8.1 Exercises

In Exercises 1-12, compute the exact value.

1. \(3^{-5}\)
2. \(4^2\)
3. \((3/2)^3\)
4. \((2/3)^1\)
5. \(6^{-2}\)
6. \(4^{-3}\)
7. \((2/3)^{-3}\)
8. \((1/3)^{-3}\)
9. \(7^1\)
10. \((3/2)^{-4}\)
11. \((5/6)^3\)
12. \(3^2\)

In Exercises 13-24, perform each of the following tasks for the given equation.

i. Load the left- and right-hand sides of the given equation into Y1 and Y2, respectively. Adjust the WINDOW parameters until all points of intersection (if any) are visible in your viewing window. Use the intersect utility in the CALC menu to determine the coordinates of any points of intersection.

ii. Make a copy of the image in your viewing window on your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax. Label each graph with its equation. Drop dashed vertical lines from each point of intersection to the x-axis, then shade and label each solution of the given equation on the x-axis. Remember to draw all lines with a ruler.

iii. Solve each problem algebraically. Use a calculator to approximate any radicals and compare these solutions with those found in parts (i) and (ii).

13. \(x^2 = 5\)
14. \(x^2 = 7\)
15. \(x^2 = -7\)
16. \(x^2 = -3\)
17. \(x^3 = -6\)
18. \(x^3 = -4\)
19. \(x^4 = 4\)
20. \(x^4 = -7\)
21. \(x^5 = 8\)
22. \(x^5 = 4\)
23. \(x^6 = -5\)
24. \(x^6 = 9\)

In Exercises 25-40, simplify the given radical expression.

25. \(\sqrt{49}\)
26. \(\sqrt{121}\)

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27. \( \sqrt{-36} \)
28. \( \sqrt{-100} \)
29. \( \sqrt[3]{27} \)
30. \( \sqrt[3]{-1} \)
31. \( \sqrt[3]{-125} \)
32. \( \sqrt[3]{64} \)
33. \( \sqrt[4]{-16} \)
34. \( \sqrt[4]{81} \)
35. \( \sqrt[4]{16} \)
36. \( \sqrt[4]{-625} \)
37. \( \sqrt[-3]{32} \)
38. \( \sqrt[5]{243} \)
39. \( \sqrt[5]{1024} \)
40. \( \sqrt[-3]{3125} \)

41. Compare and contrast \( \sqrt[2]{(-2)^2} \) and \( (\sqrt[2]{-2})^2 \).

42. Compare and contrast \( \sqrt[4]{(-3)^4} \) and \( (\sqrt[4]{-3})^4 \).

43. Compare and contrast \( \sqrt[3]{(-5)^3} \) and \( (\sqrt[3]{-5})^3 \).

44. Compare and contrast \( \sqrt[5]{(-2)^5} \) and \( (\sqrt[5]{-2})^5 \).

In Exercises 45-56, compute the exact value.

45. \( 25^{\frac{3}{2}} \)
46. \( 16^{-\frac{5}{4}} \)

47. \( 8^{\frac{4}{5}} \)
48. \( 625^{-\frac{3}{4}} \)
49. \( 16^{\frac{3}{2}} \)
50. \( 64^{\frac{3}{8}} \)
51. \( 27^{\frac{2}{3}} \)
52. \( 625^{\frac{3}{4}} \)
53. \( 256^{\frac{5}{4}} \)
54. \( 4^{-\frac{3}{2}} \)
55. \( 256^{-\frac{3}{4}} \)
56. \( 81^{-\frac{5}{4}} \)

In Exercises 57-64, simplify the product, and write your answer in the form \( x^r \).

57. \( x^\frac{5}{4}x^\frac{5}{4} \)
58. \( x^\frac{3}{4}x^{-\frac{3}{4}} \)
59. \( x^{-\frac{1}{3}}x^{\frac{5}{2}} \)
60. \( x^{-\frac{3}{5}}x^{\frac{3}{5}} \)
61. \( x^\frac{4}{7}x^{-\frac{4}{7}} \)
62. \( x^\frac{5}{4}x^{\frac{1}{7}} \)
63. \( x^{-\frac{2}{7}}x^{-\frac{3}{2}} \)
64. \( x^{-\frac{5}{4}}x^{-\frac{5}{7}} \)

In Exercises 65-72, simplify the quotient, and write your answer in the form \( x^r \).

65. \( \frac{x^{-\frac{5}{4}}}{x^{\frac{1}{5}}} \)
66. \( \frac{x^{-\frac{2}{3}}}{x^4} \)
67. \( \frac{x^{-\frac{3}{2}}}{x^{-\frac{5}{6}}} \)
68. \( \frac{x^{-\frac{5}{2}}}{2^\frac{2}{5}} \)
69. \( \frac{x^{\frac{3}{5}}}{x^{-\frac{1}{4}}} \)
70. \( \frac{x^{\frac{1}{3}}}{x^{-\frac{3}{2}}} \)
71. \( \frac{x^{-\frac{3}{4}}}{x^{\frac{1}{4}}} \)
72. \( \frac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}} \)

In Exercises 73-80, simplify the expression, and write your answer in the form \( x^r \).

73. \( \left( x^{\frac{1}{2}} \right)^{\frac{4}{3}} \)
74. \( \left( x^{-\frac{1}{2}} \right)^{-\frac{1}{2}} \)
75. \( \left( x^{-\frac{5}{4}} \right)^{\frac{1}{2}} \)
76. \( \left( x^{-\frac{1}{5}} \right)^{-\frac{3}{2}} \)
77. \( \left( x^{\frac{1}{2}} \right)^{\frac{3}{2}} \)
78. \( \left( x^{-\frac{1}{3}} \right)^{-\frac{1}{2}} \)
79. \( \left( x^{\frac{1}{5}} \right)^{-\frac{1}{2}} \)
80. \( \left( x^{\frac{2}{5}} \right)^{-\frac{1}{5}} \)
Chapter 8  Exponential and Logarithmic Functions

8.1 Solutions

1. $3^{-5} = \frac{1}{3^5} = \frac{1}{243}$

3. $\left(\frac{3}{2}\right)^3 = \frac{27}{8}$

5. $6^{-2} = \frac{1}{6^2} = \frac{1}{36}$

7. $\left(\frac{2}{3}\right)^{-3} = \frac{1}{(\frac{2}{3})^3} = \frac{1}{\frac{8}{27}} = \frac{27}{8}$

9. $7^1 = 7$

11. $\left(\frac{5}{6}\right)^3 = \frac{125}{216}$

13. Using a graphing calculator, note that the graph of $y = x^2$ intersects the graph of $y = 5$ in two places. The intersect was used to determine the $x$-values of the points of intersection. These are labeled on the $x$-axis on the image that follows.

To solve the problem algebraically, the solutions of $x^2 = 5$ are called “square roots of 5” and are denoted

$$x^2 = 5$$

$$x = \pm \sqrt{5}$$

$$x \approx \pm 2.236067977$$
A calculator was used to find the approximation. This result agrees nicely with the approximations found using the `intersect` utility above.

15. In the image that follows, note that the graph of $y = x^2$ does not intersect the graph of $y = -7$. Therefore, the equation $x^2 = -7$ has no real solutions.

To solve the problem algebraically, note that it is not possible to square a real number and obtain $-7$. Therefore, the equation

$$x^2 = -7$$

has no real solutions.

17. In the image that follows, note that the graph of $y = x^3$ intersects the graph of $y = -6$ in one location. The `intersect` utility on the graphing calculator was used to find the $x$-value of this point of intersection and this approximation is placed on the $x$-axis.

To solve the problem algebraically, note that the solution of $x^3 = -6$ is called the “cube root of $-6$ and is denoted by
\[ x^3 = -6 \]
\[ x = \sqrt[3]{-6} \approx -1.817120593 \]

A calculator was used to obtain the last approximation. Note that this approximation agrees nicely with the approximation found above with the \textit{intersect} utility.

19. Note that the graph of \( y = x^4 \) intersects the graph of \( y = 4 \) in two places. Therefore, the equation \( x^4 = 4 \) has two real solutions, which are found with the calculator’s \textit{intersect} utility and labeled on the \( x \)-axis of the image that follows.

To solve the equation algebraically, note that the solutions of \( x^4 = 4 \) are called “fourth roots of 4” and are denoted by

\[ x^4 = 4 \]
\[ x = \pm \sqrt[4]{4} \]
\[ x \approx \pm 1.414213562 \]

A calculator was used to find the last approximation. Note that these agree nicely with the results found with the \textit{intersect} utility above.

21. Note that the graph of \( y = x^5 \) intersects the graph of \( y = 8 \) in one location. Hence, the equation \( x^5 = 8 \) has one real solution, which is found using the \textit{intersect} utility and labeled on the \( x \)-axis in the image that follows.
To solve the equation algebraically, note that the solution of $x^5 = 8$ is called the “fifth root of 8” and is denoted by

$$x^5 = 8$$

$$x = \sqrt[5]{8}$$

$$x \approx 1.515716567$$

The last approximation was found by a calculator and agrees nicely with the approximation found with the `intersect` utility above.

23. Note that the graph of $y = x^6$ does not intersect the graph of $y = -5$. Hence, the equation $x^6 = -5$ has no real solutions.

To solve the equation algebraically, note that it is not possible to raise a real number to the sixth power and get $-5$. Hence, the equation

$$x^6 = -5$$

has no real solutions.

25. The notation $\sqrt{49}$ calls for the positive square root of 49. Note that $7^2 = 49$. Thus, $\sqrt{49} = 7$. 

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27. The notation $\sqrt{-36}$ calls for the positive square root of $-36$. It is not possible to square a real number and get $-36$. Therefore, $\sqrt{-36}$ is not a real number.

29. The notation $\sqrt[3]{27}$ calls for the cube root of 27. Note that $3^3 = 27$. Hence, $\sqrt[3]{27} = 3$.

31. The notation $\sqrt[5]{-125}$ calls for the cube root of $-125$. Note that $(-5)^3 = -125$. Hence, $\sqrt[5]{-125} = -5$.

33. The notation $\sqrt[4]{-16}$ calls for the positive fourth root of $-16$. Note that it is not possible to raise a real number to the fourth power and get $-16$. Thus, $\sqrt[4]{-16}$ is not a real number.


37. The notation $\sqrt[5]{-32}$ calls for the fifth root of $-32$. Note that $(-2)^5 = -32$. Hence, $\sqrt[5]{-32} = -2$.

