7.7 Exercises

For each of the rational functions given in Exercises 1-6, perform each of the following tasks.

i. Set up a coordinate system on graph paper. Label and scale each axis. Remember to draw all lines with a ruler.

ii. Plot the zero of the rational function on your coordinate system and label it with its coordinates. Plot the vertical and horizontal asymptotes on your coordinate system and label them with their equations. Use this information (and your graphing calculator) to draw the graph of $f$.

iii. Plot the horizontal line $y = k$ on your coordinate system and label this line with its equation.

iv. Use your calculator’s intersect utility to help determine the solution of $f(x) = k$. Label this point on your graph with its coordinates.

v. Solve the equation $f(x) = k$ algebraically, placing the work for this solution on your graph paper next to your coordinate system containing the graphical solution. Do the answers agree?

1. $f(x) = \frac{x - 1}{x + 2}; \quad k = 3$

2. $f(x) = \frac{x + 1}{x - 2}; \quad k = -3$

3. $f(x) = \frac{x + 1}{3 - x}; \quad k = 2$

4. $f(x) = \frac{x + 3}{2 - x}; \quad k = 2$

5. $f(x) = \frac{2x + 3}{x - 1}; \quad k = -3$

6. $f(x) = \frac{5 - 2x}{x - 1}; \quad k = 3$

In Exercises 7-14, use a strictly algebraic technique to solve the equation $f(x) = k$ for the given function and value of $k$. You are encouraged to check your result with your calculator.

7. $f(x) = \frac{16x - 9}{2x - 1}; \quad k = 8$

8. $f(x) = \frac{10x - 3}{7x + 7}; \quad k = 1$

9. $f(x) = \frac{5x + 8}{4x + 1}; \quad k = -11$

10. $f(x) = \frac{-6x - 11}{7x - 2}; \quad k = -6$

11. $f(x) = \frac{-35x}{7x + 12}; \quad k = -5$

12. $f(x) = \frac{-66x - 5}{6x - 10}; \quad k = -11$

13. $f(x) = \frac{8x + 2}{x - 11}; \quad k = 11$

14. $f(x) = \frac{36x - 7}{3x - 4}; \quad k = 12$

In Exercises 15-20, use a strictly algebraic technique to solve the given equation. You are encouraged to check your result with your calculator.

15. $\frac{x}{7} + \frac{8}{9} = -\frac{8}{7}$

16. $\frac{x}{3} + \frac{9}{2} = -\frac{3}{8}$

1 Copyrighted material. See: http://msenux.redwoods.edu/IntAlgText/
17. \(-\frac{57}{x} = 27 - \frac{40}{x^2}\)
18. \(-\frac{117}{x} = 54 + \frac{54}{x^2}\)
19. \(\frac{7}{x} = 4 - \frac{3}{x^2}\)
20. \(\frac{3}{x^2} = 5 - \frac{3}{x}\)

For each of the rational functions given in Exercises 21-26, perform each of the following tasks.

i. Set up a coordinate system on graph paper. Label and scale each axis. Remember to draw all lines with a ruler.
ii. Plot the zero of the rational function on your coordinate system and label it with its coordinates. You may use your calculator’s zero utility to find this, if you wish.
iii. Plot the vertical and horizontal asymptotes on your coordinate system and label them with their equations. Use the asymptote and zero information (and your graphing calculator) to draw the graph of \(f\).
iv. Plot the horizontal line \(y = k\) on your coordinate system and label this line with its equation.
v. Use your calculator’s intersect utility to help determine the solution of \(f(x) = k\). Label this point on your graph with its coordinates.
vi. Solve the equation \(f(x) = k\) algebraically, placing the work for this solution on your graph paper next to your coordinate system containing the graphical solution. Do the answers agree?

21. \(f(x) = \frac{1}{x} + \frac{1}{x + 5}, \ k = 9/14\)
22. \(f(x) = \frac{1}{x} + \frac{1}{x - 2}, \ k = 8/15\)

In Exercises 27-34, use a strictly algebraic technique to solve the given equation. You are encouraged to check your result with your calculator.

