\[
\frac{(x + 2)(x + 8)}{(x + 1)(x + 6)}
\]

Restricted values are 7, -1, and -6.
21. First multiply \(-3 - x\) by \(-1\) and also negate the fraction:
\[
\frac{x^2 - 9}{x^2 - 4x - 45} \cdot \frac{x - 6}{x + 3}
\]
Then factor the numerators and the denominators:
\[
-\frac{(x + 3)(x - 3)}{(x + 5)(x - 9)} \cdot \frac{x - 6}{x + 3}
\]
Finally, cancel all common factors and multiply the two remaining fractions:
\[
\frac{(x - 3)(x - 6)}{(x + 5)(x - 9)}
\]
Restricted values are \(-3, -5, \text{ and } 9\).

23. First factor the numerators and the denominators:
\[
\frac{x + 5}{(x + 8)(x + 4)} \cdot \frac{(x - 6)(x + 4)}{x + 7}
\]
Then cancel all common factors and multiply the two remaining fractions:
\[
\frac{(x + 5)(x - 6)}{(x + 8)(x + 7)}
\]
Restricted values are \(-8, -4, \text{ and } -7\).

25. First factor the numerators and the denominators:
\[
\frac{x - 5}{(x - 2)(x - 6)} \cdot \frac{(x - 6)^2}{x - 8}
\]
Then cancel all common factors and multiply the two remaining fractions:
\[
\frac{(x - 5)(x - 6)}{(x - 2)(x - 8)}
\]
Restricted values are \(2, 6, \text{ and } 8\).

27. First factor the numerators and the denominators:
\[
\frac{(x - 3)(x + 5)}{(x - 2)(x - 8)} \cdot \frac{(x - 2)(x - 5)}{(3x - 2)(x + 5)}
\]
Then cancel all common factors and multiply the two remaining fractions:
\[
\frac{(x - 3)(x - 5)}{(3x - 2)(x - 8)}
\]
Restricted values are \(2, 8, 2/3, \text{ and } -5\).
29. First factor the numerators and the denominators:
\[
\frac{(x - 2)(x + 2)}{(x + 9)(x - 7)} \cdot \frac{(x + 9)(x - 3)}{(x - 8)(x + 2)}
\]
Then cancel all common factors and multiply the two remaining fractions:
\[
\frac{(x - 2)(x - 3)}{(x - 7)(x - 8)}
\]
Restricted values are -9, 7, 8, and -2.

31. First factor the numerators and the denominators:
\[
\frac{x - 1}{(x - 7)(x + 9)} \cdot \frac{(x - 9)(x + 9)}{x + 4}
\]
Then cancel all common factors and multiply the two remaining fractions:
\[
\frac{(x - 1)(x - 9)}{(x - 7)(x + 4)}
\]
Restricted values are 7, -9, and -4.

33. First multiply $5 - x$ by $-1$ and also negate the fraction:
\[
\frac{x - 5}{x + 3} \cdot \frac{x^2 + 3x - 18}{2x^2 - 7x - 15}
\]
Then factor the numerators and the denominators:
\[
\frac{x - 5}{x + 3} \cdot \frac{(x + 6)(x - 3)}{(2x + 3)(x - 5)}
\]
Finally, cancel all common factors and multiply the two remaining fractions:
\[
\frac{(x + 6)(x - 3)}{(2x + 3)(x + 3)}
\]
Restricted values are -3, -3/2, and 5.

35. First rewrite as a multiplication problem:
\[
\frac{x^2 - 14x + 48}{x^2 + 10x + 16} \cdot \frac{x^2 - x - 72}{-24 + 11x - x^2}
\]
Then multiply $-24 + 11x - x^2$ by $-1$ and also negate the fraction:
\[
\frac{x^2 - 14x + 48}{x^2 + 10x + 16} \cdot \frac{x^2 - x - 72}{x^2 - 11x + 24}
\]
Then factor the numerators and the denominators:
\[
\frac{(x - 6)(x - 8)}{(x + 8)(x + 2)} \cdot \frac{(x + 8)(x - 9)}{(x - 3)(x - 8)}
\]
Finally, cancel all common factors and multiply the two remaining fractions:

\[
\frac{(x - 6)(x - 9)}{(x + 2)(x - 3)}
\]

Restricted values are \(-8, -2, 9, 3, \text{ and } 8\).