39. The notation $\sqrt[5]{1024}$ calls for the fifth root of 1024. Note that $4^5 = 1024$. Thus, $\sqrt[5]{1024} = 4$.

41. $\sqrt[4]{(-2)^2} = \sqrt[4]{4} = 2$. However, $\sqrt[4]{-2}$ is not defined, so $(\sqrt[4]{-2})^2$ is not a real number.

43. $\sqrt[5]{(-5)^3} = \sqrt[5]{-125} = -5$ and $(\sqrt[5]{-5})^3 = -5$. Therefore, the two expressions are equal.

45. $25^{-\frac{1}{2}} = \frac{1}{25^{\frac{1}{2}}} = \frac{1}{(25^{\frac{1}{2}})^3} = \frac{1}{5^3} = \frac{1}{125}$

47. $8^{\frac{1}{4}} = (8^{\frac{1}{4}})^4 = 2^4 = 16$

49. $16^{\frac{3}{4}} = (16^{\frac{1}{4}})^3 = 4^3 = 64$

51. $27^{\frac{2}{3}} = (27^{\frac{1}{3}})^2 = 3^2 = 9$

53. $256^{\frac{5}{4}} = (256^{\frac{1}{4}})^5 = 4^5 = 1024$

55. $256^{-\frac{3}{4}} = \frac{1}{256^{\frac{1}{4}}} = \frac{1}{(256^{\frac{1}{4}})^3} = \frac{1}{4^3} = \frac{1}{64}$

57. $x^{\frac{1}{4}} x^{\frac{1}{4}} = x^{\frac{1}{4} + \frac{1}{4}} = x^{\frac{1}{2}}$

59. $x^{-\frac{1}{3}} x^{\frac{2}{3}} = x^{-\frac{1}{3} + \frac{2}{3}} = x^{\frac{1}{3}}$

61. $x^{\frac{4}{5}} x^{-\frac{3}{5}} = x^{\frac{4}{5} - \frac{3}{5}} = x^{-\frac{1}{5}}$

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63. \( x^{-2/5} x^{-3/2} = x^{-2/5 - 3/2} = x^{-19/10} \)

65. \( \frac{x^{-3/4}}{x^{1/5}} = x^{-3/4 - 1/5} = x^{-29/20} \)

67. \( \frac{x^{-1/2}}{x^{-3/5}} = x^{-1/2 + 3/5} = x^{1/10} \)

69. \( \frac{x^{3/7}}{x^{-1/4}} = x^{3/7 + 1/4} = x^{17/28} \)

71. \( \frac{x^{-3/4}}{x^{1/2}} = x^{-3/4 - 1/2} = x^{-23/12} \)

73. \( (x^{1/2})^{3/2} = x^{(1/2)(3/2)} = x^{3/4} \)

75. \( (x^{-5/4})^{1/2} = x^{-5/4}(1/2) = x^{-5/8} \)

77. \( (x^{-1/2})^{3/2} = x^{-1/2}(3/2) = x^{-3/4} \)

79. \( (x^{1/5})^{-1/2} = x^{(1/5)(-1/2)} = x^{-1/10} \)
8.2 Exercises

1. The current population of Fortuna is 10,000 hearty souls. It is known that the population is growing at a rate of 4% per year. Assuming this rate remains constant, perform each of the following tasks.

   a. Set up an equation that models the population $P(t)$ as a function of time $t$.
   b. Use the model in the previous part to predict the population 50 years from now.
   c. Use your calculator to sketch the graph of the population over the next 50 years.

2. The population of the town of Imagination currently numbers 12,000 people. It is known that the population is growing at a rate of 6% per year. Assuming this rate remains constant, perform each of the following tasks.

   a. Set up an equation that models the population $P(t)$ as a function of time $t$.
   b. Use the model in the previous part to predict the population 40 years from now.
   c. Use your calculator to sketch the graph of the population over the next 40 years.

3. The population of the town of Despairia currently numbers 15,000 individuals. It is known that the population is decaying at a rate of 5% per year. Assuming this rate remains constant, perform each of the following tasks.

   a. Set up an equation that models the population $P(t)$ as a function of time $t$.
   b. Use the model in the previous part to predict the population 30 years from now.
   c. Use your calculator to sketch the graph of the population over the next 30 years.

4. The population of the town of Hopeless currently numbers 25,000 individuals. It is known that the population is decaying at a rate of 6% per year. Assuming this rate remains constant, perform each of the following tasks.

   a. Set up an equation that models the population $P(t)$ as a function of time $t$.
   b. Use the model in the previous part to predict the population 40 years from now.
   c. Use your calculator to sketch the graph of the population over the next 40 years.

In Exercises 5-12, perform each of the following tasks for the given function.

   a. Find the $y$-intercept of the graph of the function. Also, use your calculator to find two points on the graph to the right of the $y$-axis, and two points to the left.
   b. Using your five points from (a) as a guide, set up a coordinate system on graph paper. Choose and label appropriate scales for each axis. Plot the five points, and any additional points you feel are necessary to dis-

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c. Draw the horizontal asymptote with a dashed line, and label it with its equation.
d. Sketch the graph of the function.
e. Use interval notation to describe both the domain and range of the function.

5. \( f(x) = (2.5)^x \)
6. \( f(x) = (0.1)^x \)
7. \( f(x) = (0.75)^x \)
8. \( f(x) = (1.1)^x \)
9. \( f(x) = 3^x + 1 \)
10. \( f(x) = 4^x - 5 \)
11. \( f(x) = 2^x - 3 \)
12. \( f(x) = 5^x + 2 \)

In Exercises 13-20, the graph of an exponential function of the form \( f(x) = b^x + c \) is shown. The dashed red line is a horizontal asymptote. Determine the range of the function. Express your answer in interval notation.

13.
In Exercises 21-32, compute \( f(p) \) at the given value \( p \).

21. \( f(x) = (1/3)^x; \ p = -4 \)
22. \( f(x) = (3/4)^x; \ p = 1 \)
23. \( f(x) = 5^x; \ p = 5 \)
24. \( f(x) = (1/3)^x; \ p = 4 \)
25. \( f(x) = 4^x; \ p = -4 \)
26. \( f(x) = 5^x; \ p = -3 \)

27. \( f(x) = (5/2)^x; \ p = -3 \)
28. \( f(x) = 9^x; \ p = 3 \)
29. \( f(x) = 5^x; \ p = -4 \)
30. \( f(x) = 9^x; \ p = 0 \)
31. \( f(x) = (6/5)^x; \ p = -4 \)
32. \( f(x) = (3/5)^x; \ p = 0 \)

In Exercises 33-40, use your calculator to evaluate the function at the given value \( p \). Round your answer to the nearest hundredth.

33. \( f(x) = 10^x; \ p = -0.7 \).
34. \( f(x) = 10^x; \ p = -1.60 \).
35. \( f(x) = (2/5)^x; \ p = 3.67 \).
36. \( f(x) = 2^x; \ p = -3/4 \).
37. \( f(x) = 10^x; \ p = 2.07 \).
38. \( f(x) = 7^x; \ p = 4/3 \).
39. \( f(x) = 10^x; \ p = -1/5 \).
40. \( f(x) = (4/3)^x; \ p = 1.15 \).

41. This exercise explores the property that exponential growth functions eventually increase rapidly as \( x \) increases. Let \( f(x) = 1.05^x \). Use your graphing calculator to graph \( f \) on the intervals

(a) \([0, 10]\) and (b) \([0, 100]\).

For (a), use \( Y_{\text{min}} = 0 \) and \( Y_{\text{max}} = 10 \). For (b), use \( Y_{\text{min}} = 0 \) and \( Y_{\text{max}} = 100 \).

Make accurate copies of the images in your viewing window on your homework paper. What do you observe when you compare the two graphs?
1.

a) Let $P(t)$ represent the population $t$ years from now. Then, the initial population (at time $t = 0$) is

$$P(0) = 10000.$$  

The population is growing at a rate of 4\% per year. Therefore, the population at the end of any year will be 104\% of the population at the end of the previous year. Thus, at the end of the first year, the population will be 104\% of the initial population. In symbols,

$$P(1) = 1.04P(0) = 1.04(10000).$$

At the end of the second year, the population will be 104\% of the population at the end of the first year. In symbols,

$$P(2) = 1.04P(1) = 1.04(1.04(10000)) = (1.04)^2(10000).$$

At the end of the third year, the population will be 104\% of the population at the end of the second year. In symbols,

$$P(3) = 1.04P(2) = 1.04(1.04)^2(10000) = (1.04)^3(10000).$$

Thus, the pattern is formed. The population at the end of $t$ years is $P(t) = (1.04)^t(10000)$, or equivalently,

$$P(t) = 10000(1.04)^t.$$

b) To find the population at the end of 40 years, set $t = 40$ and compute

$$P(40) = 10000(1.04)^{40} \approx 48010.$$  

\[c)\] The graph of the population over the next 40 years is displayed in the sequence of calculator snapshots that follow. Note especially how we made appropriate settings for the domain and range in (b).
3.

a) Let $P(t)$ represent the population $t$ years from now. Then, the initial population (at time $t = 0$) is

$$P(0) = 15000.$$ 

The population is decaying at a rate of 5% per year. Therefore, the population at the end of any year will be 95% of the population at the end of the previous year. Thus, at the end of the first year, the population will be 95% of the initial population. In symbols,

$$P(1) = 0.95P(0) = 0.95(15000).$$

At the end of the second year, the population will be 95% of the population at the end of the first year. In symbols,

$$P(2) = 0.95P(1) = 0.95(0.95(15000)) = (0.95)^2(15000).$$

At the end of the third year, the population will be 95% of the population at the end of the second year. In symbols,

$$P(3) = 0.95P(2) = 0.95(0.95)^2(15000) = (0.95)^3(15000).$$

Thus, the pattern is formed. The population at the end of $t$ years is $P(t) = (0.95)^t(15000)$, or equivalently,

$$P(t) = 15000(0.95)^t.$$ 

b) To find the population at the end of 50 years, set $t = 50$ and compute

$$P(50) = 15000(0.95)^{50} \approx 1154.$$ 

c) The graph of the population over the next 50 years is displayed in the sequence of calculator snapshots that follow. Note especially how we made appropriate settings for the domain and range in (b).
5.