27. \(\frac{2}{x + 1} + \frac{4}{x + 2} = -3\)
28. \(\frac{2}{x - 5} - \frac{7}{x - 7} = 9\)
29. \(\frac{3}{x + 9} - \frac{2}{x + 7} = -3\)
30. \(\frac{3}{x + 9} - \frac{6}{x + 7} = 9\)
31. \(\frac{2}{x + 9} + \frac{2}{x + 6} = -1\)
32. \(\frac{5}{x - 6} - \frac{8}{x - 7} = -1\)
33. \(\frac{3}{x + 3} + \frac{6}{x + 2} = -2\)
34. \(\frac{2}{x - 4} - \frac{2}{x - 1} = 1\)
For each of the equations in Exercises 35-40, perform each of the following tasks.

i. Follow the lead of Example 10 in the text. Make one side of the equation equal to zero. Load the nonzero side into your calculator and draw its graph.

ii. Determine the vertical asymptotes of by analyzing the equation and the resulting graph on your calculator. Use the TABLE feature of your calculator to determine any horizontal asymptote behavior.

iii. Use the zero finding utility in the CALC menu to determine the zero of the nonzero side of the resulting equation.

iv. Set up a coordinate system on graph paper. Label and scale each axis. Remember to draw all lines with a ruler. Draw the graph of the nonzero side of the equation. Draw the vertical and horizontal asymptotes and label them with their equations. Plot the x-intercept and label it with its coordinates.

v. Use an algebraic technique to determine the solution of the equation and compare it with the solution found by the graphical analysis above.

35. \( \frac{x}{x+1} + \frac{8}{x^2-2x-3} = \frac{2}{x-3} \)

36. \( \frac{x}{x+4} - \frac{2}{x+1} = \frac{12}{x^2+5x+4} \)

37. \( \frac{x}{x+1} - \frac{4}{2x+1} = \frac{2x-1}{2x^2+3x+2} \)

38. \( \frac{2x}{x-4} - \frac{1}{x+1} = \frac{4x+24}{x^2-3x-4} \)

39. \( \frac{x}{x-2} + \frac{3}{x+2} = \frac{8}{4-x^2} \)

40. \( \frac{x}{x-1} - \frac{4}{x+1} = \frac{x-6}{1-x^2} \)

In Exercises 41-68, use a strictly algebraic technique to solve the given equation. You are encouraged to check your result with your calculator.

41. \( \frac{x}{3x-9} - \frac{9}{x} = \frac{1}{x-3} \)

42. \( \frac{5x}{x+2} + \frac{5}{x-5} = \frac{x+6}{x^2-3x-10} \)

43. \( \frac{3x}{x+2} - \frac{7}{x} = \frac{-1}{2x+4} \)

44. \( \frac{4x}{x+6} - \frac{4}{x+4} = \frac{x-4}{x^2+10x+24} \)

45. \( \frac{x}{x-5} + \frac{9}{4-x} = \frac{x+5}{x^2-9x+20} \)

46. \( \frac{6x}{x-5} - \frac{2}{x-3} = \frac{x-8}{x^2-8x+15} \)

47. \( \frac{2x}{x-4} + \frac{5}{2-x} = \frac{x+8}{x^2-6x+8} \)

48. \( \frac{x}{x-7} - \frac{8}{5-x} = \frac{x+7}{x^2-12x+35} \)

49. \( \frac{-x}{2x+2} - \frac{6}{x} = \frac{2}{x+1} \)

50. \( \frac{7x}{x+3} - \frac{4}{2-x} = \frac{x+8}{x^2+x-6} \)

51. \( \frac{2x}{x+5} - \frac{2}{6-x} = \frac{x-2}{x^2-x-30} \)

52. \( \frac{4x}{x+1} + \frac{6}{x+3} = \frac{x-9}{x^2+4x+3} \)

