8.2 Exercises

1. The current population of Fortuna is 10,000 hearty souls. It is known that the population is growing at a rate of 4% per year. Assuming this rate remains constant, perform each of the following tasks.
   a. Set up an equation that models the population \( P(t) \) as a function of time \( t \).
   b. Use the model in the previous part to predict the population 40 years from now.
   c. Use your calculator to sketch the graph of the population over the next 40 years.

2. The population of the town of Imagination currently numbers 12,000 people. It is known that the population is growing at a rate of 6% per year. Assuming this rate remains constant, perform each of the following tasks.
   a. Set up an equation that models the population \( P(t) \) as a function of time \( t \).
   b. Use the model in the previous part to predict the population 30 years from now.
   c. Use your calculator to sketch the graph of the population over the next 30 years.

3. The population of the town of Despairia currently numbers 15,000 individuals. It is known that the population is decaying at a rate of 5% per year. Assuming this rate remains constant, perform each of the following tasks.
   a. Set up an equation that models the population \( P(t) \) as a function of time \( t \).
   b. Use the model in the previous part to predict the population 50 years from now.
   c. Use your calculator to sketch the graph of the population over the next 50 years.

4. The population of the town of Hopeless currently numbers 25,000 individuals. It is known that the population is decaying at a rate of 6% per year. Assuming this rate remains constant, perform each of the following tasks.
   a. Set up an equation that models the population \( P(t) \) as a function of time \( t \).
   b. Use the model in the previous part to predict the population 40 years from now.
   c. Use your calculator to sketch the graph of the population over the next 40 years.

In Exercises 5-12, perform each of the following tasks for the given function.

a. Find the \( y \)-intercept of the graph of the function. Also, use your calculator to find two points on the graph to the right of the \( y \)-axis, and two points to the left.

b. Using your five points from (a) as a guide, set up a coordinate system on graph paper. Choose and label appropriate scales for each axis. Plot the five points, and any additional points you feel are necessary to dis-
carn the shape of the graph.

c. Draw the horizontal asymptote with a dashed line, and label it with its equation.
d. Sketch the graph of the function.
e. Use interval notation to describe both the domain and range of the function.

5. \( f(x) = (2.5)^x \)

6. \( f(x) = (0.1)^x \)

7. \( f(x) = (0.75)^x \)

8. \( f(x) = (1.1)^x \)

9. \( f(x) = 3^x + 1 \)

10. \( f(x) = 4^x - 5 \)

11. \( f(x) = 2^x - 3 \)

12. \( f(x) = 5^x + 2 \)

In Exercises 13-20, the graph of an exponential function of the form \( f(x) = b^x + c \) is shown. The dashed red line is a horizontal asymptote. Determine the range of the function. Express your answer in interval notation.

13. 

14. 

15. 

16. 

17. 

18. 

19. 

20.
In Exercises 21-32, compute \( f(p) \) at the given value \( p \).

21. \( f(x) = (1/3)^x; \ p = -4 \)
22. \( f(x) = (3/4)^x; \ p = 1 \)
23. \( f(x) = 5^x; \ p = 5 \)
24. \( f(x) = (1/3)^x; \ p = 4 \)
25. \( f(x) = 4^x; \ p = -4 \)
26. \( f(x) = 5^x; \ p = -3 \)

27. \( f(x) = (5/2)^x; \ p = -3 \)
28. \( f(x) = 9^x; \ p = 3 \)
29. \( f(x) = 5^x; \ p = -4 \)
30. \( f(x) = 9^x; \ p = 0 \)
31. \( f(x) = (6/5)^x; \ p = -4 \)
32. \( f(x) = (3/5)^x; \ p = 0 \)

In Exercises 33-40, use your calculator to evaluate the function at the given value \( p \). Round your answer to the nearest hundredth.

33. \( f(x) = 10^x; \ p = -0.7 \).
34. \( f(x) = 10^x; \ p = -1.60 \).
35. \( f(x) = (2/5)^x; \ p = 3.67 \).
36. \( f(x) = 2^x; \ p = -3/4 \).
37. \( f(x) = 10^x; \ p = 2.07 \).
38. \( f(x) = 7^x; \ p = 4/3 \).
39. \( f(x) = 10^x; \ p = -1/5 \).
40. \( f(x) = (4/3)^x; \ p = 1.15 \).

41. This exercise explores the property that exponential growth functions eventually increase rapidly as \( x \) increases. Let \( f(x) = 1.05^x \). Use your graphing calculator to graph \( f \) on the intervals 
   (a) \([0, 10]\) and (b) \([0, 100]\).
   For (a), use \( Y_{\text{min}} = 0 \) and \( Y_{\text{max}} = 10 \).
   For (b), use \( Y_{\text{min}} = 0 \) and \( Y_{\text{max}} = 100 \).
   Make accurate copies of the images in your viewing window on your homework paper. What do you observe when you compare the two graphs?
8.2 Answers

1. 
   a) \( P(t) = 10000(1.04)^t \)
   b) \( P(40) \approx 48101 \)
   c) 

5. 
   a) The \( y \)-intercept is \((0, 1)\). Evaluate the function at \( x = 1, 2, -1, -2 \) to obtain the points \((1, 2.5), (2, 6.25), (-1, 0.4), (-2, 0.16)\) (other answers are possible).
   
   b) See the graph in part (d).
   
   c) The horizontal asymptote is \( y = 0 \). See the graph in part (d).
   
   d) 

3. 
   a) \( P(t) = 15000(0.95)^t \)
   b) \( P(50) \approx 1154 \)
   c) 

   e) Domain = \((−\infty, \infty)\), Range = \((0, \infty)\)
7. a) The \( y \)-intercept is \((0, 1)\). Evaluate the function at \( x = 1, 2, -1, -2 \) to obtain the points \((1, 0.75), (2, 0.56), (-1, 1.34), (-2, 1.78)\) (other answers are possible).

b) See the graph in part (d).

c) The horizontal asymptote is \( y = 0 \). See the graph in part (d).

d) [Graph]

e) Domain = \((-\infty, \infty)\), Range = \((0, \infty)\)

9. a) The \( y \)-intercept is \((0, 2)\). Evaluate the function at \( x = 1, 2, -1, -2 \) to obtain the points \((1, 4), (2, 10), (-1, 1.34), (-2, 1.11)\) (other answers are possible).

b) See the graph in part (d).

c) The horizontal asymptote is \( y = 1 \). See the graph in part (d).

d) [Graph]

e) Domain = \((-\infty, \infty)\), Range = \((-3, \infty)\)

11. a) The \( y \)-intercept is \((0, -2)\). Evaluate the function at \( x = 1, 2, -1, -2 \) to obtain the points \((1, -1), (2, 1), (-1, -2.5), (-2, -2.75)\) (other answers are possible).

b) See the graph in part (d).

c) The horizontal asymptote is \( y = -3 \). See the graph in part (d).

d) [Graph]

e) Domain = \((-\infty, \infty)\), Range = \((-3, \infty)\)

13. \((-1, \infty)\)

15. \((2, \infty)\)

17. \((2, \infty)\)
19. \((-2, \infty)\)

21. 81

23. 3125

25. \(\frac{1}{256}\)

27. \(\frac{8}{125}\)

29. \(\frac{1}{625}\)

31. \(\frac{625}{1296}\)

33. 0.20

35. 0.03

37. 117.49

39. 0.63

41.

a) The graph on the interval \([0, 10]\) increases very slowly. In fact, the graph looks almost linear.

b) The graph on the interval \([0, 100]\) increases slowly at first, but then increases very rapidly on the second half of the interval.