a) The y-intercept is (0, 1). Evaluate the function at \( x = 1, 2, -1, -2 \) to obtain the points \((1, 2.5), (2, 6.25), (-1, 0.4), (-2, 0.16)\) (other answers are possible).

b) See the graph in part (d).

c) Since the base 1.5 is larger than 1, this is an exponential growth function. Therefore, \( y = 0 \) is a horizontal asymptote on the left side of the graph. See the graph in part (d).

d) 

\[
\begin{align*}
\text{Domain} &= (-\infty, \infty), \quad \text{Range} = (0, \infty)
\end{align*}
\]

7.

a) The y-intercept is (0, 1). Evaluate the function at \( x = 1, 2, -1, -2 \) to obtain the points \((1, 0.75), (2, 0.56), (-1, 1.34), (-2, 1.78)\) (other answers are possible).

b) See the graph in part (d).

c) Since the base 0.75 is smaller than 1, this is an exponential decay function. Therefore, \( y = 0 \) is a horizontal asymptote on the right side of the graph. See the graph in part (d).

d)
Section 8.2  Exponential Functions

9.

a) The $y$-intercept is $(0,2)$. Evaluate the function at $x = 1, 2, -1, -2$ to obtain the points $(1, 4), (2, 10), (-1, 1.34), (-2, 1.11)$ (other answers are possible).

b) See the graph in part (d).

c) The graph of $f$ can be obtained from the graph of $p(x) = 3^x$ by a vertical shift up 1 unit. Therefore, the horizontal asymptote $y = 0$ of the graph of $p$ will also be shifted up 1 unit, so the graph of $f$ has a horizontal asymptote $y = 1$. See the graph in part (d).

d) 

![Graph of f(x) = 3^x + 1]

e) Domain = $(-\infty, \infty)$, Range = $(0, \infty)$

11.

a) The $y$-intercept is $(0, -2)$. Evaluate the function at $x = 1, 2, -1, -2$ to obtain the points $(1, -1), (2, 1), (-1, -2.5), (-2, -2.75)$ (other answers are possible).

b) See the graph in part (d).

c) The graph of $f$ can be obtained from the graph of $p(x) = 2^x$ by a vertical shift down 3 units. Therefore, the horizontal asymptote $y = 0$ of the graph of $p$ will also be shifted down 3 units, so the graph of $f$ has a horizontal asymptote $y = -3$. See the graph in part (d).
Chapter 8  Exponential and Logarithmic Functions

d) $f(x) = 2^x - 3$

\[ y = 5 \]

\[ x = 5 \]

\[ y = -3 \]

\[ f(x) = 2^x - 3 \]

\[ \text{Domain} = (-\infty, \infty); \text{Range} = (-3, \infty) \]

13. Project all points on the graph onto the $y$-axis. This is shaded in red in the figure below. Thus, the range is the set of all real numbers greater than $-1$. In interval notation, the range equals $(-1, \infty)$.

\[ y = 5 \]

\[ x = 5 \]

\[ y = -3 \]

15. Project all points on the graph onto the $y$-axis. This is shaded in red in the figure below. Thus, the range is the set of all real numbers greater than $2$. In interval notation, the range equals $(2, \infty)$.
17. Project all points on the graph onto the $y$-axis. This is shaded in red in the figure below. Thus, the range is the set of all real numbers greater than 2. In interval notation, the range equals $(2, \infty)$.

19. Project all points on the graph onto the $y$-axis. This is shaded in red in the figure below. Thus, the range is the set of all real numbers greater than $-2$. In interval notation, the range equals $(-2, \infty)$.

21. $f(-4) = \left(\frac{1}{3}\right)^{-4} = \frac{1}{\left(\frac{1}{3}\right)^4} = \frac{1}{\frac{1}{81}} = 81$

23. $f(5) = 5^5 = 3125$

25. $f(-4) = 4^{-4} = \frac{1}{4^4} = \frac{1}{256}$

27. $f(-3) = \left(\frac{5}{2}\right)^{-3} = \frac{1}{\left(\frac{5}{2}\right)^3} = \frac{1}{\frac{125}{8}} = \frac{8}{125}$

29. $f(-4) = 5^{-4} = \frac{1}{5^4} = \frac{1}{625}$

31. $f(-4) = \left(\frac{6}{5}\right)^{-4} = \frac{1}{\left(\frac{6}{5}\right)^4} = \frac{1}{\frac{1296}{625}} = \frac{625}{1296}$

33. Using a calculator, $f(-0.7) = 10^{-0.7} \approx 0.20$.

35. Using a calculator, $f(3.67) = \left(\frac{2}{5}\right)^{3.67} \approx 0.03$. 

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37. Using a calculator, \( f(2.07) = 10^{2.07} \approx 117.49 \).

39. Using a calculator, \( f(-1/5) = 10^{-1/5} \approx 0.63 \).

41. a) The graph on the interval \([0, 10]\) increases very slowly. In fact, the graph looks almost linear.

![Graph on interval [0, 10]](image1)

b) The graph on the interval \([0, 100]\) increases slowly at first, but then increases very rapidly on the second half of the interval.

![Graph on interval [0, 100]](image2)
8.3 Exercises

1. Suppose that you invest $15,000 at 7% interest compounded monthly. How much money will be in your account in 4 years? Round your answer to the nearest cent.

2. Suppose that you invest $14,000 at 3% interest compounded monthly. How much money will be in your account in 7 years? Round your answer to the nearest cent.

3. Suppose that you invest $14,000 at 4% interest compounded daily. How much money will be in your account in 6 years? Round your answer to the nearest cent.

4. Suppose that you invest $15,000 at 8% interest compounded monthly. How much money will be in your account in 8 years? Round your answer to the nearest cent.

5. Suppose that you invest $4,000 at 3% interest compounded monthly. How much money will be in your account in 7 years? Round your answer to the nearest cent.

6. Suppose that you invest $3,000 at 5% interest compounded monthly. How much money will be in your account in 4 years? Round your answer to the nearest cent.

7. Suppose that you invest $1,000 at 3% interest compounded monthly. How much money will be in your account in 4 years? Round your answer to the nearest cent.

8. Suppose that you invest $19,000 at 2% interest compounded daily. How much money will be in your account in 9 years? Round your answer to the nearest cent.

9. Suppose that you can invest money at 4% interest compounded monthly. How much should you invest in order to have $20,000 in 2 years? Round your answer to the nearest cent.

10. Suppose that you can invest money at 6% interest compounded daily. How much should you invest in order to have $1,000 in 2 years? Round your answer to the nearest cent.

11. Suppose that you can invest money at 3% interest compounded daily. How much should you invest in order to have $20,000 in 3 years? Round your answer to the nearest cent.

12. Suppose that you can invest money at 3% interest compounded monthly. How much should you invest in order to have $10,000 in 7 years? Round your answer to the nearest cent.

13. Suppose that you can invest money at 9% interest compounded daily. How much should you invest in order to have $4,000 in 9 years? Round your answer to the nearest cent.

14. Suppose that you can invest money at 8% interest compounded daily. How much should you invest in order to have $18,000 in 6 years? Round your answer to the nearest cent.

---

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15. Suppose that you can invest money at 8% interest compounded daily. How much should you invest in order to have $17,000 in 6 years? Round your answer to the nearest cent.

16. Suppose that you can invest money at 9% interest compounded daily. How much should you invest in order to have $5,000 in 7 years? Round your answer to the nearest cent.

In Exercises 17-24, evaluate the function at the given value \( p \). Round your answer to the nearest hundredth.

17. \( f(x) = e^x; \ p = 1.57 \).

18. \( f(x) = e^x; \ p = 2.61 \).

19. \( f(x) = e^x; \ p = 3.07 \).

20. \( f(x) = e^x; \ p = -4.33 \).

21. \( f(x) = e^x; \ p = 1.42 \).

22. \( f(x) = e^x; \ p = -0.8 \).

23. \( f(x) = e^x; \ p = 4.75 \).

24. \( f(x) = e^x; \ p = 3.60 \).

25. Suppose that you invest $3,000 at 4% interest compounded continuously. How much money will be in your account in 9 years? Round your answer to the nearest cent.

26. Suppose that you invest $8,000 at 8% interest compounded continuously. How much money will be in your account in 7 years? Round your answer to the nearest cent.

27. Suppose that you invest $1,000 at 2% interest compounded continuously. How much money will be in your account in 3 years? Round your answer to the nearest cent.

28. Suppose that you invest $3,000 at 8% interest compounded continuously. How much money will be in your account in 4 years? Round your answer to the nearest cent.

29. Suppose that you invest $15,000 at 2% interest compounded continuously. How much money will be in your account in 4 years? Round your answer to the nearest cent.

30. Suppose that you invest $8,000 at 2% interest compounded continuously. How much money will be in your account in 6 years? Round your answer to the nearest cent.

31. Suppose that you invest $13,000 at 9% interest compounded continuously. How much money will be in your account in 8 years? Round your answer to the nearest cent.

32. Suppose that you invest $16,000 at 4% interest compounded continuously. How much money will be in your account in 6 years? Round your answer to the nearest cent.

33. Suppose that you can invest money at 6% interest compounded continuously. How much should you invest in order to have $17,000 in 9 years? Round your answer to the nearest cent.

34. Suppose that you can invest money at 8% interest compounded continuously. How much should you invest in order to have $5,000 in 6 years? Round your answer to the nearest cent.
35. Suppose that you can invest money at 8% interest compounded continuously. How much should you invest in order to have $10,000 in 6 years? Round your answer to the nearest cent.

36. Suppose that you can invest money at 6% interest compounded continuously. How much should you invest in order to have $17,000 in 13 years? Round your answer to the nearest cent.

37. Suppose that you can invest money at 2% interest compounded continuously. How much should you invest in order to have $13,000 in 8 years? Round your answer to the nearest cent.

38. Suppose that you can invest money at 9% interest compounded continuously. How much should you invest in order to have $10,000 in 15 years? Round your answer to the nearest cent.

39. Suppose that you can invest money at 7% interest compounded continuously. How much should you invest in order to have $18,000 in 10 years? Round your answer to the nearest cent.

