9.6 Exercises

In Exercises 1-8, state whether or not the given triple is a Pythagorean Triple. Give a reason for your answer.

1. (8, 15, 17)
2. (7, 24, 25)
3. (8, 9, 17)
4. (4, 9, 13)
5. (12, 35, 37)
6. (12, 17, 29)
7. (11, 17, 28)
8. (11, 60, 61)

In Exercises 9-16, set up an equation to model the problem constraints and solve. Use your answer to find the missing side of the given right triangle. Include a sketch with your solution and check your result.

9.

10.

11.

12.

13.

1 Copyrighted material. See: http://msenux.redwoods.edu/IntAlgText/
14. 

\[ \sqrt{3} \]

15. 

\[ \sqrt{2} \]

16. 

\[ \sqrt{2} \]

In Exercises 17-20, set up an equation that models the problem constraints. Solve the equation and use the result to answer the question. Look back and check your result.

17. The legs of a right triangle are consecutive positive integers. The hypotenuse has length 5. What are the lengths of the legs?

18. The legs of a right triangle are consecutive even integers. The hypotenuse has length 10. What are the lengths of the legs?

19. One leg of a right triangle is 1 centimeter less than twice the length of the first leg. If the length of the hypotenuse is 17 centimeters, find the lengths of the legs.

20. One leg of a right triangle is 3 feet longer than 3 times the length of the first leg. The length of the hypotenuse is 25 feet. Find the lengths of the legs.

21. Pythagoras is credited with the following formulae that can be used to generate Pythagorean Triples.

\[
\begin{align*}
a &= m \\
b &= \frac{m^2 - 1}{2} \\
c &= \frac{m^2 + 1}{2}
\end{align*}
\]

Use the technique of Example 6 to demonstrate that the formulae given above will generate Pythagorean Triples, provided that \( m \) is an odd positive integer larger than one. Secondly, generate at least 3 instances of Pythagorean Triples with Pythagoras’s formula.

22. Plato (380 BC) is credited with the following formulae that can be used to generate Pythagorean Triples.

\[
\begin{align*}
a &= 2m \\
b &= m^2 - 1 \\
c &= m^2 + 1
\end{align*}
\]

Use the technique of Example 6 to demonstrate that the formulae given above will generate Pythagorean Triples, provided that \( m \) is a positive integer larger than 1. Secondly, generate at least 3 instances of Pythagorean Triples with Plato’s formula.
In Exercises 23-28, set up an equation that models the problem constraints. Solve the equation and use the result to answer the question. Look back and check your result.

23. Fritz and Greta are planting a 12-foot by 18-foot rectangular garden, and are laying it out using string. They would like to know the length of a diagonal to make sure that right angles are formed. Find the length of a diagonal. Approximate your answer to within 0.1 feet.

24. Angelina and Markos are planting a 20-foot by 28-foot rectangular garden, and are laying it out using string. They would like to know the length of a diagonal to make sure that right angles are formed. Find the length of a diagonal. Approximate your answer to within 0.1 feet.

25. The base of a 36-foot long guy wire is located 16 feet from the base of the telephone pole that it is anchoring. How high up the pole does the guy wire reach? Approximate your answer to within 0.1 feet.

26. The base of a 35-foot long guy wire is located 10 feet from the base of the telephone pole that it is anchoring. How high up the pole does the guy wire reach? Approximate your answer to within 0.1 feet.

27. A stereo receiver is in a corner of a 13-foot by 16-foot rectangular room. Speaker wire will run under a rug, diagonally, to a speaker in the far corner. If 3 feet of slack is required on each end, how long a piece of wire should be purchased? Approximate your answer to within 0.1 feet.

28. A stereo receiver is in a corner of a 10-foot by 15-foot rectangular room. Speaker wire will run under a rug, diagonally, to a speaker in the far corner. If 4 feet of slack is required on each end, how long a piece of wire should be purchased? Approximate your answer to within 0.1 feet.

In Exercises 29-38, use the distance formula to find the exact distance between the given points.

