Computing Large Powers

Logarithms were originally used to compute large products and powers. Prior to the age of calculators and computers, mathematics students spent many hours learning and practicing these procedures. In current times, most of these computations can be done easily on a calculator, so the original use of logarithms is usually not taught anymore.

However, calculators are still limited. They cannot compute large powers such as $253^{789}$ (try it!), and most computer programs can’t either (all such tools have a limit on the size of the computations they can perform).

So how can we compute large powers such as these? The idea is to use our knowledge of the properties of logarithmic and exponential functions. Here is the procedure:

1. First, let $y = 253^{789}$, and take the log of both sides:
   \[
   \log(y) = \log(253^{789}) \\
   = 789 \log(253) \quad \text{(property of logs)} \\
   \approx 1896.062091 \quad \text{(calculator approximation)}
   \]

2. Now the idea is to exponentiate both sides, using the function $10^x$. However, your calculator still cannot compute $10^{1896.062091}$ (try it). So now we separate out the integer part, and our final answer will be in scientific notation:
   \[
   y = 10^{\log(y)} = 10^{1896.062091} = 10^{1896 + 0.062091} = 10^{1896} \cdot 10^{0.062091} \\
   \approx 10^{1896} \cdot 1.153694972 \quad \text{(calculator approximation)}
   \]

Thus, the final answer is approximately $1.153695 \cdot 10^{1896}$.

Here is one additional example:

**Example 1.** Compute the value $2^{400}$, and express your answer in scientific notation.

1. Let $y = 2^{400}$, and take the log of both sides:
   \[
   \log(y) = \log(2^{400}) \\
   = 400 \log(2) \quad \text{(property of logs)} \\
   \approx 120.4119983 \quad \text{(calculator approximation)}
   \]

2. Exponentiate both sides, using the function $10^x$ and separating out the integer part of the exponent:
\[
y = 10^{\log(y)} = 10^{120.4119983} = 10^{120+0.4119983} = 10^{120} \cdot 10^{0.4119983}
\]
\[
\approx 10^{120} \cdot 2.582250083 \quad \text{(calculator approximation)}
\]

The final answer is approximately \(2.582250 \cdot 10^{120}\).