40. Suppose that you can invest money at 9% interest compounded continuously. How much should you invest in order to have $14,000 in 12 years? Round your answer to the nearest cent.
8.3 Solutions

1. \[ P(t) = 15000 \left(1 + \frac{0.07}{12}\right)^{12t} \]

\[ \implies P(4) = 15000 \left(1 + \frac{0.07}{12}\right)^{(12)(4)} \approx 19830.8081682804 \]

3. \[ P(t) = 14000 \left(1 + \frac{0.04}{365}\right)^{365t} \]

\[ \implies P(6) = 14000 \left(1 + \frac{0.04}{365}\right)^{(365)(6)} \approx 17797.2540739749 \]

5. \[ P(t) = 4000 \left(1 + \frac{0.03}{12}\right)^{12t} \]

\[ \implies P(7) = 4000 \left(1 + \frac{0.03}{12}\right)^{(12)(7)} \approx 4933.41920219694 \]

7. \[ P(t) = 1000 \left(1 + \frac{0.03}{12}\right)^{12t} \]

\[ \implies P(4) = 1000 \left(1 + \frac{0.03}{12}\right)^{(12)(4)} \approx 1127.32802103993 \]

9. \[ P(t) = P_0 \left(1 + \frac{0.04}{12}\right)^{12t} \implies P(2) = P_0 \left(1 + \frac{0.04}{12}\right)^{(12)(2)} \]

\[ \implies 20000 = P_0 \left(1 + \frac{0.04}{12}\right)^{(12)(2)} \]

\[ \implies 20000 \left(1 + \frac{0.04}{12}\right)^{-(12)(2)} = P_0 \]

\[ \implies P_0 \approx 18464.7832780335 \]
11. 

\[ P(t) = P_0 \left( 1 + \frac{0.03}{365} \right)^{365t} \implies P(3) = P_0 \left( 1 + \frac{0.03}{365} \right)^{(365)(3)} \]

\[ \implies 20000 = P_0 \left( 1 + \frac{0.03}{365} \right)^{(365)(3)} \]

\[ \implies 20000 \left( 1 + \frac{0.03}{365} \right)^{-(365)(3)} = P_0 \]

\[ \implies P_0 \approx 18278.6913077117 \]

13. 

\[ P(t) = P_0 \left( 1 + \frac{0.09}{365} \right)^{365t} \implies P(9) = P_0 \left( 1 + \frac{0.09}{365} \right)^{(365)(9)} \]

\[ \implies 4000 = P_0 \left( 1 + \frac{0.09}{365} \right)^{(365)(9)} \]

\[ \implies 4000 \left( 1 + \frac{0.09}{365} \right)^{-(365)(9)} = P_0 \]

\[ \implies P_0 \approx 1779.6099440245 \]

15. 

\[ P(t) = P_0 \left( 1 + \frac{0.08}{365} \right)^{365t} \implies P(6) = P_0 \left( 1 + \frac{0.08}{365} \right)^{(365)(6)} \]

\[ \implies 17000 = P_0 \left( 1 + \frac{0.08}{365} \right)^{(365)(6)} \]

\[ \implies 17000 \left( 1 + \frac{0.08}{365} \right)^{-(365)(6)} = P_0 \]

\[ \implies P_0 \approx 10519.8709393407 \]

17. Using a calculator, \( f(1.57) = e^{1.57} \approx 4.81 \).

19. Using a calculator, \( f(3.07) = e^{3.07} \approx 21.54 \).

21. Using a calculator, \( f(1.42) = e^{1.42} \approx 4.14 \).

23. Using a calculator, \( f(4.75) = e^{4.75} \approx 115.58 \).
25. 

\[ P(t) = 3000e^{0.04t} \implies P(9) = 3000e^{(0.04)(9)} \]
\[ \implies P(9) \approx 4299.98824368102 \]

27. 

\[ P(t) = 1000e^{0.02t} \implies P(3) = 1000e^{(0.02)(3)} \]
\[ \implies P(3) \approx 1061.83654654536 \]

29. 

\[ P(t) = 15000e^{0.02t} \implies P(4) = 15000e^{(0.02)(4)} \]
\[ \implies P(4) \approx 16249.3060151244 \]

31. 

\[ P(t) = 13000e^{0.09t} \implies P(8) = 13000e^{(0.09)(8)} \]
\[ \implies P(8) \approx 26707.6317383705 \]

33. 

\[ P(t) = P_0e^{0.06t} \implies P(9) = P_0e^{(0.06)(9)} \]
\[ \implies 17000 = P_0e^{(0.06)(9)} \]
\[ \implies 17000e^{-(0.06)(9)} = P_0 \]
\[ \implies P_0 \approx 9906.72029035782 \]

35. 

\[ P(t) = P_0e^{0.08t} \implies P(6) = P_0e^{(0.08)(6)} \]
\[ \implies 10000 = P_0e^{(0.08)(6)} \]
\[ \implies 10000e^{-(0.08)(6)} = P_0 \]
\[ \implies P_0 \approx 6187.83391806141 \]

37. 

\[ P(t) = P_0e^{0.02t} \implies P(8) = P_0e^{(0.02)(8)} \]
\[ \implies 13000 = P_0e^{(0.02)(8)} \]
\[ \implies 13000e^{-(0.02)(8)} = P_0 \]
\[ \implies P_0 \approx 11077.8692565607 \]
39.

\[ P(t) = P_0 e^{0.07t} \implies P(10) = P_0 e^{(0.07)(10)} \]
\[ \implies 18000 = P_0 e^{(0.07)(10)} \]
\[ \implies 18000 e^{-(0.07)(10)} = P_0 \]
\[ \implies P_0 \approx 8938.53546824537 \]
8.4 Exercises

In Exercises 1-12, use the graph to determine whether the function is one-to-one.

1.

2.

3.

4.

5.

6.

7.

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In Exercises 13-28, evaluate the composition $g(f(x))$ and simplify your answer.

13. $g(x) = \frac{9}{x}$, $f(x) = -2x^2 + 5x - 2$
14. $f(x) = -\frac{5}{x}$, $g(x) = -4x^2 + x - 1$
15. $g(x) = 2\sqrt{x}$, $f(x) = -x - 3$
16. $f(x) = 3x^2 - 3x - 5$, $g(x) = \frac{6}{x}$
17. $g(x) = 3\sqrt{x}$, $f(x) = 4x + 1$
18. $f(x) = -3x - 5$, $g(x) = -x - 2$
19. $g(x) = -5x^2 + 3x - 4$, $f(x) = \frac{5}{x}$
20. $g(x) = 3x + 3$, $f(x) = 4x^2 - 2x - 2$
21. $g(x) = 6\sqrt{x}$, $f(x) = -4x + 4$
22. $g(x) = 5x - 3$, $f(x) = -2x - 4$
23. $g(x) = 3\sqrt{x}$, $f(x) = -2x + 1$
24. $g(x) = \frac{3}{x}$, $f(x) = -5x^2 - 5x - 4$
25. $f(x) = \frac{5}{x}$, $g(x) = -x + 1$
26. $f(x) = 4x^2 + 3x - 4$, $g(x) = \frac{2}{x}$
27. \( g(x) = -5x + 1, \ f(x) = -3x - 2 \)

28. \( g(x) = 3x^2 + 4x - 3, \ f(x) = \frac{8}{x} \)

In Exercises 29-36, first copy the given graph of the one-to-one function \( f(x) \) onto your graph paper. Then on the same coordinate system, sketch the graph of the inverse function \( f^{-1}(x) \).

29.

30.

31.

32.

33.

34.

35.
In Exercises 37-68, find the formula for the inverse function \( f^{-1}(x) \).

37. \( f(x) = 5x^3 - 5 \)
38. \( f(x) = 4x^7 - 3 \)
39. \( f(x) = \frac{-9x - 3}{7x + 6} \)
40. \( f(x) = 6x - 4 \)
41. \( f(x) = 7x - 9 \)
42. \( f(x) = 7x + 4 \)
43. \( f(x) = 3x^5 - 9 \)
44. \( f(x) = 6x + 7 \)
45. \( f(x) = \frac{4x + 2}{4x + 3} \)
46. \( f(x) = 5x^7 + 4 \)
47. \( f(x) = \frac{4x - 1}{2x + 2} \)
48. \( f(x) = \sqrt{8x - 3} \)
49. \( f(x) = \sqrt{-6x - 4} \)
50. \( f(x) = \frac{8x - 7}{3x - 6} \)
51. \( f(x) = \sqrt{-3x - 5} \)
52. \( f(x) = \frac{3x + 7}{2x + 8} \)
53. \( f(x) = \frac{9x - 3}{9x + 7} \)
54. \( f(x) = \frac{3\sqrt{6x + 7}}{2x + 8} \)
55. \( f(x) = -5x + 2 \)
56. \( f(x) = 6x + 8 \)
57. \( f(x) = 9x^9 + 5 \)
58. \( f(x) = 4x^5 - 9 \)
59. \( f(x) = \sqrt{9x - 7} \)
60. \( f(x) = x^4, x \leq 0 \)
61. \( f(x) = x^4, x \geq 0 \)
62. \( f(x) = x^2 - 1, x \leq 0 \)
63. \( f(x) = x^2 + 2, x \geq 0 \)
64. \( f(x) = x^4 + 3, x \leq 0 \)
65. \( f(x) = x^4 - 5, x \geq 0 \)
66. \( f(x) = (x - 1)^2, x \leq 1 \)
67. \( f(x) = (x + 2)^2, x \geq -2 \)
8.4 Solutions

1. The graph fails the horizontal line test. For example, the horizontal line $y = 1$ cuts the graph in more than one place. Therefore, the function is not one-to-one.

3. The graph fails the horizontal line test. For example, the horizontal line $y = 0$ cuts the graph in more than one place. Therefore, the function is not one-to-one.

5. The graph fails the horizontal line test. For example, the horizontal line $y = 4$ cuts the graph in more than one place. Therefore, the function is not one-to-one.

7. The graph meets the horizontal line test. Every horizontal line intersects the graph no more than once. Therefore, the function is one-to-one.

9. The graph meets the horizontal line test. Every horizontal line intersects the graph no more than once. Therefore, the function is one-to-one.
11. The graph meets the horizontal line test. Every horizontal line intersects the graph no more than once. Therefore, the function is one-to-one.