29. \((-8, -9)\) and \((6, -6)\)

30. \((1, 0)\) and \((-9, -2)\)

31. \((-9, 1)\) and \((-8, 7)\)

32. \((0, 9)\) and \((3, 1)\)

33. \((6, -5)\) and \((-9, -2)\)

34. \((-9, 6)\) and \((1, 4)\)

35. \((-7, 7)\) and \((-3, 6)\)

36. \((-7, -6)\) and \((-2, -4)\)

37. \((4, -3)\) and \((-9, 6)\)

38. \((-7, -1)\) and \((4, -5)\)

In Exercises 39-42, set up an equation that models the problem constraints. Solve the equation and use the result to answer the question. Look back and check your result.

39. Find \(k\) so that the point \((4, k)\) is \(2\sqrt{2}\) units away from the point \((2, 1)\).

40. Find \(k\) so that the point \((k, 1)\) is \(2\sqrt{2}\) units away from the point \((0, -1)\).
41. Find \( k \) so that the point \((k, 1)\) is \(\sqrt{17}\) units away from the point \((2, -3)\).

42. Find \( k \) so that the point \((-1, k)\) is \(\sqrt{13}\) units away from the point \((-4, -3)\).

43. Set up a coordinate system on a sheet of graph paper. Label and scale each axis. Plot the points \(P(0, 5)\) and \(Q(4, -3)\) on your coordinate system.

a) Plot several points that are equidistant from the points \(P\) and \(Q\) on your coordinate system. What graph do you get if you plot all points that are equidistant from the points \(P\) and \(Q\)? Determine the equation of the graph by examining the resulting image on your coordinate system.

b) Use the distance formula to find the equation of the graph of all points that are equidistant from the points \(P\) and \(Q\). Hint: Let \((x, y)\) represent an arbitrary point on the graph of all points equidistant from points \(P\) and \(Q\). Calculate the distances from the point \((x, y)\) to the points \(P\) and \(Q\) separately, then set them equal and simplify the resulting equation. Note that this analytical approach should provide an equation that matches that found by the graphical approach in part (a).

44. Set up a coordinate system on a sheet of graph paper. Label and scale each axis. Plot the point \(P(0, 2)\) and label it with its coordinates. Draw the line \(y = -2\) and label it with its equation.

a) Plot several points that are equidistant from the point \(P\) and the line \(y = -2\) on your coordinate system. What graph do you get if you plot all points that are equidistant from the points \(P\) and the line \(y = -2\).

b) Use the distance formula to find the equation of the graph of all points that are equidistant from the points \(P\) and the line \(y = -2\). Hint: Let \((x, y)\) represent an arbitrary point on the graph of all points equidistant from points \(P\) and the line \(y = -2\). Calculate the distances from the point \((x, y)\) to the points \(P\) and the line \(y = -2\) separately, then set them equal and simplify the resulting equation.
45. Copy the following figure onto a sheet of graph paper. Cut the pieces of the first figure out with a pair of scissors, then rearrange them to form the second figure. Explain how this proves the Pythagorean Theorem.

46. Compare this image to the one that follows and explain how this proves the Pythagorean Theorem.
9.6 Answers

1. Yes, because $8^2 + 15^2 = 17^2$

3. No, because $8^2 + 9^2 \neq 17^2$

5. Yes, because $12^2 + 35^2 = 37^2$

7. No, because $11^2 + 17^2 \neq 28^2$

9. 4

11. $4\sqrt{3}$

13. $2\sqrt{2}$

15. $5\sqrt{3}$

17. The legs have lengths 3 and 4.

19. The legs have lengths 8 and 15 centimeters.

21. (3, 4, 5), (5, 12, 13), and (7, 24, 25), with $m = 3, 5,$ and 7, respectively.

23. 21.63 ft

25. 32.25 ft

27. 26.62 ft

29. $\sqrt{205}$

31. $\sqrt{37}$

33. $\sqrt{234} = 3\sqrt{26}$

35. $\sqrt{17}$

37. $\sqrt{250} = 5\sqrt{10}$

39. $k = 3, -1.$

41. $k = 1, 3.$

43. 

a) In the figure that follows, $XP = XQ.$

b) $y = (1/2)x$