37. First rewrite as a multiplication problem:

\[
\frac{x^2 - 1}{x^2 - 7x + 12} \cdot \frac{-24 + 10x - x^2}{x^2 + 6x + 5}
\]

Then multiply \(-24 + 10x - x^2\) by \(-1\) and also negate the fraction:

\[
-\frac{x^2 - 1}{x^2 - 7x + 12} \cdot \frac{x^2 - 10x + 24}{x^2 + 6x + 5}
\]

Then factor the numerators and the denominators:

\[
-\frac{(x - 1)(x + 1)}{(x - 4)(x - 3)} \cdot \frac{(x - 4)(x - 6)}{(x + 5)(x + 1)}
\]

Finally, cancel all common factors and multiply the two remaining fractions:

\[
-\frac{(x - 1)(x - 6)}{(x - 3)(x + 5)}
\]

Restricted values are \(4, 3, 6, -5, \text{ and } -1\).

39. First factor the numerators and the denominators, and rewrite as a multiplication problem:

\[
\frac{(x - 5)(x + 5)}{x + 1} \cdot \frac{x - 3}{(5x - 2)(x + 5)}
\]

Then cancel all common factors and multiply the two remaining fractions:

\[
\frac{(x - 5)(x - 3)}{(5x - 2)(x + 1)}
\]

Restricted values are \(-1, 2/5, -5, \text{ and } 3\).

41. First factor the numerators and the denominators, and rewrite as a multiplication problem:

\[
\frac{(x + 7)(x + 3)}{x - 4} \cdot \frac{x + 8}{x(x + 3)}
\]

Then cancel all common factors and multiply the two remaining fractions:

\[
\frac{(x + 7)(x + 8)}{x(x - 4)}
\]

Restricted values are \(4, 0, -3, \text{ and } -8\).
43. First factor the numerators and the denominators, and rewrite as a multiplication problem:

\[
\frac{(x + 2)(x - 8)}{(x + 7)(x - 6)} \cdot \frac{(x + 7)(x + 5)}{(x + 8)(x - 8)}
\]

Then cancel all common factors and multiply the two remaining fractions:

\[
\frac{(x + 2)(x + 5)}{(x - 6)(x + 8)}
\]

Restricted values are -7, 6, -5, -8, and 8.

45. First factor the numerators and the denominators, and rewrite as a multiplication problem:

\[
\frac{(x + 7)(x + 5)}{x + 4} \cdot \frac{x + 9}{(x + 5)^2}
\]

Then cancel all common factors and multiply the two remaining fractions:

\[
\frac{(x + 7)(x + 9)}{(x + 4)(x + 5)}
\]

Restricted values are -4, -5, and -9.

47. First rewrite as a multiplication problem:

\[
\frac{x^2 + x - 30}{x^2 + 5x - 36} \cdot \frac{x + 8}{-6 - x}
\]

Then multiply \(-6 - x\) by -1 and also negate the fraction:

\[
\frac{x^2 + x - 30}{x^2 + 5x - 36} \cdot \frac{x + 8}{x + 6}
\]

Then factor the numerators and the denominators:

\[
\frac{(x + 6)(x - 5)}{(x - 4)(x + 9)} \cdot \frac{x + 8}{x + 6}
\]

Finally, cancel all common factors and multiply the two remaining fractions:

\[
\frac{(x - 5)(x + 8)}{(x - 4)(x + 9)}
\]

Restricted values are 4, -9, -8, and -6.
49. First rewrite as a multiplication problem:
\[
\frac{x^2 - 9x + 8}{x^2 - 9} \cdot \frac{-15 - 8x - x^2}{x^2 - 8x}
\]
Then multiply \(-15 - 8x - x^2\) by \(-1\) and also negate the fraction:
\[
-\frac{x^2 - 9x + 8}{x^2 - 9} \cdot \frac{x^2 + 8x + 15}{x^2 - 8x}
\]
Then factor the numerators and the denominators:
\[
-\frac{(x - 1)(x - 8)}{(x + 3)(x - 3)} \cdot \frac{(x + 3)(x + 5)}{x(x - 8)}
\]
Finally, cancel all common factors and multiply the two remaining fractions:
\[
\frac{(x - 1)(x + 5)}{x(x - 3)}
\]
Restricted values are \(-3, 3, -5, 0,\) and \(8\).

51. First factor the numerators and the denominators, and rewrite as a multiplication problem:
\[
\frac{(x + 2)(x - 2)}{x + 8} \cdot \frac{x + 3}{(x + 8)(x - 2)}
\]
Then cancel all common factors and multiply the two remaining fractions:
\[
\frac{(x + 2)(x + 3)}{(x + 8)(x - 8)}
\]
Restricted values are \(-8, 8, 2,\) and \(-3\).