13. Substitute $f(x)$ for $x$ in the expression $\frac{9}{x}$ and simplify:

$$g(f(x)) = g(-2x^2 + 5x - 2) = -\frac{9}{2x^2 - 5x + 2}$$

15. Substitute $f(x)$ for $x$ in the expression $2\sqrt{x}$:

$$g(f(x)) = 2\sqrt{-x - 3}$$

17. Substitute $f(x)$ for $x$ in the expression $3\sqrt{x}$:

$$g(f(x)) = 3\sqrt{4x + 1}$$

19. Substitute $f(x)$ for $x$ in the expression $-5x^2 + 3x - 4$ and simplify:

$$g(f(x)) = g\left(\frac{5}{x}\right) = -5\left(\frac{5}{x}\right)^2 + 3\left(\frac{5}{x}\right) - 4 = -\frac{125}{x^2} + \frac{15}{x} - 4$$

21. Substitute $f(x)$ for $x$ in the expression $6\sqrt{x}$:

$$g(f(x)) = 6\sqrt{-4x + 4}$$

23. Substitute $f(x)$ for $x$ in the expression $3\sqrt{x}$:

$$g(f(x)) = 3\sqrt{-2x + 1}$$

25. Substitute $f(x)$ for $x$ in the expression $-x + 1$ and simplify:

$$g(f(x)) = g(\frac{5}{x}) = -\frac{5}{x} + 1$$

27. Substitute $f(x)$ for $x$ in the expression $-5x + 1$ and simplify:

$$g(f(x)) = g(-3x - 2)
= -5(-3x - 2) + 1
= 15x + 10 + 1
= 15x + 11$$
29. If you reflect the given graph across the line $y = x$ (pictured in black), you obtain the inverse, shown in red.

31. If you reflect the given graph across the line $y = x$ (pictured in black), you obtain the inverse, shown in red.

33. If you reflect the given graph across the line $y = x$ (pictured in black), you obtain the inverse, shown in red.

35. If you reflect the given graph across the line $y = x$ (pictured in black), you obtain the inverse, shown in red.
37. Start with the equation \( y = 5x^3 - 5 \).
Interchange \( x \) and \( y \): \( x = 5y^3 - 5 \).
Then solve for \( y \): \( y = \sqrt[3]{\frac{x+5}{5}} \).

39. Start with the equation \( y = -\frac{9x - 3}{7x + 6} \).
Interchange \( x \) and \( y \): \( x = -\frac{9y - 3}{7y + 6} \).
Then solve for \( y \):
\[
(x(7y + 6)) = -9y + 3 \iff (7x + 9)y = -6x + 3 \iff y = -\frac{6x - 3}{7x + 9}
\]

41. Start with the equation \( y = 7x - 9 \).
Interchange \( x \) and \( y \): \( x = 7y - 9 \).
Then solve for \( y \): \( y = \frac{x+9}{7} \).

43. Start with the equation \( y = 3x^5 - 9 \).
Interchange \( x \) and \( y \): \( x = 3y^5 - 9 \).
Then solve for \( y \): \( y = \sqrt[5]{\frac{x+9}{3}} \).

45. Start with the equation \( y = \frac{4x + 2}{4x + 3} \).
Interchange \( x \) and \( y \): \( x = \frac{4y + 2}{4y + 3} \).
Then solve for \( y \):
\[
x(4y + 3) = 4y + 2 \iff (4x - 4)y = -3x + 2 \iff y = -\frac{3x - 2}{4x - 4}
\]

47. Start with the equation \( y = \frac{4x - 1}{2x + 2} \).
Interchange \( x \) and \( y \): \( x = \frac{4y - 1}{2y + 2} \).
Then solve for \( y \):
\[
x(2y + 2) = 4y - 1 \iff (2x - 4)y = -2x - 1 \iff y = \frac{2x + 1}{2x - 4}
\]

49. Start with the equation \( y = \sqrt[3]{-6x - 4} \).
Interchange \( x \) and \( y \): \( x = \sqrt[3]{-6y - 4} \).
Then solve for \( y \): \( y = -\frac{x^3 + 4}{6} \).

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51. Start with the equation \( y = \sqrt[7]{-3x - 5} \).
Interchange \( x \) and \( y \): \( x = \sqrt[7]{-3y - 5} \).
Then solve for \( y \): \( y = -\frac{x^7 + 5}{3} \).

53. Start with the equation \( y = \sqrt[7]{6x + 7} \).
Interchange \( x \) and \( y \): \( x = \sqrt[7]{6y + 7} \).
Then solve for \( y \): \( y = \frac{x^7 - 7}{6} \).

55. Start with the equation \( y = -5x + 2 \).
Interchange \( x \) and \( y \): \( x = -5y + 2 \).
Then solve for \( y \): \( y = -\frac{x - 2}{5} \).

57. Start with the equation \( y = 9x^9 + 5 \).
Interchange \( x \) and \( y \): \( x = 9y^9 + 5 \).
Then solve for \( y \): \( y = \sqrt[9]{\frac{x - 5}{9}} \).

59. Start with the equation \( y = \frac{9x - 3}{9x + 7} \).
Interchange \( x \) and \( y \): \( x = \frac{9y - 3}{9y + 7} \).
Then solve for \( y \):

\[
x(9y + 7) = 9y - 3 \implies (9x - 9)y = -7x - 3 \implies y = -\frac{7x + 3}{9x - 9}
\]

61. Start with the equation \( y = x^4 \) with the domain condition \( x \leq 0 \).
Interchange \( x \) and \( y \): \( x = y^4, y \leq 0 \).
Solve for \( y \): \( y = \pm \sqrt[4]{x}, y \leq 0 \).
The condition \( y \leq 0 \) then implies that \( y = -\sqrt[4]{x} \).

63. Start with the equation \( y = x^2 - 1 \) with the domain condition \( x \leq 0 \).
Interchange \( x \) and \( y \): \( x = y^2 - 1, y \leq 0 \).
Solve for \( y \): \( y = \pm \sqrt{x + 1}, y \leq 0 \).
The condition \( y \leq 0 \) then implies that \( y = -\sqrt{x + 1} \).

65. Start with the equation \( y = x^4 + 3 \) with the domain condition \( x \leq 0 \).
Interchange \( x \) and \( y \): \( x = y^4 + 3, y \leq 0 \).
Solve for \( y \): \( y = \pm \sqrt[4]{x - 3}, y \leq 0 \).
The condition \( y \leq 0 \) then implies that \( y = -\sqrt[4]{x - 3} \).
67. Start with the equation $y = (x - 1)^2$ with the domain condition $x \leq 1$. Interchange $x$ and $y$: $x = (y - 1)^2$, $y \leq 1$. Solve for $y$: $y = \pm \sqrt{x} + 1$, $y \leq 1$. The condition $y \leq 1$ then implies that $y = -\sqrt{x} + 1$. 
8.5 Exercises

In Exercises 1-18, find the exact value of the function at the given value $b$.

1. $f(x) = \log_3(x); \quad b = \sqrt[3]{3}$.
2. $f(x) = \log_5(x); \quad b = 3125$.
3. $f(x) = \log_2(x); \quad b = \frac{1}{16}$.
4. $f(x) = \log_2(x); \quad b = 4$.
5. $f(x) = \log_5(x); \quad b = 5$.
6. $f(x) = \log_2(x); \quad b = 8$.
7. $f(x) = \log_2(x); \quad b = 32$.
8. $f(x) = \log_4(x); \quad b = \frac{1}{16}$.
9. $f(x) = \log_5(x); \quad b = \frac{1}{3125}$.
10. $f(x) = \log_5(x); \quad b = \frac{1}{25}$.
11. $f(x) = \log_5(x); \quad b = \frac{\sqrt{5}}{5}$.
12. $f(x) = \log_3(x); \quad b = \frac{\sqrt[3]{3}}{3}$.
13. $f(x) = \log_6(x); \quad b = \frac{\sqrt[6]{6}}{6}$.
14. $f(x) = \log_5(x); \quad b = \frac{\sqrt[5]{5}}{5}$.
15. $f(x) = \log_2(x); \quad b = \frac{\sqrt{2}}{2}$.
16. $f(x) = \log_4(x); \quad b = \frac{1}{4}$.
17. $f(x) = \log_3(x); \quad b = \frac{1}{9}$.
18. $f(x) = \log_4(x); \quad b = 64$.

In Exercises 19-26, use a calculator to evaluate the function at the given value $p$. Round your answer to the nearest hundredth.

19. $f(x) = \ln(x); \quad p = 10.06$.
20. $f(x) = \ln(x); \quad p = 9.87$.
21. $f(x) = \ln(x); \quad p = 2.40$.
22. $f(x) = \ln(x); \quad p = 9.30$.
23. $f(x) = \log(x); \quad p = 7.68$.
24. $f(x) = \log(x); \quad p = 652.22$.
25. $f(x) = \log(x); \quad p = 6.47$.
26. $f(x) = \log(x); \quad p = 86.19$.

In Exercises 27-34, solve the given equation, and round your answer to the nearest hundredth.

27. $13 = e^{8x}$
28. $2 = 8e^x$
29. $19 = 10^{8x}$
30. $17 = 10^{2x}$
31. $7 = 6(10)^x$
32. $7 = e^{9x}$
33. $13 = 8e^x$
34. $5 = 7(10)^x$

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In Exercises 35–42, the graph of a logarithmic function of the form \( f(x) = \log_b(x - a) \) is shown. The dashed red line is a vertical asymptote. Determine the domain of the function. Express your answer in interval notation.

35.

36.

37.

38.

39.

40.

41.
42.
8.5 **Solutions**

1. \( f(\sqrt[5]{3}) = \log_3(\sqrt[5]{3}) = \log_3(3^{\frac{1}{5}}) = \frac{1}{5} \).

3. 
   \[
f\left(\frac{1}{16}\right) = \log_2\left(\frac{1}{16}\right) \\
   = \log_2\left(\frac{1}{2^4}\right) \\
   = \log_2\left(\left(\frac{1}{2}\right)^4\right) \\
   = \log_2\left(2^{-4}\right) \\
   = -4
   \]

5. \( f(5) = \log_5(5) = \log_5(5^1) = 1 \).

7. \( f(32) = \log_2(32) = \log_2(2^5) = 5 \).

9. 
   \[
f\left(\frac{1}{3125}\right) = \log_5\left(\frac{1}{3125}\right) \\
   = \log_5\left(\frac{1}{5^5}\right) \\
   = \log_5\left(\left(\frac{1}{5}\right)^5\right) \\
   = \log_5\left(5^{-5}\right) \\
   = -5
   \]

11. \( f(\sqrt[6]{5}) = \log_5(\sqrt[6]{5}) = \log_5(5^{\frac{1}{6}}) = \frac{1}{6} \).