53. \( \frac{x}{x+7} - \frac{2}{x+5} = \frac{x+1}{x^2+12x+35} \)

54. \( \frac{5x}{6x+4} + \frac{6}{x} = \frac{1}{3x+2} \)

55. \( \frac{2x}{3x+9} - \frac{4}{x} = \frac{-2}{x+3} \)
56. \( \frac{7x}{x+1} - \frac{4}{x+2} = \frac{x+6}{x^2 + 3x + 2} \)

57. \( \frac{x}{2x-8} + \frac{8}{x} = \frac{2}{x-4} \)

58. \( \frac{3x}{x-6} + \frac{6}{x-6} = \frac{x+2}{x^2 - 12x + 36} \)

59. \( \frac{x}{x+2} + \frac{2}{x} = \frac{5}{2x+4} \)

60. \( \frac{4x}{x-2} + \frac{2}{2-x} = \frac{x+4}{x^2 - 4x + 4} \)

61. \( \frac{2x}{3x-9} - \frac{3}{x} = -\frac{2}{x-3} \)

62. \( \frac{2x}{x+1} - \frac{2}{x} = \frac{1}{2x + 2} \)

63. \( \frac{x}{x+1} + \frac{5}{x} = \frac{1}{4x + 4} \)

64. \( \frac{2x}{x-4} - \frac{8}{x-7} = \frac{x+2}{x^2 - 11x + 28} \)

65. \( -\frac{9x}{8x-2} + \frac{2}{x} = -\frac{2}{4x-1} \)

66. \( \frac{2x}{x-3} - \frac{4}{4-x} = \frac{x-9}{x^2 - 7x + 12} \)

67. \( \frac{4x}{x+6} - \frac{5}{7-x} = \frac{x-5}{x^2 - x - 42} \)

68. \( \frac{x}{x-1} - \frac{4}{x} = \frac{1}{5x - 5} \)
### 7.7 Answers

1. \( x = -\frac{7}{2} \)

2. \( x = \frac{5}{3} \)

3. \( x = 0 \)

4. \( x = \frac{5}{3} \)

5. \( x = 1 \)

6. \( x = 0 \)

7. None

8. \( \frac{19}{49} \)

9. None

10. \( \frac{128}{9} \)

11. None

12. \( \frac{8}{3}, \frac{5}{9} \)

13. \( \frac{7 + \sqrt{97}}{8}, \frac{7 - \sqrt{97}}{8} \)

14. \( x = -\frac{35}{9} \) or \( x = 2 \)

15. \( \frac{7 + \sqrt{97}}{8}, \frac{7 - \sqrt{97}}{8} \)

16. \( x = -3 \) or \( x = 3 \)
25. \( x = \frac{1 + \sqrt{65}}{4}, \frac{1 - \sqrt{65}}{4} \)

27. \( \frac{-15 + \sqrt{57}}{6}, \frac{-15 - \sqrt{57}}{6} \)

29. \( \frac{-49 + \sqrt{97}}{6}, \frac{-49 - \sqrt{97}}{6} \)

31. \(-7, -12\)

33. \( \frac{-19 + \sqrt{73}}{4}, \frac{-19 - \sqrt{73}}{4} \)

35. \( x = 2 \)

37. \( x = 3 \)

39. \( x = \frac{-5 + \sqrt{17}}{2}, \frac{-5 - \sqrt{17}}{2} \)

41. \( 27 \)

43. \( \frac{7}{2}, -\frac{4}{3} \)

45. 10

47. 3

49. \(-6, -2\)

51. 4, \frac{3}{2}

53. 3

55. 6

57. \(-16\)
59. \(-\frac{9 + \sqrt{17}}{4}, \frac{-9 - \sqrt{17}}{4}\)

61. \(-\frac{9}{2}\)

63. \(-\frac{19 + \sqrt{41}}{8}, \frac{-19 - \sqrt{41}}{8}\)

65. \(\frac{2}{9}, 2\)

67. \(\frac{7}{2}, \frac{5}{2}\)