53. First rewrite as a multiplication problem:
\[
\frac{x^2 + 5x + 6}{x^2 - 36} \cdot \frac{-6 - x}{x - 7}
\]
Then multiply \(-6 - x\) by \(-1\) and also negate the fraction:
\[
-\frac{x^2 + 5x + 6}{x^2 - 36} \cdot \frac{x + 6}{x - 7}
\]
Then factor the numerators and the denominators:
\[
-\frac{(x + 2)(x + 3)}{(x - 6)(x + 6)} \cdot \frac{x + 6}{x - 7}
\]
Finally, cancel all common factors and multiply the two remaining fractions:
\[
\frac{(x + 2)(x + 3)}{(x - 6)(x - 7)}
\]
Restricted values are \(6, -6,\) and \(7\).
55. First factor the numerators and the denominators, and rewrite as a multiplication problem:

\[
\frac{x + 3}{(x - 2)(x + 6)} \cdot \frac{(x - 6)(x + 6)}{x - 4}
\]

Then cancel all common factors and multiply the two remaining fractions:

\[
\frac{(x + 3)(x - 6)}{(x - 2)(x - 4)}
\]

Restricted values are 2, −6, 4, and 6.

57. First factor the numerators and the denominators, and rewrite as a multiplication problem:

\[
\frac{(x - 7)(x - 4)}{(x + 2)(x + 3)} \cdot \frac{(x + 2)(x - 3)}{(7x - 2)(x - 4)}
\]

Then cancel all common factors and multiply the two remaining fractions:

\[
\frac{(x - 7)(x - 3)}{(7x - 2)(x + 3)}
\]

Restricted values are −2, −3, 3, 2/7, and 4.

59.

\[
\frac{f(x)}{g(x)} = \frac{x^2 - 7x + 10}{x^2 + 4x - 21} \cdot \frac{x^2 + 15x + 56}{5x - x^2}
\]

First multiply \(5x - x^2\) by −1 and also negate the fraction:

\[
\frac{-x^2 - 7x + 10}{x^2 + 4x - 21} \cdot \frac{x^2 + 15x + 56}{x^2 - 5x}
\]

Then factor the numerators and the denominators:

\[
\frac{(x - 2)(x - 5)}{(x + 7)(x - 3)} \cdot \frac{(x + 7)(x + 8)}{x(x - 5)}
\]

Finally, cancel all common factors and multiply the two remaining fractions:

\[
\frac{- (x - 2)(x + 8)}{x(x - 3)}
\]

Restricted values are −7, 3, −8, 0, and 5.
61.

\[
\frac{f(x)}{g(x)} = \frac{x^2 + 12x + 35}{x^2 + 4x - 32} \cdot \frac{x^2 + 8x}{x^2 - 2x - 35}
\]

Factor the numerators and the denominators:

\[
\frac{(x + 7)(x + 5)}{(x + 8)(x - 4)} \cdot \frac{x(x + 8)}{(x - 7)(x + 5)}
\]

Then cancel all common factors and multiply the two remaining fractions:

\[
\frac{x(x + 7)}{(x - 4)(x - 7)}
\]

Restricted values are \(-8, 4, 0, 7, \) and \(-5\).

63.

\[
f(x)g(x) = \frac{x^2 + x - 20}{x} \cdot \frac{x - 1}{x^2 - 2x - 35}
\]

First factor the numerators and the denominators:

\[
\frac{(x - 4)(x + 5)}{x} \cdot \frac{x - 1}{(x - 7)(x + 5)}
\]

Then cancel all common factors and multiply the two remaining fractions:

\[
\frac{(x - 4)(x - 1)}{x(x - 7)}
\]

Restricted values are \(0, 7, \) and \(-5\).

65.

\[
f(x)g(x) = \frac{x + 5}{-6 - x} \cdot \frac{x^2 + 8x + 12}{x^2 - 49}
\]

First multiply \(-6 - x\) by \(-1\) and also negate the fraction:

\[
\frac{-x + 5}{x + 6} \cdot \frac{x^2 + 8x + 12}{x^2 - 49}
\]

Then factor the numerators and the denominators:

\[
\frac{-x + 5}{x + 6} \cdot \frac{(x + 6)(x + 2)}{(x + 7)(x - 7)}
\]

Finally, cancel all common factors and multiply the two remaining fractions:

\[
\frac{(x + 5)(x + 2)}{(x + 7)(x - 7)}
\]

Restricted values are \(-6, -7, \) and \(7\).