13. \( f(\sqrt[6]{6}) = \log_6(\sqrt[6]{6}) = \log_6(6^{\frac{1}{6}}) = \frac{1}{6} \).

15. \( f(\sqrt[6]{2}) = \log_2(\sqrt[6]{2}) = \log_2(2^{\frac{1}{6}}) = \frac{1}{6} \).

17. 

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\[ f \left( \frac{1}{9} \right) = \log_3 \left( \frac{1}{9} \right) \]
\[ = \log_3 \left( \frac{1}{3^2} \right) \]
\[ = \log_3 \left( \left( \frac{1}{3} \right)^2 \right) \]
\[ = \log_3 \left( 3^{-2} \right) \]
\[ = -2 \]

19. Using a calculator, \( f(10.06) = \ln(10.06) \approx 2.31 \).

21. Using a calculator, \( f(2.40) = \ln(2.40) \approx 0.88 \).

23. Using a calculator, \( f(7.68) = \log(7.68) \approx 0.89 \).

25. Using a calculator, \( f(6.47) = \log(6.47) \approx 0.81 \).

27.
\[ 13 = e^{8x} \implies \ln(13) = \ln(e^{8x}) \]
\[ \implies \ln(13) = 8x \]
\[ \implies x = \frac{\ln(13)}{8} \approx 0.320618669682692 \]

29.
\[ 19 = 10^{8x} \implies \log(19) = \log(10^{8x}) \]
\[ \implies \log(19) = 8x \]
\[ \implies x = \frac{\log(19)}{8} \approx 0.159844200119104 \]

31.
\[ 7 = 6(10)^x \implies \frac{7}{6} = 10^x \]
\[ \implies \log \left( \frac{7}{6} \right) = \log(10^x) \]
\[ \implies \log \left( \frac{7}{6} \right) = x \]
\[ \implies x = \log \left( \frac{7}{6} \right) \approx 0.0669467896306132 \]
33. \[
13 = 8e^x \implies \frac{13}{8} = e^x \\
\implies \ln\left(\frac{13}{8}\right) = \ln(e^x) \\
\implies \ln\left(\frac{13}{8}\right) = x \\
\implies x = \ln\left(\frac{13}{8}\right) \approx 0.485507815781701
\]

35. Project all points on the graph onto the x-axis. This is shaded in red in the figure below. Thus, the domain is the set of all real numbers greater than 0. In interval notation, the domain equals \((0, \infty)\).

37. Project all points on the graph onto the x-axis. This is shaded in red in the figure below. Thus, the domain is the set of all real numbers greater than \(-1\). In interval notation, the domain equals \((-1, \infty)\).
39. Project all points on the graph onto the $x$-axis. This is shaded in red in the figure below. Thus, the domain is the set of all real numbers greater than 0. In interval notation, the domain equals $(0, \infty)$.

41. Project all points on the graph onto the $x$-axis. This is shaded in red in the figure below. Thus, the domain is the set of all real numbers greater than $-3$. In interval notation, the domain equals $(-3, \infty)$. 
8.6 Exercises

In Exercises 1-10, use a calculator to evaluate the function at the given value $p$. Round your answer to the nearest hundredth.

1. $f(x) = \log_4(x)$; $p = 57.60$.
2. $f(x) = \log_4(x)$; $p = 11.22$.
3. $f(x) = \log_7(x)$; $p = 2.98$.
4. $f(x) = \log_3(x)$; $p = 2.27$.
5. $f(x) = \log_6(x)$; $p = 2.56$.
6. $f(x) = \log_8(x)$; $p = 289.27$.
7. $f(x) = \log_8(x)$; $p = 302.67$.
8. $f(x) = \log_5(x)$; $p = 15.70$.
9. $f(x) = \log_8(x)$; $p = 46.13$.
10. $f(x) = \log_4(x)$; $p = 15.59$.

In Exercises 11-18, perform each of the following tasks.

a) Approximate the solution of the given equation using your graphing calculator. Load each side of the equation into the Y= menu of your calculator. Adjust the WINDOW parameters so that the point of intersection of the graphs is visible in the viewing window. Use the intersect utility in the CALC menu of your calculator to determine the x-coordinate of the point of intersection. Then make an accurate copy of the image in your viewing window on your homework paper.

b) Solve the given equation algebraically, and round your answer to the nearest hundredth.

11. $20 = 3(1.2)^x$
12. $15 = 2(1.8)^x$
13. $14 = 1.4^{5x}$
14. $16 = 1.8^{4x}$
15. $-4 = 0.2^x - 9$
16. $12 = 2.9^x + 2$
17. $13 = 0.1^{x+1}$
18. $19 = 1.2^{x-6}$

In Exercises 19-34, solve the given equation algebraically, and round your answer to the nearest hundredth.

19. $20 = e^{x-3}$
20. $-4 = e^x - 9$
21. $23 = 0.9^x + 9$
22. $10 = e^x + 7$
23. $19 = e^x + 5$
24. $4 = 7(2.3)^x$
25. $18 = e^{x+4}$
26. $15 = e^{x+6}$
27. $8 = 2.7^{3x}$

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28. \(7 = e^{x+1}\)

29. \(7 = 1.1^{8x}\)

30. \(6 = 0.2^{x-8}\)

31. \(-7 = 1.3^x - 9\)

32. \(11 = 3(0.7)^x\)

33. \(23 = e^x + 9\)

34. \(20 = 3.2^{x+1}\)

35. Suppose that you invest $17,000 at 6% interest compounded daily. How many years will it take for your investment to double? Round your answer to the nearest hundredth.

36. Suppose that you invest $6,000 at 9% interest compounded continuously. How many years will it take for your investment to double? Round your answer to the nearest hundredth.

37. Suppose that you invest $16,000 at 6% interest compounded daily. How many years will it take for your investment to reach $26,000? Round your answer to the nearest hundredth.

38. Suppose that you invest $15,000 at 5% interest compounded monthly. How many years will it take for your investment to double? Round your answer to the nearest hundredth.

39. Suppose that you invest $18,000 at 3% interest compounded monthly. How many years will it take for your investment to double? Round your answer to the nearest hundredth.

40. Suppose that you invest $7,000 at 5% interest compounded daily. How many years will it take for your investment to reach $13,000? Round your answer to the nearest hundredth.

41. Suppose that you invest $16,000 at 9% interest compounded continuously. How many years will it take for your investment to double? Round your answer to the nearest hundredth.

42. Suppose that you invest $16,000 at 2% interest compounded continuously. How many years will it take for your investment to reach $25,000? Round your answer to the nearest hundredth.

43. Suppose that you invest $2,000 at 5% interest compounded continuously. How many years will it take for your investment to reach $10,000? Round your answer to the nearest hundredth.

44. Suppose that you invest $4,000 at 6% interest compounded continuously. How many years will it take for your investment to reach $10,000? Round your answer to the nearest hundredth.

45. Suppose that you invest $4,000 at 3% interest compounded daily. How many years will it take for your investment to reach $14,000? Round your answer to the nearest hundredth.

46. Suppose that you invest $13,000 at 2% interest compounded monthly. How many years will it take for your investment to reach $20,000? Round your answer to the nearest hundredth.

47. Suppose that you invest $20,000 at 7% interest compounded continuously. How many years will it take for your investment to reach $30,000? Round your answer to the nearest hundredth.
48. Suppose that you invest $16,000 at 4% interest compounded continuously. How many years will it take for your investment to double? Round your answer to the nearest hundredth.

49. Suppose that you invest $8,000 at 8% interest compounded continuously. How many years will it take for your investment to double? Round your answer to the nearest hundredth.

50. Suppose that you invest $3,000 at 3% interest compounded daily. How many years will it take for your investment to double? Round your answer to the nearest hundredth.
8.6 Solutions

1. Using a calculator,
\[ f(57.60) = \log_4(57.60) = \frac{\log(57.60)}{\log 4} \approx 2.92399845327748 \]

3. Using a calculator,
\[ f(2.98) = \log_7(2.98) = \frac{\log(2.98)}{\log 7} \approx 0.561137574130754 \]

5. Using a calculator,
\[ f(2.56) = \log_6(2.56) = \frac{\log(2.56)}{\log 6} \approx 0.524628039999395 \]

7. Using a calculator,
\[ f(302.67) = \log_8(302.67) = \frac{\log(302.67)}{\log 8} \approx 2.74720062506472 \]

9. Using a calculator,
\[ f(46.13) = \log_8(46.13) = \frac{\log(46.13)}{\log 8} \approx 1.8425446243031 \]

11. a) The graphical solution is shown below.

b) Here is an algebraic solution:
\[ 20 = 3(1.2)^x \implies \frac{20}{3} = 1.2^x \]
\[ \implies \log \left( \frac{20}{3} \right) = \log(1.2^x) \]
\[ \implies \log \left( \frac{20}{3} \right) = x \cdot \log(1.2) \]
\[ \implies x = \frac{\log \left( \frac{20}{3} \right)}{\log(1.2)} \approx 10.4053520507718 \]
13. a) The graphical solution is shown below.

b) Here is an algebraic solution:

\[ 14 = 1.4^{5x} \implies \log(14) = \log(1.4^{5x}) \]
\[ = \log(14) = 5x \cdot \log(1.4) \]
\[ \implies x = \frac{\log(14)}{5 \log(1.4)} \approx 1.5686627557246 \]

15. a) The graphical solution is shown below.

b) Here is an algebraic solution:

\[ -4 = 0.2^x - 9 \implies 5 = 0.2^x \]
\[ \implies \log(5) = \log(0.2^x) \]
\[ \implies \log(5) = x \cdot \log(0.2) \]
\[ \implies x = \frac{\log(5)}{\log(0.2)} \approx -1.00 \]

17. a) The graphical solution is shown below.
b) Here is an algebraic solution:

\[ 13 = 0.1^{x+1} \implies \log(13) = \log(0.1^{x+1}) \]
\[ \implies \log(13) = (x + 1) \log(0.1) \]
\[ \implies \frac{\log(13)}{\log(0.1)} = x + 1 \]
\[ \implies x = \frac{\log(13)}{\log(0.1)} - 1 \approx -2.11394335230684 \]

19.

\[ 20 = e^{x-3} \implies \ln(20) = \ln(e^{x-3}) \]
\[ \implies \ln(20) = x - 3 \]
\[ \implies x = \ln(20) + 3 \approx 5.99573227355399 \]

21.

\[ 23 = 0.9^x + 9 \implies 14 = 0.9^x \]
\[ \implies \log(14) = \log(0.9^x) \]
\[ \implies \log(14) = x \cdot \log(0.9) \]
\[ \implies x = \frac{\log(14)}{\log(0.9)} \approx -25.0478778804195 \]

23.

\[ 19 = e^x + 5 \implies 14 = e^x \]
\[ \implies \ln(14) = \ln(e^x) \]
\[ \implies \ln(14) = x \]
\[ \implies x = \ln(14) \approx 2.63905732961526 \]

25.

\[ 18 = e^{x+4} \implies \ln(18) = \ln(e^{x+4}) \]
\[ \implies \ln(18) = x + 4 \]
\[ \implies x = \ln(18) - 4 \approx -1.10962824210384 \]
27. \[ 8 = 2.7^{3x} \implies \log(8) = \log(2.7^{3x}) \]
\[ \implies \log(8) = 3x \cdot \log(2.7) \]
\[ \implies x = \frac{\log(8)}{3 \log(2.7)} \approx 0.69785647455816 \]

29. \[ 7 = 1.1^{8x} \implies \log(7) = \log(1.1^{8x}) \]
\[ \implies \log(7) = 8x \cdot \log(1.1) \]
\[ \implies x = \frac{\log(7)}{8 \log(1.1)} \approx 2.55207543550219 \]

31. \[ -7 = 1.3^x - 9 \implies 2 = 1.3^x \]
\[ \implies \log(2) = \log(1.3^x) \]
\[ \implies \log(2) = x \cdot \log(1.3) \]
\[ \implies x = \frac{\log(2)}{\log(1.3)} \approx 2.64192679581114 \]

33. \[ 23 = e^x + 9 \implies 14 = e^x \]
\[ \implies \ln(14) = \ln(e^x) \]
\[ \implies \ln(14) = x \]
\[ \implies x = \ln(14) \approx 2.63905732961526 \]

35. \[ P(t) = 17000 \left( 1 + \frac{0.06}{365} \right)^{365t} \]. Therefore,
\[ 34000 = 17000 \left( 1 + \frac{0.06}{365} \right)^{365t} \implies 2 = \left( 1 + \frac{0.06}{365} \right)^{365t} \]
\[ \implies \log(2) = \log \left( \left( 1 + \frac{0.06}{365} \right)^{365t} \right) \]
\[ \implies \log(2) = 365t \log \left( 1 + \frac{0.06}{365} \right) \]
\[ \implies t = \frac{\log(2)}{365 \log \left( 1 + \frac{0.06}{365} \right)} \approx 11.5534025000063 \]
37. \( P(t) = 16000 \left(1 + \frac{0.06}{365}\right)^{365t} \). Therefore,

\[
26000 = 16000 \left(1 + \frac{0.06}{365}\right)^{365t} \implies \frac{13}{8} = \left(1 + \frac{0.06}{365}\right)^{365t}
\]

\[
\implies \log \left(\frac{13}{8}\right) = \log \left(1 + \frac{0.06}{365}\right)^{365t}
\]

\[
\implies \log \left(\frac{13}{8}\right) = 365t \log \left(1 + \frac{0.06}{365}\right)
\]

\[
\implies t = \frac{\log \left(\frac{13}{8}\right)}{365 \log \left(1 + \frac{0.06}{365}\right)} \approx 8.09246199067501
\]

39. \( P(t) = 18000 \left(1 + \frac{0.03}{12}\right)^{12t} \). Therefore,

\[
36000 = 18000 \left(1 + \frac{0.03}{12}\right)^{12t} \implies 2 = \left(1 + \frac{0.03}{12}\right)^{12t}
\]

\[
\implies \log(2) = \log \left(1 + \frac{0.03}{12}\right)^{12t}
\]

\[
\implies \log(2) = 12t \log \left(1 + \frac{0.03}{12}\right)
\]

\[
\implies t = \frac{\log(2)}{12 \log \left(1 + \frac{0.03}{12}\right)} \approx 23.1337751324019
\]

41. \( P(t) = 16000e^{0.09t} \). Therefore,

\[
32000 = 16000e^{0.09t} \implies 2 = e^{0.09t}
\]

\[
\implies \ln(2) = 0.09t
\]

\[
\implies t = \frac{\ln(2)}{0.09} \approx 7.70163533955495
\]

43. \( P(t) = 2000e^{0.05t} \). Therefore,

\[
10000 = 2000e^{0.05t} \implies 5 = e^{0.05t}
\]

\[
\implies \ln(5) = 0.05t
\]

\[
\implies t = \frac{\ln(5)}{0.05} \approx 32.188758248682
\]
45. \( P(t) = 4000 \left(1 + \frac{0.03}{365}\right)^{365t} \). Therefore,

\[
14000 = 4000 \left(1 + \frac{0.03}{365}\right)^{365t} \implies \frac{7}{2} = \left(1 + \frac{0.03}{365}\right)^{365t} \\
\implies \log\left(\frac{7}{2}\right) = \log\left(1 + \frac{0.03}{365}\right)^{365t} \\
\implies \log\left(\frac{7}{2}\right) = 365t \log\left(1 + \frac{0.03}{365}\right) \\
\implies t = \frac{\log\left(\frac{7}{2}\right)}{365 \log\left(1 + \frac{0.03}{365}\right)} \approx 41.7604817066046
\]

47. \( P(t) = 20000e^{0.07t} \). Therefore,

\[
30000 = 20000e^{0.07t} \implies \frac{3}{2} = e^{0.07t} \\
\implies \ln\left(\frac{3}{2}\right) = 0.07t \\
\implies t = \frac{\ln\left(\frac{3}{2}\right)}{0.07} \approx 5.79235868725949
\]

49. \( P(t) = 8000e^{0.08t} \). Therefore,

\[
16000 = 8000e^{0.08t} \implies 2 = e^{0.08t} \\
\implies \ln(2) = 0.08t \\
\implies t = \frac{\ln(2)}{0.08} \approx 8.66433975699932
\]
8.7 Exercises

1. Suppose that the population of a certain town grows at an annual rate of 6%. If the population is currently 5,000, what will it be in 7 years? Round your answer to the nearest integer.

2. Suppose that the population of a certain town grows at an annual rate of 5%. If the population is currently 2,000, how many years will it take for it to double? Round your answer to the nearest hundredth.

3. Suppose that a certain radioactive isotope has an annual decay rate of 7.2%. How many years will it take for a 227 gram sample to decay to 93 grams? Round your answer to the nearest hundredth.

4. Suppose that a certain radioactive isotope has an annual decay rate of 6.8%. How many years will it take for a 399 gram sample to decay to 157 grams? Round your answer to the nearest hundredth.

5. Suppose that the population of a certain town grows at an annual rate of 8%. If the population is currently 4,000, how many years will it take for it to double? Round your answer to the nearest hundredth.

6. Suppose that a certain radioactive isotope has an annual decay rate of 19.2%. Starting with a 443 gram sample, how many grams will be left after 9 years? Round your answer to the nearest hundredth.

7. Suppose that a certain radioactive isotope has an annual decay rate of 17.4%. What is the half-life (in years) of the isotope? Round your answer to the nearest hundredth.

8. Suppose that the population of a certain town grows at an annual rate of 7%. If the population is currently 8,000, how many years will it take for it to reach 18,000? Round your answer to the nearest hundredth.

9. Suppose that a certain radioactive isotope has an annual decay rate of 17.3%. Starting with a 214 gram sample, how many grams will be left after 5 years? Round your answer to the nearest hundredth.

10. Suppose that the population of a certain town grows at an annual rate of 7%. If the population grows to 2,000 in 7 years, what was the original population? Round your answer to the nearest integer.

11. Suppose that the population of a certain town grows at an annual rate of 3%. If the population is currently 3,000, how many years will it take for it to double? Round your answer to the nearest hundredth.

12. Suppose that a certain radioactive isotope has an annual decay rate of 12.5%. Starting with a 127 gram sample, how many grams will be left after 6 years? Round your answer to the nearest hundredth.

13. Suppose that a certain radioactive isotope has an annual decay rate of 13.1%.

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Starting with a 353 gram sample, how many grams will be left after 7 years? Round your answer to the nearest hundredth.

14. Suppose that the population of a certain town grows at an annual rate of 2%. If the population grows to 9,000 in 4 years, what was the original population? Round your answer to the nearest integer.

15. Suppose that the population of a certain town grows at an annual rate of 2%. If the population is currently 7,000, how many years will it take for it to double? Round your answer to the nearest hundredth.

16. Suppose that a certain radioactive isotope has an annual decay rate of 5.3%. How many years will it take for a 217 gram sample to decay to 84 grams? Round your answer to the nearest hundredth.

17. Suppose that a certain radioactive isotope has an annual decay rate of 18.7%. How many years will it take for a 324 gram sample to decay to 163 grams? Round your answer to the nearest hundredth.

18. Suppose that the population of a certain town grows at an annual rate of 8%. If the population is currently 8,000, how many years will it take for it to reach 18,000? Round your answer to the nearest hundredth.

19. Suppose that a certain radioactive isotope has an annual decay rate of 2.3%. If a particular sample decays to 25 grams after 8 years, how big (in grams) was the original sample? Round your answer to the nearest hundredth.

20. Suppose that the population of a certain town grows at an annual rate of 4%. If the population is currently 7,000, how many years will it take for it to reach 17,000? Round your answer to the nearest hundredth.

21. Suppose that a certain radioactive isotope has an annual decay rate of 9.8%. If a particular sample decays to 11 grams after 6 years, how big (in grams) was the original sample? Round your answer to the nearest hundredth.

22. Suppose that the population of a certain town grows at an annual rate of 5%. If the population grows to 6,000 in 3 years, what was the original population? Round your answer to the nearest integer.

23. Suppose that the population of a certain town grows at an annual rate of 8%. If the population is currently 6,000, what will it be in 5 years? Round your answer to the nearest integer.

24. Suppose that a certain radioactive isotope has an annual decay rate of 15.8%. What is the half-life (in years) of the isotope? Round your answer to the nearest hundredth.

25. Suppose that the population of a certain town grows at an annual rate of 9%. If the population grows to 7,000 in 5 years, what was the original population? Round your answer to the nearest integer.

26. Suppose that a certain radioactive isotope has an annual decay rate of 18.6%. If a particular sample decays to 41 grams after 3 years, how big (in grams) was the original sample? Round your answer to the nearest hundredth.
27. Suppose that a certain radioactive isotope has an annual decay rate of 5.2%. What is the half-life (in years) of the isotope? Round your answer to the nearest hundredth.

28. Suppose that a certain radioactive isotope has an annual decay rate of 6.5%. What is the half-life (in years) of the isotope? Round your answer to the nearest hundredth.

29. Suppose that the population of a certain town grows at an annual rate of 8%. If the population is currently 2,000, how many years will it take for it to reach 7,000? Round your answer to the nearest hundredth.

30. Suppose that a certain radioactive isotope has an annual decay rate of 3.7%. If a particular sample decays to 47 grams after 8 years, how big (in grams) was the original sample? Round your answer to the nearest hundredth.

31. Suppose that the population of a certain town grows at an annual rate of 6%. If the population is currently 7,000, what will it be in 7 years? Round your answer to the nearest integer.

32. Suppose that the population of a certain town grows at an annual rate of 4%. If the population is currently 1,000, what will it be in 3 years? Round your answer to the nearest integer.

In Exercises 33-40, use the fact that the decay rate of carbon-14 is 0.012%. Round your answer to the nearest year.

34. Suppose that only 5.2% of the normal amount of carbon-14 remains in a fragment of bone. How old is the bone?

35. Suppose that 90.1% of the normal amount of carbon-14 remains in a piece of wood. How old is the wood?

36. Suppose that 83.6% of the normal amount of carbon-14 remains in a piece of cloth. How old is the cloth?

37. Suppose that only 6.2% of the normal amount of carbon-14 remains in a fragment of bone. How old is the bone?

38. Suppose that only 1.3% of the normal amount of carbon-14 remains in a fragment of bone. How old is the bone?

39. Suppose that 96.7% of the normal amount of carbon-14 remains in a piece of cloth. How old is the cloth?

40. Suppose that 84.9% of the normal amount of carbon-14 remains in a piece of wood. How old is the wood?
8.7 Solutions

1. 
\[ P(t) = 5000e^{0.06t} \implies P(7) = 5000e^{(0.06)(7)} \]
   \[ \implies P(7) \approx 7609.80777809317 \]

3. 
\[ P(t) = 227e^{-0.072t}. \text{ Therefore,} \]
\[ 93 = 227e^{-0.072t} \implies \frac{93}{227} = e^{-0.072t} \]
\[ \implies \ln \left( \frac{93}{227} \right) = -0.072t \]
\[ \implies t = \frac{\ln \left( \frac{93}{227} \right)}{-0.072} \approx 12.3937572823354 \]

5. 
\[ P(t) = 4000e^{0.08t}. \text{ Therefore,} \]
\[ 8000 = 4000e^{0.08t} \implies 2 = e^{0.08t} \]
\[ \implies \ln(2) = 0.08t \]
\[ \implies t = \frac{\ln(2)}{0.08} \approx 8.66433975699932 \]

7. 
\[ P(t) = P_0e^{-0.174t}. \text{ Therefore,} \]
\[ \frac{P_0}{2} = P_0e^{-0.174t} \implies \frac{1}{2} = e^{-0.174t} \]
\[ \implies \ln(0.5) = -0.174t \]
\[ \implies t = \frac{\ln(0.5)}{-0.174} \approx 3.9836044859767 \]

9. 
\[ P(t) = 214e^{-0.173t} \implies P(5) = 214e^{-(0.173)(5)} \]
\[ \implies P(5) \approx 90.105032622467 \]
11. \( P(t) = 3000e^{0.03t} \). Therefore,
\[
6000 = 3000e^{0.03t} \implies 2 = e^{0.03t} \\
\implies \ln(2) = 0.03t \\
\implies t = \frac{\ln(2)}{0.03} \approx 23.1049060186648
\]

13. \( P(t) = 353e^{-0.131t} \implies P(7) = 353e^{-(0.131)(7)} \)
\[
\implies P(7) \approx 141.099886848361
\]

15. \( P(t) = 7000e^{0.02t} \). Therefore,
\[
14000 = 7000e^{0.02t} \implies 2 = e^{0.02t} \\
\implies \ln(2) = 0.02t \\
\implies t = \frac{\ln(2)}{0.02} \approx 34.6573590279973
\]

17. \( P(t) = 324e^{-0.187t} \). Therefore,
\[
163 = 324e^{-0.187t} \implies \frac{163}{324} = e^{-0.187t} \\
\implies \ln\left(\frac{163}{324}\right) = -0.187t \\
\implies t = \frac{\ln\left(\frac{163}{324}\right)}{-0.187} \approx 3.67376104270357
\]

19. \( P(t) = P_0e^{-0.023t} \). Therefore,
\[
25 = P_0e^{-(0.023)(8)} \implies 25e^{(0.023)(8)} = P_0 \\
\implies P_0 \approx 30.0503955774075
\]

21. \( P(t) = P_0e^{-0.098t} \). Therefore,
\[
11 = P_0e^{-(0.098)(6)} \implies 11e^{(0.098)(6)} = P_0 \\
\implies P_0 \approx 19.8042244855375
\]

23. \( P(t) = 6000e^{0.08t} \implies P(5) = 6000e^{(0.08)(5)} \)
\[
\implies P(5) \approx 8950.94818584762
\]
25. \( P(t) = P_0e^{0.09t} \). Therefore,
\[
7000 = P_0e^{0.09(5)} \implies 7000e^{-(0.09)(5)} = P_0 \\
\implies P_0 \approx 4463.39706135241
\]

27. \( P(t) = P_0e^{-0.052t} \). Therefore,
\[
\frac{P_0}{2} = P_0e^{-0.052t} \implies \frac{1}{2} = e^{-0.052t} \\
\implies \ln(0.5) = -0.052t \\
\implies t = \frac{\ln(0.5)}{-0.052} \approx 13.3297534723066
\]

29. \( P(t) = 2000e^{0.08t} \). Therefore,
\[
7000 = 2000e^{0.08t} \implies \frac{7}{2} = e^{0.08t} \\
\implies \ln\left(\frac{7}{2}\right) = 0.08t \\
\implies t = \frac{\ln\left(\frac{7}{2}\right)}{0.08} \approx 15.6595371061921
\]

31. 
\[
P(t) = 7000e^{0.06t} \implies P(7) = 7000e^{(0.06)(7)} \\
\implies P(7) \approx 10653.7308893304
\]

33. \( P(t) = P_0e^{-0.00012t} \). Therefore,
\[
0.086P_0 = P_0e^{-0.00012t} \implies 0.086P_0 = P_0e^{-0.00012t} \\
\implies \ln(0.086) = -0.00012t \\
\implies t = \frac{\ln(0.086)}{-0.00012} \approx 20445.0665227386
\]

35. \( P(t) = P_0e^{-0.00012t} \). Therefore,
\[
0.901P_0 = P_0e^{-0.00012t} \implies 0.901P_0 = P_0e^{-0.00012t} \\
\implies \ln(0.901) = -0.00012t \\
\implies t = \frac{\ln(0.901)}{-0.00012} \approx 868.750178114994
\]
37. \( P(t) = P_0 e^{-0.00012t} \). Therefore,
\[
0.062P_0 = P_0 e^{-0.00012t} \implies 0.062P_0 = P_0 e^{-0.00012t} \\
\implies \ln (0.062) = -0.00012t \\
\implies t = \frac{\ln(0.062)}{-0.00012} \approx 23171.8407828087
\]

39. \( P(t) = P_0 e^{-0.00012t} \). Therefore,
\[
0.967P_0 = P_0 e^{-0.00012t} \implies 0.967P_0 = P_0 e^{-0.00012t} \\
\implies \ln (0.967) = -0.00012t \\
\implies t = \frac{\ln(0.967)}{-0.00012} \approx 279.639862740355
8.8 Exercises

In Exercises 1-10, compute the value of the expression. Express your answer in scientific notation $c \cdot 10^n$.

1. $131^{808}$
2. $132^{759}$
3. $148^{524}$
4. $143^{697}$
5. $187^{642}$
6. $198^{693}$
7. $162^{803}$
8. $142^{569}$
9. $134^{550}$
10. $153^{827}$

\footnote{Copyrighted material. See: http://msenux.redwoods.edu/IntAlgText/}
8.8 Solutions

1. 

\[ y = 131^{808} \implies \log(y) = 808 \log(131) \]

\[ \implies \log(y) = 1710.75520688986 \]

\[ \implies y = 10^{1710.75520688986} \]

\[ \implies y = 10^{1710+0.755206889857391} \]

\[ \implies y = 10^{1710}10^{0.755206889857391} \]

\[ \implies y \approx 5.691 \cdot 10^{1710} \]

3. 

\[ y = 148^{524} \implies \log(y) = 524 \log(148) \]

\[ \implies \log(y) = 1137.21713886696 \]

\[ \implies y = 10^{1137.21713886696} \]

\[ \implies y = 10^{1137+0.217138866957384} \]

\[ \implies y = 10^{1137}10^{0.217138866957384} \]

\[ \implies y \approx 1.649 \cdot 10^{1137} \]

5. 

\[ y = 187^{642} \implies \log(y) = 642 \log(187) \]

\[ \implies \log(y) = 1458.52231139643 \]

\[ \implies y = 10^{1458.52231139643} \]

\[ \implies y = 10^{1458+0.522311396432315} \]

\[ \implies y = 10^{1458}10^{0.522311396432315} \]

\[ \implies y \approx 3.329 \cdot 10^{1458} \]
7. 

\[ y = 162^{803} \implies \log(y) = 803 \log(162) \]
\[ \implies \log(y) = 1774.24055667773 \]
\[ \implies y = 10^{1774.24055667773} \]
\[ \implies y = 10^{1774+0.240556677732457} \]
\[ \implies y = 10^{1774.10^{0.240556677732457}} \]
\[ \implies y \approx 1.740 \cdot 10^{1774} \]

9. 

\[ y = 134^{550} \implies \log(y) = 550 \log(134) \]
\[ \implies \log(y) = 1169.90763910064 \]
\[ \implies y = 10^{1169.90763910064} \]
\[ \implies y = 10^{1169+0.907639100644019} \]
\[ \implies y = 10^{1169.10^{0.907639100644019}} \]
\[ \implies y \approx 8.084 \cdot 10^{1169} \]